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Section II
Superstructure Design

9
Reinforced Concrete Bridges

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9.1 Introduction

The raw materials of concrete, consisting of water, fine aggregate, coarse aggregate, and cement, can be found in most areas of the world and can be mixed to form a variety of structural shapes. The great availability and flexibility of concrete material and reinforcing bars have made the reinforced concrete bridge a very competitive alternative. Reinforced concrete bridges may consist of precast concrete elements, which are fabricated at a production plant and then transported for erection at the job site, or cast-in-place concrete, which is formed and cast directly in its setting location. Cast-in-place concrete structures are often constructed monolithically and continuously. They usually provide a relatively low maintenance cost and better earthquake-resistance performance. Cast-in-place concrete structures, however, may not be a good choice when the project is on a fast-track construction schedule or when the available falsework opening clearance is limited. In this chapter, various structural types and design considerations for conventional cast-in-place, reinforced concrete highway bridge are discussed. Two design examples of a simply supported slab bridge and a two-span box girder bridge are also presented. All design specifications referenced in this chapter are based on 1994 AASHTO LRFD (Load and Resistance Factor Design) Bridge Design Specifications [1].
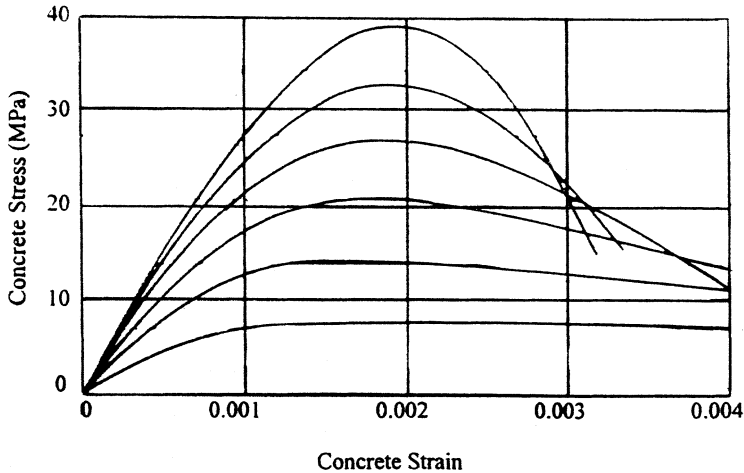
FIGURE 9.1 Typical stress–strain curves for concrete under uniaxial compression loading.

9.2   Materials

9.2.1   Concrete

1 Compressive Strength

The compressive strength of concrete (f'_c) at 28 days after placement is usually obtained from a standard 150-mm-diameter by 300-mm-high cylinder loaded longitudinally to failure. Figure 9.1 shows typical stress–strain curves from unconfined concrete cylinders under uniaxial compression loading. The strain at the peak compression stress f'_c is approximately 0.002 and maximum usable strain is about 0.003. The concrete modulus of elasticity, E_c, may be calculated as

$$E_c = 0.043\gamma_c^{1.5}\sqrt{f'_c}\ \text{MPa} \tag{9.1}$$

where γ_c is the density of concrete (kg/m³) and f'_c is the specified strength of concrete (MPa). For normal-weight concrete (γ_c = 2300 kg/m³), E_c may be calculated as $4800\sqrt{f'_c}$ MPa.

The concrete compressive strength or class of concrete should be specified in the contract documents for each bridge component. A typical specification for different classes of concrete and their corresponding specified compressive strengths is shown in Table 9.1. These classes are intended for use as follows:

- Class A concrete is generally used for all elements of structures and specially for concrete exposed to salt water.
- Class B concrete is used in footings, pedestals, massive pier shafts, and gravity walls.
- Class C concrete is used in thin sections under 100 mm in thickness, such as reinforced railings and for filler in steel grid floors.
- Class P concrete is used when strengths exceeding 28 MPa are required.
- Class S concrete is used for concrete deposited under water in cofferdams to seal out water.

Both concrete compressive strengths and water–cement ratios are specified in Table 9.1 for different concrete classes. This is because the water–cement ratio is a dominant factor contributing to both durability and strength, while simply obtaining the required concrete compressive strength to satisfy the design assumptions may not ensure adequate durability.

TABLE 9.1   Concrete Mix Characteristics by Class[1]

Class of Concrete	Minimum Cement Content (kg/m³)	Maximum Water–Cement Ratio (kg/kg)	Air Content Range, %	Coarse Aggregate per AASHTO M43 (square size of openings, mm)	28-day Compressive Strength, f_c' MPa
A	362	0.49	—	25 to 4.75	28
A(AE)	362	0.45	6.0 ± 1.5	25 to 4.75	28
B	307	0.58	—	50 to 4.75	17
B(AE)	307	0.55	5.0 ± 1.5	50 to 4.75	17
C	390	0.49	—	12.5 to 4.75	28
C(AE)	390	0.45	7.0 ± 1.5	12.5 to 4.75	28
P	334	0.49	As specified elsewhere	25 to 4.75 or 19 to 4.75	As specified elsewhere
S	390	0.58	—	25 to 4.75	—
Low-density	334	As specified in the contract documents			

Notes:
1. AASHTO Table C5.4.2.1-1 (From AASHTO LRFD Bridge Design Specifications, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)
2. Concrete strengths above 70 MPa need to have laboratory testing verification. Concrete strengths below 16 MPa should not be used.
3. The sum of portland cement and other cementitious materials should not exceed 475 kg/m3.
4. Air-entrained concrete (AE) can improve durability when subjected to freeze–thaw action and to scaling caused by chemicals applied for snow and ice removal.

2. Tensile Strength

The tensile strength of concrete can be measured directly from tension loading. However, fixtures for holding the specimens are difficult to apply uniform axial tension loading and sometimes will even introduce unwanted secondary stresses. The direct tension test method is therefore usually used to determine the cracking strength of concrete caused by effects other than flexure. For most regular concrete, the direct tensile strength may be estimated as 10% of the compressive strength.

The tensile strength of concrete may be obtained indirectly by the split tensile strength method. The splitting tensile stress (f_s) at which a cylinder is placed horizontally in a testing machine and loaded along a diameter until split failure can be calculated as

$$f_s = 2P/(\pi LD) \qquad (9.2)$$

where P is the total applied load that splits the cylinder, L is the length of cylinder, and D is the diameter of the cylinder.

The tensile strength of concrete can also be evaluated by means of bending tests conducted on plain concrete beams. The flexural tensile stress, known as the modulus of rupture (f_r) is computed from the flexural formula M/S, where M is the applied failure bending moment and S is the elastic section modulus of the beam. Modulus of rupture (f_r) in MPa can be calculated as

$$f_r = \begin{cases} 0.63\sqrt{f_c'} & \text{for normal-weight concrete} \\ 0.52\sqrt{f_c'} & \text{for sand–low-density concrete} \\ 0.45\sqrt{f_c'} & \text{for all–low-density concrete} \end{cases} \qquad (9.3)$$

TABLE 9.2 Steel Deformed Bar Sizes and Weight (ASTM A615M and A706M)

Bar Number	Nominal Dimensions		
	Diameter, mm	Area, mm^2	Unit Weight, kg/m
10	9.5	71	0.560
13	12.7	129	0.994
16	15.9	199	1.552
19	19.1	284	2.235
22	22.2	387	3.042
25	25.4	510	3.973
29	28.7	645	5.060
32	32.3	819	6.404
36	35.8	1006	7.907
43	43.0	1452	11.38
57	57.3	2581	20.24

Both the splitting tensile stress (f_s) and flexural tensile stress (f_r) overestimate the tensile cracking stress determined by a direct tension test. However, concrete in tension is usually ignored in strength calculations of reinforced concrete members because the tensile strength of concrete is low. The modulus of elasticity for concrete in tension may be assumed to be the same as in compression.

3. Creep and Shrinkage

Both creep and shrinkage of concrete are time-dependent deformations and are discussed in Chapter 10.

9.2.2   Steel Reinforcement

Deformed steel bars are commonly employed as reinforcement in most reinforced concrete bridge construction. The surface of a steel bar is rolled with lugs or protrusions called deformations in order to restrict longitudinal movement between the bars and the surrounding concrete. Reinforcing bars, rolled according to ASTM A615/A615M specifications (billet steel) [2], are widely used in construction. ASTM A706/A706M low-alloy steel deformed bars (Grade 420 only) [2] are specified for special applications where extensive welding of reinforcement or controlled ductility for earthquake-resistant, reinforced concrete structures or both are of importance.

1. Bar Shape and Size

Deformed steel bars are approximately numbered based on the amount of millimeters of the nominal diameter of the bar. The nominal dimensions of a deformed bar are equivalent to those of a plain round bar which has the same mass per meter as the deformed bar. Table 9.2 lists a range of deformed bar sizes according to the ASTM specifications.

2. Stress–Strain Curve

The behavior of steel reinforcement is usually characterized by the stress–strain curve under uniaxial tension loading. Typical stress–strain curves for steel Grade 300 and 420 are shown in Figure 9.2. The curves exhibit an initial linear elastic portion with a slope calculated as the modulus of elasticity of steel reinforcement E_s = 200,000 MPa; a yield plateau in which the strain increases (from ε_y to ε_h) with little or no increase in yield stress (f_y); a strain-hardening range in which stress again increases with strain until the maximum stress (f_u) at a strain (ε_u) is reached; and finally a range in which the stress drops off until fracture occurs at a breaking strain of ε_b.
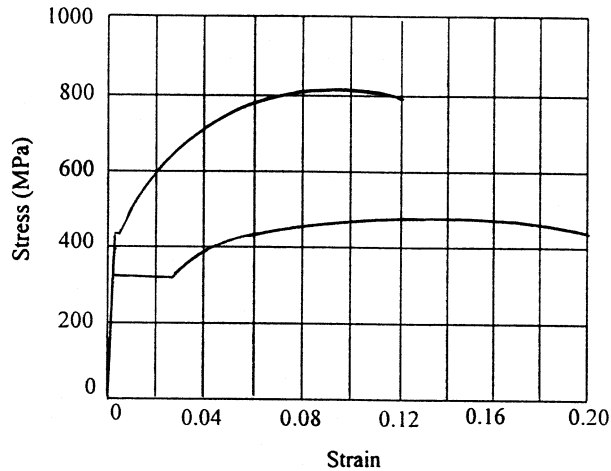
FIGURE 9.2 Typical stress–strain curves for steel reinforcement.
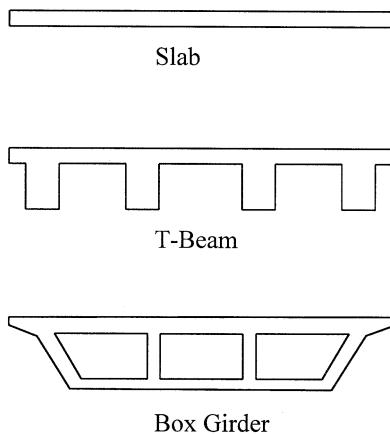

Slab

T-Beam

Box Girder

FIGURE 9.3 Typical reinforced concrete sections in bridge superstructures.

9.3 Bridge Types

Reinforced concrete sections, used in the bridge superstructures, usually consist of slabs, T-beams (deck girders), and box girders (Figure 9.3). Safety, cost-effectiveness, and aesthetics are generally the controlling factors in the selection of the proper type of bridges [3]. Occasionally, the selection is complicated by other considerations such as the deflection limit, life-cycle cost, traffic maintenance during construction stages, construction scheduling and worker safety, feasibility of falsework layout, passage of flood debris, seismicity at the site, suitability for future widening, and commitments made to officials and individuals of the community. In some cases, a prestressed concrete or steel bridge may be a better choice.

9.3.1 Slab Bridges

Longitudinally reinforced slab bridges have the simplest superstructure configuration and the neatest appearance. They generally require more reinforcing steel and structural concrete than do girder-type

bridges of the same span. However, the design details and formworks are easier and less expensive. It has been found economical for simply supported spans up to 9 m and for continuous spans up to 12 m.

9.3.2 T-Beam Bridges

The T-beam construction consists of a transversely reinforced slab deck which spans across to the longitudinal support girders. These require a more-complicated formwork, particularly for skewed bridges, compared to the other superstructure forms. T-beam bridges are generally more economical for spans of 12 to 18 m. The girder stem thickness usually varies from 35 to 55 cm and is controlled by the required horizontal spacing of the positive moment reinforcement. Optimum lateral spacing of longitudinal girders is typically between 1.8 and 3.0 m for a minimum cost of formwork and structural materials. However, where vertical supports for the formwork are difficult and expensive, girder spacing can be increased accordingly.

9.3.3 Box-Girder Bridges

Box-girder bridges contain top deck, vertical web, and bottom slab and are often used for spans of 15 to 36 m with girders spaced at 1.5 times the structure depth. Beyond this range, it is probably more economical to consider a different type of bridge, such as post-tensioned box girder or steel girder superstructure. This is because of the massive increase in volume and materials. They can be viewed as T-beam structures for both positive and negative moments. The high torsional strength of the box girder makes it particularly suitable for sharp curve alignment, skewed piers and abutments, superelevation, and transitions such as interchange ramp structures.

9.4 Design Considerations

9.4.1 Basic Design Theory

The AASHTO LRFD Specifications (1994) [1] were developed in a reliability-based limit state design format. Limit state is defined as the limiting condition of acceptable performance for which the bridge or component was designed. In order to achieve the objective for a safe design, each bridge member and connection is required to examine some, or all, of the service, fatigue, strength, and extreme event limit states. All applicable limit states shall be considered of equal importance. The basic requirement for bridge design in the LRFD format for each limit state is as follows:

$$\eta \; \Sigma \gamma_i \; Q_i \leq \phi \; R_n \tag{9.4}$$

where η = load modifier to account for bridge ductility, redundancy, and operational importance, γ_i = load factor for load component i, Q_i = nominal force effect for load component i, ϕ = resistance factor, and R_n = nominal resistance. The margin of safety for a bridge design is provided by ensuring the bridge has sufficient capacity to resist various loading combinations in different limit states.

The load factors, γ, which often have values larger than one, account for the loading uncertainties and their probabilities of occurrence during bridges design life. The resistance factors, ϕ, which are typically less than unity at the strength limit state and equal to unity for all other limit states, account for material variabilities and model uncertainties. Table 9.3 lists the resistance factors in the strength limit state for conventional concrete construction. The load modifiers, η, which are equal to unity for all non-strength-limit states, account for structure ductility, redundancy, and operational importance. They are related to the bridge physical strength and the effects of a bridge being out of service. Detailed load resistance factor design theory and philosophy are discussed in Chapter 5.

TABLE 9.3 Resistance Factors φ in the Strength Limit State for Conventional Construction

Strength Limit State	Resistance Factors φ
For flexural and tension of reinforced concrete	0.90
For shear and torsion	
Normal weight concrete	0.90
Lightweight concrete	0.70
For axial compression with spirals and ties	0.75
(except for Seismic Zones 3 and 4 at the extreme event limit state)	
For bearing on concrete	0.79
For compression in strut-and-tie models	0.70

Notes:
1. AASHTO 5.5.4.2.1 (From AASHTO LRFD Bridge Design Specifications, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)
2. For compression members with flexural, the value of φ may be increased linearly to the value for flexural as the factored axial load resistance, ϕP_n, decreases from 0.10 $f_c' A_g$ to 0.

9.4.2 Design Limit States

1. Service Limit States

For concrete structures, service limit states correspond to the restrictions on cracking width and deformations under service conditions. They are intended to ensure that the bridge will behave and perform acceptably during its service life.

a. Control of Cracking

Cracking may occur in the tension zone for reinforced concrete members due to the low tensile strength of concrete. Such cracks may occur perpendicular to the axis of the members under axial tension or flexural bending loading without significant shear force, or inclined to the axis of the members with significant shear force. The cracks can be controlled by distributing steel reinforcements over the maximum tension zone in order to limit the maximum allowable crack widths at the surface of the concrete for given types of environment. The tensile stress in the steel reinforcement (f_s) at the service limit state should not exceed

$$f_{sa} = \frac{Z}{(d_c A)^{1/3}} \leq 0.6 f_y \tag{9.5}$$

where d_c (mm) is the concrete cover measured from extreme tension fiber to the center of the closest bars and should not to be taken greater than 50 mm; A (mm²) is the concrete area having the same centroid as the principal tensile reinforcement divided by the number of bars; Z (N/mm) should not exceed 30,000 for members in moderate exposure conditions, 23,000 in severe exposure conditions, and 17,500 for buried structures. Several smaller tension bars at moderate spacing can provide more effective crack control by increasing f_{sa} rather than installing a few larger bars of equivalent area.

When flanges of reinforced concrete T-beams and box girders are in tension, the flexural tension reinforcement should be distributed over the lesser of the effective flange width or a width equal to ¹⁄₁₀ of the span in order to avoid the wide spacing of the bars. If the effective flange width exceeds ¹⁄₁₀ of the span length, additional longitudinal reinforcement, with an area not less than 0.4% of the excess slab area, should be provided in the outer portions of the flange.

For flexural members with web depth exceeding 900 mm, longitudinal skin reinforcements should be uniformly distributed along both side faces for a height of $d/2$ nearest the flexural tension reinforcement for controlling cracking in the web. Without such auxiliary steel, the width of the

TABLE 9.4 Traditional Minimum Depths for Constant Depth Superstructures

| Bridge Types | Minimum Depth (Including Deck) | |
	Simple Spans	Continuous Spans
Slabs	$\dfrac{1.2\,(S+3000)}{30}$	$\dfrac{(S+3000)}{30} \geq 165\ \text{mm}$
T-beams	$0.070L$	$0.065L$
Box beams	$0.060L$	$0.055L$
Pedestrian structure beams	$0.035L$	$0.033L$

Notes:

1. AASHTO Table 2.5.2.6.3-1 (From AASHTO LRFD Bridge Design Specifications, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)
2. S (mm) is the slab span length and L (mm) is the span length.
3. When variable-depth members are used, values may be adjusted to account for change in relative stiffness of positive and negative moment sections.

cracks in the web may greatly exceed the crack widths at the level of the flexural tension reinforcement. The area of skin reinforcement (A_{sk}) in mm²/mm of height on each side face should satisfy

$$A_{sk} \geq 0.001\ (d_e - 760) \leq \frac{A_s}{1200} \tag{9.6}$$

where d_e (mm) is the flexural depth from extreme compression fiber to the centroid of the tensile reinforcement and A_s (mm²) is the area of tensile reinforcement and prestressing steel. The maximum spacing of the skin reinforcement shall not exceed $d/6$ or 300 mm.

b. Control of Deformations

Service-load deformations in bridge elements need to be limited to avoid the structural behavior which differs from the assumed design conditions and to ease the psychological effects on motorists. Service-load deformations may not be a potential source of collapse mechanisms but usually cause some undesirable effects, such as the deterioration of wearing surfaces and local cracking in concrete slab which could impair serviceability and durability. AASHTO LRFD [1] provides two alternative criteria for controlling the deflections:

Limiting Computed Deflections (*AASHTO 2.5.2.6.2*):

Vehicular load, general	Span length/800
Vehicular and/or pedestrian loads	Span length/1000
Vehicular load on cantilever arms	Span length/300
Vehicular and/or pedestrian loads on cantilever arms	Span length/1000

Limiting Span-to-Depth Ratios (*AASHTO 2.5.2.6.3*): For superstructures with constant depth, Table 9.4 shows the typical minimum depth recommendation for a given span length.

Deflections of bridges can be estimated in two steps: (1) instantaneous deflections which occur at the first loading and (2) long-time deflections which occur with time due to the creep and shrinkage of the concrete.

Instantaneous deflections may be computed by using the elastic theory equations. The modulus of elasticity for concrete can be calculated from Eq. (9.1). The moment of inertia of a section can be taken as either the uncracked gross moment of inertia (I_g) for uncracked elements or the effective moment of inertia (I_e) for cracked elements. The effective moment of inertia can be calculated as

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} \leq I_g \tag{9.7}$$

and

$$M_{cr} = f_r \frac{I_g}{y_t} \tag{9.8}$$

where M_{cr} is the moment at first cracking, f_r is the modulus of rupture, y_t is the distance from the neutral axis to the extreme tension fiber, I_{cr} is the moment of inertia of the cracked section transformed to concrete (see Section 9.4.6), and M_a is the maximum moment in a component at the stage for which deformation is computed. For prismatic members, the effective moment of inertia may be calculated at midspan for simple or continuous bridges and at support for cantilevers. For continuous nonprismatic members, the moment of inertia may be calculated as the average of the critical positive and negative moment sections.

Long-time deflections may be calculated as the instantaneous deflection multiplied by the following:

If the instantaneous deflection is based on I_g: 4.0
If the instantaneous deflection is based on I_e: $3.0 - 1.2\left(A_s' / A_s\right) \geq 1.6$

where A_s' is area of compression reinforcement and A_s is the area of tension reinforcement.

2. Fatigue Limit States

Fatigue limit states are used to limit stress in steel reinforcements to control concrete crack growth under repetitive truck loading in order to prevent early fracture failure before the design service life of a bridge. Fatigue loading consists of one design truck with a constant spacing of 9000 mm between the 145-kN axles. Fatigue is considered at regions where compressive stress due to permanent loads is less than two times the maximum tensile live-load stress resulting from the fatigue-load combination. Allowable fatigue stress range in straight reinforcement is limited to

$$f_f = 145 - 0.33 f_{min} + 55\left(\frac{r}{h}\right) \tag{9.9}$$

where f_{min} (MPa) is the minimum stress in reinforcement from fatigue loading (positive for tension and negative for compression stress) and r/h is the ratio of the base radius to the height of rolled-on transverse deformations (0.3 may be used if the actual value in not known).

The cracked section properties should be used for fatigue. Gross section properties may be used when the sum of stresses, due to unfactored permanent loads, plus 1.5 times the fatigue load is not to exceed the tensile stress of $0.25\sqrt{f_c'}$.

3. Strength Limit States and Extreme Event Limit States

For reinforced concrete structures, strength and extreme event limit states are used to ensure that strength and stability are provided to resist specified statistically significant load combinations. A detailed discussion for these limit states is covered in Chapter 5.

9.4.3 Flexural Strength

Figure 9.4 shows a doubly reinforced concrete beam when flexural strength is reached and the depth of neutral axis falls outside the compression flange ($c > h_f$). Assume that both tension and compression steel are yielding and the concrete compression stress block is in a rectangular shape. ε_{cu} is the maximum strain at the extreme concrete compression fiber and is about 0.003 for unconfined concrete.

Concrete compression force in the web;

$$C_w = 0.85\, f_c'\, ab_w = 0.85\beta_1\, f_c'\, cb_w \tag{9.10}$$
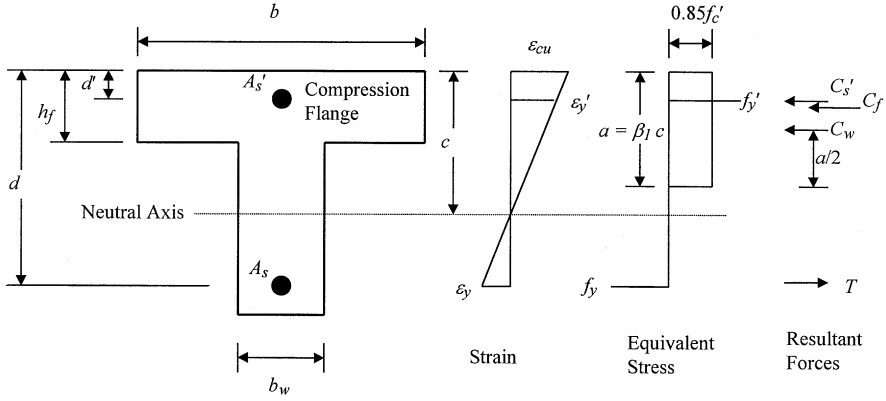
FIGURE 9.4 Reinforced concrete beam when flexural strength is reached.

where

$$a = c \, \beta_1 \tag{9.11}$$

Concrete compression force in the flange:

$$C_f = 0.85\beta_1 \, f_c' \, (b - b_w)h_f \tag{9.12}$$

Compression force in the steel:

$$C_s' = A_s' f_y' \tag{9.13}$$

Tension force in the steel:

$$T = A_s f_y \tag{9.14}$$

From the equilibrium of the forces in the beam, we have

$$C_w + C_f + C_s' = T \tag{9.15}$$

The depth of the neutral axis can be solved as

$$c = \frac{A_s f_y - A_s' f_y' - 0.85\beta_1 f_c' (b - b_w) h_f}{0.85\beta_1 f_c' b_w} \geq h_f \tag{9.16}$$

The nominal flexural strength is

$$M_n = A_s f_y \left(d - \frac{a}{2} \right) + A_s' f_y' \left(\frac{a}{2} - d' \right) + 0.85\beta_1 f_c' (b - b_w) h_f \left(\frac{a}{2} - \frac{h_f}{2} \right) \tag{9.17}$$

where A_s is the area of tension steel, A_s' is the area of compression steel, b is the width of the effective flange, b_w is the width of the web, d is the distance between the centroid of tension steel and the most compressed concrete fiber, d' is the distance between the centroid of compression steel and the most compressed concrete fiber, and h_f is the thickness of the effective flange. The concrete stress factor, β_1 can be calculated as

$$\beta_1 = \begin{cases} 0.85 & \text{for } f_c' \le 28 \text{ MPa} \\ 0.85 - 0.05\left(\dfrac{f_c' - 28}{7}\right) & \text{for } 28 \text{ MPa} \le f_c' \le 56 \text{ MPa} \\ 0.65 & \text{for } f_c' \ge 56 \text{ MPa} \end{cases} \qquad (9.18)$$

Limits for reinforcement are

- Maximum tensile reinforcement:

$$\frac{c}{d} \le 0.42 \qquad (9.19)$$

When Eq. (9.19) is not satisfied, the reinforced concrete sections become overreinforced and will have sudden brittle compression failure if they are not well confined.

- Minimum tensile reinforcement:

$$\rho_{\min} \ge 0.03 \frac{f_c'}{f_y'}, \quad \text{where } \rho_{\min} = \text{ratio of tension steel to gross area} \qquad (9.20)$$

When Eq. (9.20) is not satisfied, the reinforced concrete sections become underreinforced and will have sudden tension steel fracture failure.

The strain diagram can be used to verify compression steel yielding assumption.

$$f_s' = f_y' \quad \text{if} \quad \varepsilon_s' = \varepsilon_{cu}\left(\frac{c - d'}{c}\right) \ge \frac{f_y'}{E_s} \qquad (9.21)$$

If compression steel is not yielding as checked from Eqs. (9.21). The depth of neutral axis, c, and value of nominal flexural strength, M_n, calculated from Eqs. (9.16) and (9.17) are incorrect. The actual forces applied in compression steel reinforcement can be calculated as

$$C_s' = A_s' f_s' = A_s \varepsilon_s' E_s' = A_s \varepsilon_{cu}\left(\frac{d - c}{c}\right) E_s' \qquad (9.22)$$

The depth of neutral axis, c, can be solved by substituting Eqs. (9.22) into forces equilibrium Eq. (9.15). The flexural strength, M_n, can then be obtained from Eq. (9.17) with the actual applied compression steel forces. In a typical beam design, the tension steel will always be yielding and the compression steel is close to reaching yielding strength as well.

If the depth of the neutral axis falls within the compression flange ($x \le h_f$) or for sections without compression flange, then the depth of the neutral axis, c, and the value of nominal flexural strength, M_n, can be calculated by setting b_w equal to b.

9.4.4 Shear Strength

1. Strut-and-Tie Model

The strut-and-tie model should be used for shear and torsion designs of bridge components at locations near discontinuities, such as regions adjacent to abrupt changes in the cross section, openings, and dapped ends. The model should also be used for designing deep footings and pile
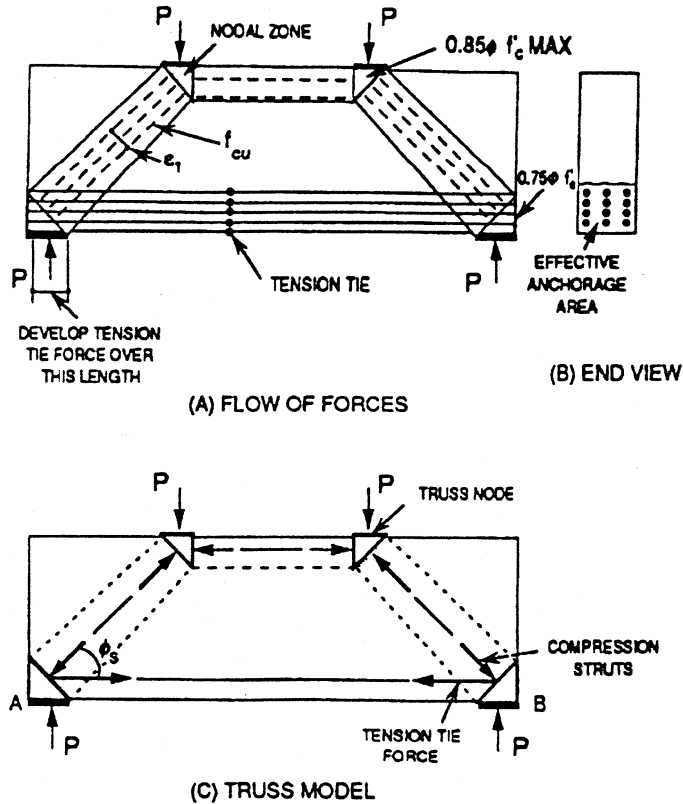
FIGURE 9.5 Strut-and-tie model for a deep beam. (*Source*: AASHTO LRFD Bridge Design Specifications, Figure 5.6.3.2-1, © 1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)

caps or in other situations where the distance between the centers of the applied load and the supporting reactions is less than about twice the member thickness. Figure 9.5 shows a strut-and-tie model for a deep beam that is composed of steel tension ties and concrete compressive struts. These are interconnected at nodes to form a truss capable of carrying all applied loads to the supports.

2. Sectional Design Model

The sectional design model can be used for the shear and torsion design for regions of bridge members where plane sections remain plane after loading. It was developed by Collins and Mitchell [4] and is based on the modified compression field theory. The general shear design procedure for reinforced concrete members, containing transverse web reinforcement, is as follows:

- Calculate the effective shear depth d_v:
  Effective shear depth is calculated between the resultants of the tensile and compressive forces due to flexure. This should not be less than the greater of $0.9d_e$ or $0.72h$, where d_e is the effective depth from extreme compression fiber to the centroid of the tensile reinforcement and h is the overall depth of a member.

- Calculate shear stress:

$$v = \frac{V_u}{\phi b_v d_v} \tag{9.23}$$

where b_v is the equivalent web width and V_u is the factored shear demand envelope from the strength limit state.

- Calculate v/f_c', if this ratio is greater than 0.25, then a larger web section needs to be used.
- Assume an angle of inclination of the diagonal compressive stresses, θ, and calculate the strain in the flexural tension reinforcement:

$$\varepsilon_x = \frac{\dfrac{M_u}{d_v} + 0.5 V_u \cot\theta}{E_s A_s} \tag{9.24}$$

where M_u is the factored moment demand. It is conservative to take M_u enveloped from the strength limit state that will occur at that section, rather than a moment coincident with V_u.

- Use the calculated v/f_c' and ε_x to find θ from Figure 9.6 and compare it with the value assumed. Repeat the above procedure until the assumed θ is reasonably close to the value found from Figure 9.6. Then record the value of β, a factor which indicates the ability of diagonally cracked concrete to transmit tension.
- Calculate the required transverse web reinforcement strength, V_s:

$$V_s = \frac{V_u}{\phi} - V_c = \frac{V_u}{\phi} - 0.083\beta\sqrt{f_c'}\,b_v d_v \tag{9.25}$$

where V_c is the nominal concrete shear resistance.

- Calculate the required spacing for the transverse web reinforcement:

$$s \le \frac{A_v f_y d_v \cot\theta}{V_s} \tag{9.26}$$

where A_v is the area of a transverse web reinforcement within distance s.

Check for the minimum transverse web reinforcement requirement:

$$A_v \ge 0.083\sqrt{f_c'}\,\frac{b_v S}{f_y} \quad \text{or} \quad s \le \frac{A_v f_y}{0.083\sqrt{f_c'}\,b_v} \tag{9.27}$$

Check for the maximum spacing requirement for transverse web reinforcements:

$$\text{if } V_u < 0.1\,f_c'\,b_v d_v, \quad \text{then } s \le 0.8 d_v \le 600 \text{ mm} \tag{9.28}$$

$$\text{if } V_u \ge 0.1\,f_c'\,b_v d_v, \quad \text{then } s \le 0.4 d_v \le 300 \text{ mm} \tag{9.29}$$

- Check the adequacy of the longitudinal reinforcements to avoid yielding due to the combined loading of moment, axial load, and shear.

$$A_s f_y \ge \frac{M_u}{d_v \phi} + \left(\frac{V_u}{\phi} - 0.5\,V_s\right)\cot\theta \tag{9.30}$$

If the above equation is not satisfied, then you need either to add more longitudinal reinforcement or to increase the amount of transverse web reinforcement.
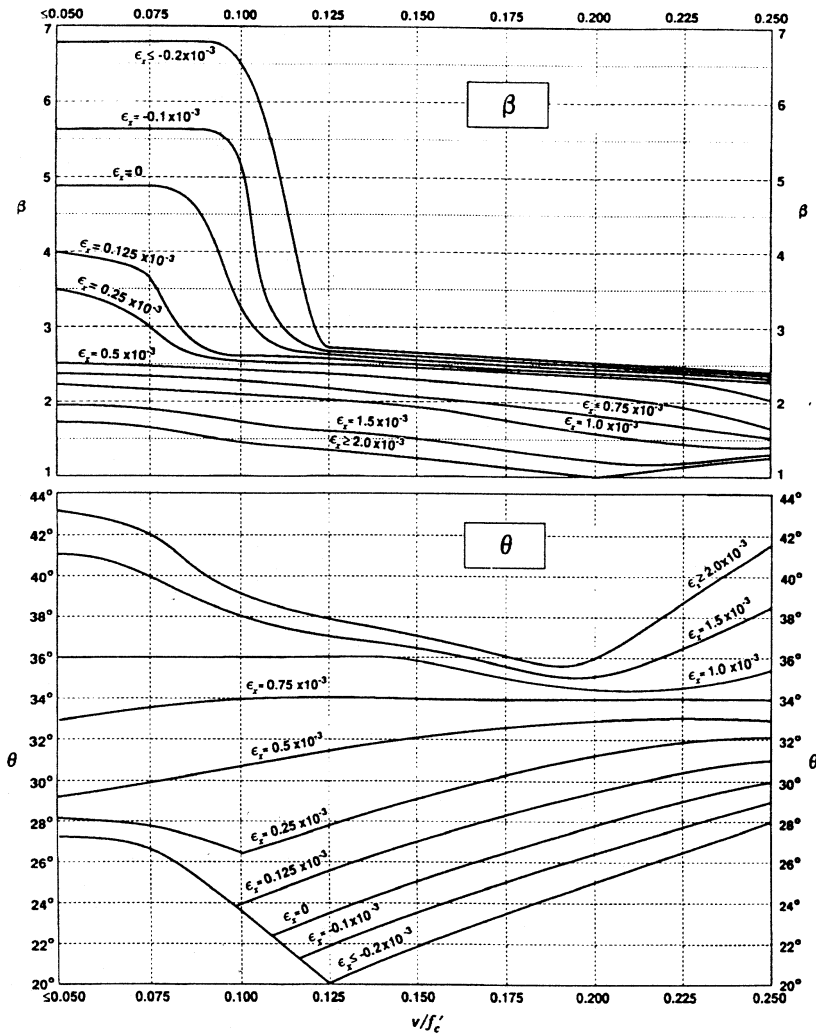
FIGURE 9.6 Values of θ and β for sections with transverse web reinforcement. (*Source*: AASHTO LRFD Bridge Design Specifications, Figure 5.8.3.4.2-1, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)

9.4.5 Skewed Concrete Bridges

Shear, in the exterior beam at the obtuse corner of the bridge, needs to be adjusted when the line of support is skewed. The value of the correction factor obtained from AASHTO Table 4.6.2.2.3c-1, needs to be applied to live-load distribution factors for shear. In determining end shear in multibeam bridges, all beams should be treated like the beam at the obtuse corner, including interior beams.

Moment load distribution factors in longitudinal beams on skew supports may be reduced according to AASHTO Table 4.6.2.2.2e-1, when the line supports are skewed and the difference between skew angles of two adjacent lines of supports does not exceed 10°.
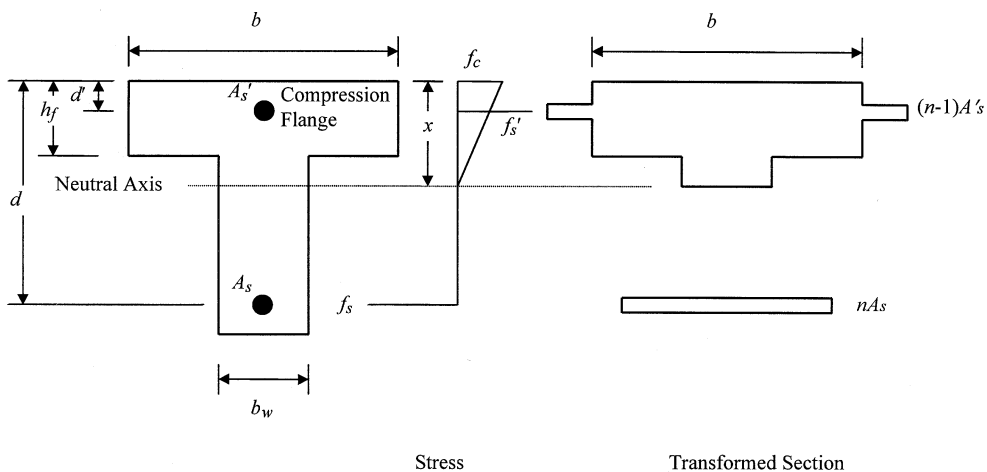
9.4.6 Design Information

1. Stress Analysis at Service Limit States [5]

A reinforced concrete beam subject to flexural bending moment is shown in Figure 9.7 and x is the distance between the neutral axis and the extreme compressed concrete fiber. Assume the neutral axis
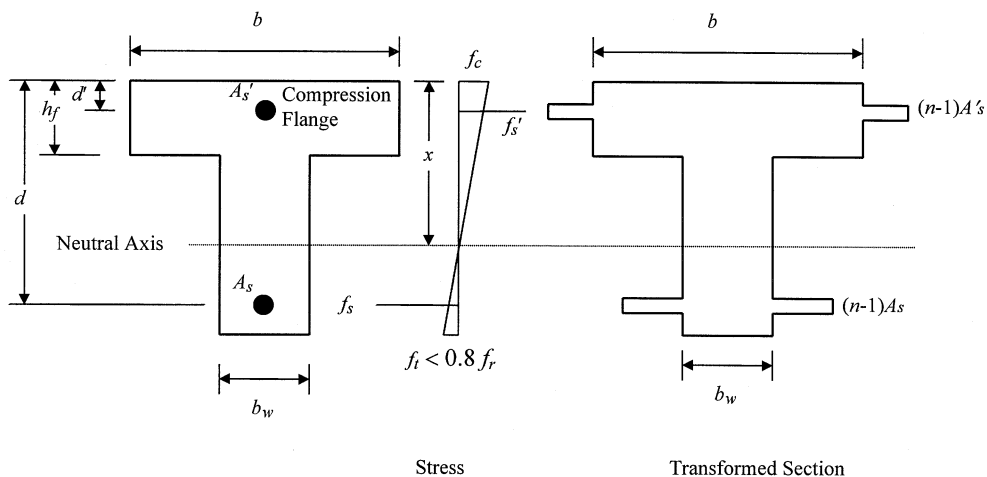
(a) Stress and Transformed Section Before Cracking

(b) Stress and Transformed Section After Cracking

FIGURE 9.7 Reinforced concrete beam for working stress analysis.

falls within the web ($x > h_f$) and the stress in extreme tension concrete fiber is greater than 80% of the concrete modulus of rupture ($f_t \geq 0.8\, f_r$). The depth of neutral axis, x, can be solved through the following quadratic equation by using the cracked transformed section method (see Figure 9.7).

$$b(x)\left(\frac{x}{2}\right) - \left(b - b_w\right)\left(x - h_f\right)\left(\frac{x - h_f}{2}\right) + (n-1)A_s'(x - d') = nA_s(d - x) \qquad (9.31)$$

$$x = \sqrt{B^2 + C} - B \qquad (9.32)$$

where

$$B = \frac{1}{b_w}\left[h_f(b - b_w) + nA_s + (n-1)A_s' \right] \tag{9.33}$$

$$C = \frac{2}{b_w}\left[\frac{h_f^2}{2}(b - b_w) + ndA_s + (n-1)d'A_s' \right] \tag{9.34}$$

and the moment of inertia of the cracked transformed section about the neutral axis:

$$I_{cr} = \frac{1}{3}bx^3 - \frac{1}{3}(b - b_w)\left(x - h_f\right)^3 + nA_s(d - x)^2 + (n-1)A_s'(x - d')^2 \tag{9.35}$$

if the calculated neutral axis falls within the compression flange ($x \le h_f$) or for sections without compression flange, the depth of neutral axis, x, and cracked moment of inertia, I_{cr}, can be calculated by setting b_w equal to b.

Stress in extreme compressed concrete fiber:

$$f_c = \frac{Mx}{I_{cr}} \tag{9.36}$$

Stress in compression steel:

$$f_s' = \frac{nM(x - d')}{I_{cr}} = nf_c\left(1 - \frac{d'}{x}\right) \tag{9.37}$$

Stress in tension steel:

$$f_s = \frac{nM(d - x)}{I_{cr}} = nf_c\left(\frac{d}{x} - 1\right) \tag{9.38}$$

where

$$n = \frac{E_s}{E_c} \tag{9.39}$$

and M is moment demand enveloped from the service limit state.

2. Effective Flange Width (AASHTO 4.6.2.6)

When reinforced concrete slab and girders are constructed monolithically, the effective flange width (b_{eff}) of a concrete slab, which will interact with girders in composite action, may be calculated as

For interior beams:

$$b_{eff}^I = \text{the smallest of} \begin{cases} \dfrac{l_{eff}}{4} \\ 12t_s + b_w \\ \text{the average spacing of adjacent beams} \end{cases} \tag{9.40}$$

TABLE 9.5 Cover for Unprotected Main Reinforcing Steel (mm)

Situation	Cover (mm)
Direct exposure to salt water	100
Cast against earth	75
Coastal	75
Exposure to deicing salt	60
Deck surface subject to tire stud or chain wear	60
Exterior other than above	50
Interior other than above	
• Up to No. 36 Bar	40
• No. 43 and No. 57 Bars	50
Bottom of CIP slab	
• Up to No. 36 Bar	25
• No. 43 and No. 57 Bars	50

Notes:
1. Minimum cover to main bars, including bars protected by epoxy coating, shall be 25 mm.
2. Cover to epoxy-coated steel may be used as interior exposure situation.
3. Cover to ties and stirrups may be 12 mm less than the value specified here, but shall not be less than 25 mm.
4. Modification factors for water:cement ratio, *w/c*, shall be the following:

     for $w/c \leq 0.40$ modification factor = 0.8
     for $w/c \geq 0.40$ modification factor = 1.2

Source: AASHTO Table C5.12.3-1. (From AASHTO LRFD Bridge Design Specifications, ©1994 by the American Association of State Highway and Transportation Officials, Washington, D.C. With permission.)

For exterior beams:

$$b_{\text{eff}}^E = \frac{1}{2}\, b_{\text{eff}}^I + \text{the smallest of} \begin{cases} \dfrac{l_{\text{eff}}}{8} \\[2ex] 6t_s + \dfrac{b_w}{2} \\[2ex] \text{the width of overhang} \end{cases} \qquad (9.41)$$

where the effective span length (l_{eff}) may be calculated as the actual span for simply supported spans. Also, the distance between the points of permanent load inflection for continuous spans of either positive or negative moments (t_s) is the average thickness of the slab, and b_w is the greater of web thickness or one half the width of the top flange of the girder.

3. Concrete Cover (AASHTO 5.12.3)

Concrete cover for unprotected main reinforcing steel should not be less than that specified in Table 9.5 and modified for the water:cement ratio.

9.4.7 Details of Reinforcement

Table 9.6 shows basic tension, compression, and hook development length for Grade 300 and Grade 420 deformed steel reinforcement (AASHTO 5.11.2). Table 9.7 shows the minimum center-to-center spacing between parallel reinforcing bars (AASHTO 5.10.3).

TABLE 9.6 Basic Rebar Development Lengths for Grade 300 and 420 (AASHTO 5.11.2)

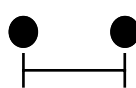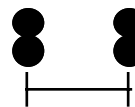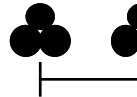| Bar Size	f_c'								
	28 MPa			35 MPa			42 MPa		
	Tension	Compression	Hook	Tension	Compression	Hook	Tension	Compression	Hook
Grade 300, f_y = 300 MPa									
13	230	175	240	230	170	215	230	170	200
16	290	220	300	290	210	270	290	210	245
19	345	260	365	345	255	325	345	255	295
22	440	305	420	400	295	375	400	295	345
25	580	350	480	520	335	430	475	335	395
29	735	395	545	655	380	485	600	380	445
32	930	440	610	835	430	550	760	430	500
36	1145	490	680	1020	475	605	935	475	555
43	1420	585	815	1270	570	730	1160	570	665
57	1930	780	1085	1725	765	970	1575	760	885
Grade 420, f_y = 420 MPa									
13	320	245	255	320	235	225	320	235	210
16	405	305	320	405	295	285	405	295	260
19	485	365	380	485	355	340	485	355	310
22	615	425	445	560	410	395	560	410	360
25	810	485	505	725	470	455	665	470	415
29	1025	550	570	920	530	510	840	530	465
32	1300	615	645	1165	600	575	1065	600	525
36	1600	685	710	1430	665	635	1305	665	580
43	1985	820	855	1775	795	765	1620	795	700
57	2700	1095	1140	2415	1060	1020	2205	1060	930

Notes:
1. Numbers are rounded up to nearest 5 mm.
2. Basic hook development length has included reinforcement yield strength modification factor.
3. Minimum tension development length (AASHTO 5.11.2.1). Maximum of (1) basic tension development length times appropriate modification factors (AASHTO 5.11.2.1.2 and 5.11.2.1.3) and (2) 300 mm.
4. Minimum compression development length (AASHTO 5.11.2.2). Maximum of (1) basic compression development length times appropriate modification factors (AASHTO 5.11.2.2.2) and (2) 200 mm.
5. Minimum hook development length (AASHTO 5.11.2.4). Maximum of (1) basic hook development length times appropriate modification factors (AASHTO 5.11.2.4.2), (2) eight bar diameters, and (3) 150 mm.

Except at supports of simple spans and at the free ends of cantilevers, reinforcement (AASHTO 5.11.1.2) should be extended beyond the point at which it is no longer required to resist the flexural demand for a distance of

$$\text{the largest of } \begin{cases} \text{the effective depth of the member} \\ \text{15 times the nominal diameter of a bar} \\ \text{0.05 times the clear span length} \end{cases} \tag{9.42}$$

Continuing reinforcement shall extend not less than the development length beyond the point where bent or terminated tension reinforcement is no longer required for resisting the flexural demand.

TABLE 9.7   Minimum Rebar Spacing for CIP Concrete (mm) (AASHTO 5.10.3)

Bar Size	Minimum Spacing			
13	51	51	63	63
16	54	56	70	70
19	57	68	76	83
22	60	78	82	96
25	64	90	90	110
29	72	101	101	124
32	81	114	114	140
36	90	127	127	155
43	108	152	152	
57	143	203	203	

Notes:
1. Clear distance between bars should not be less than 1.5 times the maximum size of the course aggregate.
2. Note 1 does not need to be verified when maximum size of the course aggregate grading is less than 25 mm.
3. Bars spaced less than $3d_b$ on center require modification of development length (AASHTO 5.11.2.1.2).

For negative moment reinforcement, in addition to the above requirement for bar cutoff, it must be extended to a length beyond the inflection point for a distance of

$$\text{the largest of} \begin{cases} \text{the effective depth of the member} \\ \text{12 times the nominal diameter of a bar} \\ \text{0.0625 times the clear span length} \end{cases} \tag{9.43}$$

9.5   Design Examples

9.5.1   Solid Slab Bridge Design

Given
A simple span concrete slab bridge with clear span length (*S*) of 9150 mm is shown in Figure 9.8. The total width (*W*) is 10,700 mm, and the roadway is 9640 wide (W_R) with 75 mm (d_W) of future wearing surface.

The material properties are as follows: Density of wearing surface ρ_w = 2250 kg/m³; concrete density ρ_c = 2400 kg/m³; concrete strength f'_c = 28 MPa, E_c = 26 750 MPa; reinforcement f_y = 420 MPa, E_s = 200,000 MPa; n = 8.

Requirements
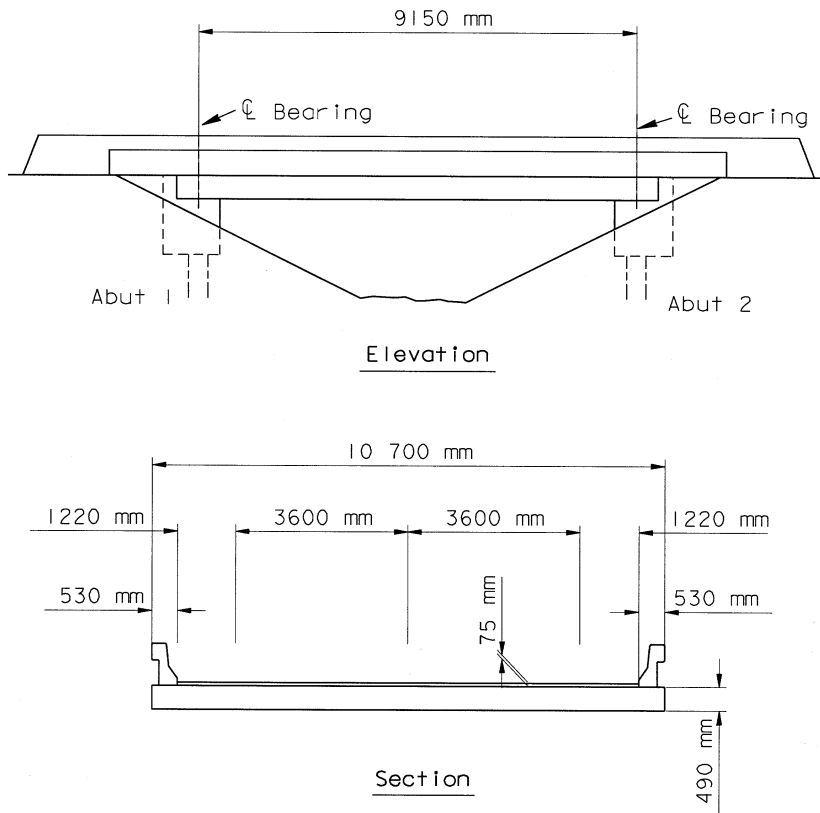Design the slab reinforcement base on AASHTO-LRFD (1994) Strength I and Service I (cracks) Limit States.

FIGURE 9.8   Solid slab bridge design example.

Solution

1. **Select Deck Thickness (Table 9.4)**

$$h_{\min} = 1.2\left(\frac{S+3000}{30}\right) = 1.2\left(\frac{9150+3000}{30}\right) = 486\,\text{mm}$$

Use $h = 490$ mm

2. **Determine Live Load Equivalent Strip Width (AASHTO 4.6.2.3 and 4.6.2.1.4b)**
   a. *Interior strip width*:
      i. Single-lane loaded:

$$E_{\text{interior}} = 250 + 0.42\sqrt{L_1 W_1}$$

   L_1 = lesser of actual span length and 18,000 mm
   W_1 = lesser of actual width or 9000 mm for single lane loading or 18,000 mm
             for multilane loading

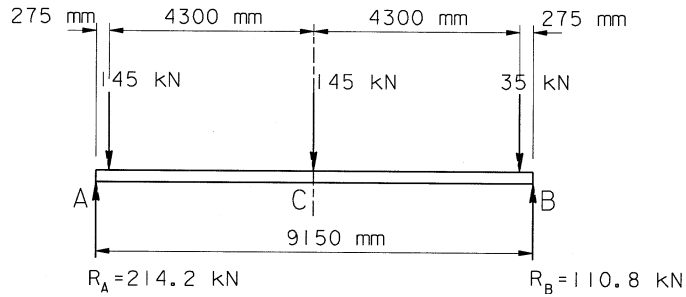$$E_{\text{interior}} = 250 + 0.42\sqrt{(9150)(9000)} = 4061\,\text{mm}$$

FIGURE 9.9 Position of design truck for maximum moment.

ii. Multilane loaded:

$$N_L = INT\left(\frac{W}{3600}\right) = INT\left(\frac{10,700}{3600}\right) = 2$$

$$\frac{W}{N_L} = \frac{10,700}{2} = 5350 \text{ mm}$$

$$E_{\text{interior}} = 2100 + 0.12\sqrt{L_1 W_1} = 2100 + \sqrt{(9150)(10,700)} = 3287 \text{ mm} < 5350 \text{ mm}$$

$$\underline{\text{Use } E_{\text{interior}} = 3287 \text{ mm}}$$

b. *Edge strip width*:

E_{edge} = the distance between the edge of the deck and the inside face of the barrier
+ 300 mm + ½ strip width < full strip or 1800 mm

$$E_{\text{edge}} = 530 + 300 + \frac{3287}{2} = 2324 \text{ mm} > 1800 \text{ mm}$$

$$\underline{\text{Use } E_{\text{edge}} = 1800 \text{ mm}}$$

3. **Dead Load**

Slab: $W_{\text{slab}} = (0.49)(2400)(9.81)(10^{-3}) = 11.54 \text{ kN/m}^2$
Future wearing: $W_{\text{fw}} = (0.075)(2250)(9.81)(10^{-3}) = 1.66 \text{ kN/m}^2$
Assume 0.24 m³ concrete per linear meter of concrete barrier
Concrete barrier: $W_{\text{barrier}} = (0.24)(2400)(9.81)(10^{-3}) = 5.65 \text{ kN/m}^2$

4. **Calculate Live-Load Moments**

Moment at midspan will control the design.
a. *Moment due to the design truck* (see Figure 9.9):

$$M_{\text{LL-Truck}} = (214.2)(4.575) - (145)(4.3) = 356.47 \text{ kN·m}$$

b. *Moment due to the design tandem* (see Figure 9.10):

$$M_{\text{LL-Tandem}} = (95.58)(4.575) = 437.28 \text{ kN·m}.$$
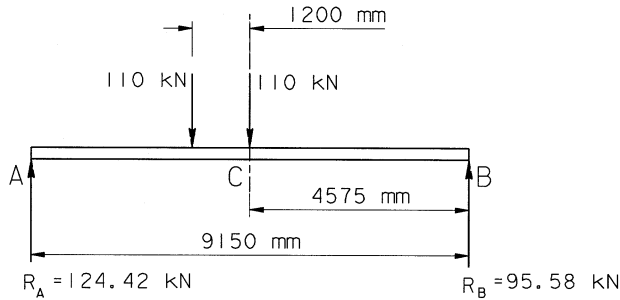
$$\underline{\text{Design Tandem Controls}}$$

FIGURE 9.10 Position of tandem for maximum moment.

c. *Moment due to lane load*:

$$M_{\text{LL-Lane}} = \frac{(9.3)(9.15)^2}{8} = 97.32 \text{ kN·m}$$

5. **Determine Load Factors (AASHTO Table 3.4.1-1) and Load Combinations (AASHTO 1.3.3-5)**
   a. *Strength I Limit State load factors*:

   Weight of superstructure (DC): 1.25
   Weight of wearing surface (DW): 1.50
   Live Load (LL): 1.75
   $\eta_d = 0.95, \eta_R = 1.05, \eta_I = 0.95$
   $\eta = (0.95)(1.05)(0.95) = 0.948 \leq 0.95$

   <u>Use $\eta = 0.95$</u>

   b. *Interior strip moment* (1 m wide) (AASHTO 3.6.2.1 and 3.6.1.2.4):

   Dynamic load factor   IM = 0.33

Lane load	$M_{\text{LL-Lane}} = \left(\dfrac{97.32}{3.287}\right) = 29.61 \text{ kN·m}$
Live load	$M_{\text{LL+IM}} = (1+0.33)\left(\dfrac{437.28}{3.287}\right) + 29.61 = 206.54 \text{ kN·m}$
Future wearing	$M_{\text{DW}} = \dfrac{W_{fw}L^2}{8} = \dfrac{(1.66)(9.15)^2}{8} = 17.37 \text{ kN·m}$
Dead load	$M_{\text{DC}} = \dfrac{W_{\text{slab}}L^2}{8} = \dfrac{(11.54)(9.15)^2}{8} = 120.77 \text{ kN·m}$
Factored moment	$M_U = \eta[1.25(M_{\text{DC}}) + 1.50(M_{\text{DW}}) + 1.75(M_{\text{LL+IM}})]$
	$= (0.95) [1.25 (120.77) + (1.50) (17.37) + (1.75) (206.54)]$
	$= 511.54 \text{ kN·m}$

   c. *Edge strip moment* (1 m wide) (AASHTO Table 3.6.1.1.2-1):

   End strip is limited to half lane width, use multiple presence factor 1.2 and half design lane load.

Lane load
$$M_{\text{LL-Lane}} = (1.2)\left(\frac{1}{2}\right)\left(\frac{97.3}{1.8}\right) = 32.44 \ \text{kN·m}$$

Live load
$$M_{\text{LL+IM}} = (1 + 0.33)(1.2)\left(\frac{1}{2}\right)\left(\frac{437.28}{1.8}\right) + 32.44 = 226.3 \ \text{kN·m}$$

Dead load
$$M_{\text{DC}} = \left(11.54 + \frac{5.65}{1.8}\right)\left(\frac{9.15^2}{8}\right) = 153.63 \ \text{kN·m}$$

Future wearing
$$M_{\text{DW}} = (1.66)\left(\frac{1.8 - 0.53}{1.8}\right)\left(\frac{9.15^2}{8}\right) = 12.25 \ \text{kN·m}$$

Factored moment
$$M_U = (0.95)[(1.25)(153.63) + (1.50)(12.25) + (1.75)(226.3)] = 579.12 \ \text{kN·m}$$

6. Reinforcement Design

a. *Interior strip*:

Assume No. 25 bars, $d = 490 - 25 - \left(\frac{25}{2}\right) = 452.5$ mm.

The required reinforcements are calculated using Eqs. (9.11), (9.16), and (9.17).

Neglect the compression steel and set $b_w = b$ for sections without compression flange.

$$M_u = \phi A_s f_y \left(d - \frac{a}{2}\right) \quad \text{and} \quad a = c\beta_1 = \frac{A_s f_y}{0.85 f_c' b_w}$$

A_s can be solved by substituting a into M_u or

$$R_u = \frac{M_u}{\phi b d^2} = \frac{511.54 \times 10^6}{(0.9)(1000)(452.5)^2} = 2.766 \ \text{N/mm}$$

$$m = \frac{f_y}{(0.85) f_c'} = \frac{420}{(0.85)(28)} = 17.647$$

$$\rho = \frac{1}{m}\left[1 - \sqrt{1 - \frac{2mR_u}{f_y}}\right] = \frac{1}{17.647}\left[1 - \sqrt{1 - \frac{2(17.647)(2.776)}{420}}\right] = 0.00705$$

Required reinforced steel $A_s = \rho b d = (0.00705)(1000)(452.5) = 3189 \ \text{mm}^2/\text{m}$.

Maximum allowed spacing of No. 25 bar = 510/3189 = 0.160 m.

Try No. 25 bars at 150 mm.

i. Check limits for reinforcement:

$\beta_1 = 0.85$ for $f_c' = 28$ MPa; see Eq. (9.18)

$$c = \frac{A_s f_y}{0.85 \beta_1 f_c' b_w} = \frac{(510)(420)}{0.85(0.85)(28)(150)} = 70.6 \ \text{mm}$$

from Eqs. (9.19),

$$\frac{c}{d} = \frac{70.79}{452.5} = 0.156 \leq 0.42 \qquad \qquad \text{OK}$$

from Eqs. (9.20),

$$\rho_{min} = \frac{510}{(150)(452.5)} = 0.007\ 51 \geq (0.03)\left(\frac{28}{420}\right) = 0.002 \qquad \text{OK}$$

ii. Check crack control:

Service load moment M_{sa} = 1.0[1.0(M_{DC}) + 1.0(M_{DW}) + 1.0(M_{LL+IM})]
= [120.77 + 17.37 + (176.93 + 29.61)]
= 344.68 kN·m

$$0.8f_r = 0.8(0.63\sqrt{f_c'}) = 0.8(0.63)\sqrt{28} = 2.66 \text{ MPa}$$

$$f_c = \frac{M_{sa}}{S} = \frac{344,680}{\frac{1}{6}(490)^2} = 8.61\,\text{MPa} \geq 0.8f_r;\,, \text{ Section is cracked}$$

Cracked moment of inertia can be calculated by using Eqs. (9.32) to (9.35).

n = 8, b = 150.0 mm, A_s = 510 mm, d = 452.5 mm.

$$B = \frac{1}{b}(nA_s) = \frac{1}{150}(8)(510) = 27.2$$

$$C = \frac{2}{b}(ndA_s) = \frac{2}{150}(8)(452.5)(510) = 24616$$

$$x = \sqrt{B^2 + C} - B = \sqrt{(27.2)^2 + (24616)} - (27.2) = 132 \text{ mm}$$

$$I_{cr} = \frac{1}{3}bx^3 + nA_s(d-x)^2 = \frac{1}{3}(150)(132)^3 + (8)(510)(452.5-132)^2 = 534.1\times10^6\,\text{mm}^4$$

From Eq. (9.38) $\quad f_s = n\frac{M_{sa}(d-x)}{I_{cr}} = (8)\frac{(344,680)(452.5-132)}{534.1\times10^6} = 248 \text{ MPa}$

Allowable tensile stress in the reinforcement can be calculated from Eq. (9.5) with Z = 23,000 N/mm for moderate exposure and

$$d_c = 25 + \frac{25}{2} = 37.5 \text{ mm}$$

$$A = 2d_c \times \text{bar spacing} = (2)(37.5)(150) = 11,250 \text{ mm}$$

$$f_{sa} = \frac{Z}{(d_cA)^{1/3}} \leq 0.6f_y$$

$$f_{sa} = \frac{23,000}{[(37.5)(11,250)]^{1/3}} = \frac{23,000}{75} = 307 \text{ MPa} \geq 0.6f_y = 0.6\ (420) = 252 \text{ MPa}$$

$$f_s = 248\,\text{MPa} \leq f_{sa} = 252\,\text{MPa}, \qquad \text{OK}$$

Use No. 25 Bar @150 mm for interior strip

b. *Edge strip*:

By similar procedure, Edge Strip Use No. 25 bar at 125 mm

7. **Determine Distribution Reinforcement (AASHTO 5.14.4.1)**
   The bottom transverse reinforcement may be calculated as a percentage of the main reinforcement for positive moment:

$$\frac{1750}{\sqrt{L}} \leq 50\,\%, \text{ that is, } \frac{1750}{\sqrt{9150}} = 18.3\% \leq 50\%$$

   a. *Interior strip*:
      Main reinforcement: No. 25 at 150 mm,

$$A_s = \frac{510}{150} = 3.40 \text{ mm}^2/\text{mm}.$$

   Required transverse reinforcement = $(0.183)(3.40) = 0.622$ mm²/mm

   Use No. 16 @ 300 mm transverse bottom bars,

$$A_s = \frac{199}{300} = 0.663 \text{ mm}^2/\text{mm}$$

   b. *End strip*:
      Main reinforcement: No. 25 at 125 mm,

$$A_s = \frac{510}{125} = 4.08 \text{ mm}^2/\text{mm}$$

   Required transverse reinforcement = $(0.183)(4.08) = 0.746$ mm²/mm

   Use No. 16 at 250 mm, $A_s = 0.79$ mm²/mm.

   For construction consideration, Use No. 16 @250 mm across entire width of the bridge.

8. **Determine Shrinkage and Temperature Reinforcement (AASHTO 5.10.8)**
   Temperature

$$A_s \geq 0.75\frac{A_g}{f_y} = 0.75\frac{(1)(490)}{420} = 0.875 \text{ mm}^2/\text{mm in each direction}$$

   Top layer = 0.875/2 = 0.438 mm²/mm

   Use No. 13 @ 300 mm transverse top bars, $A_s = 0.430$ mm²/mm

9. **Design Sketch**
   See Figure 9.11 for design sketch in transverse section.

10. **Summary**
    To complete the design, loading combinations for all limit states need to be checked. Design practice should also give consideration to long-term deflection, cracking in the support area for longer or continuous spans. For large skew bridges, alteration in main rebar placement is essential.
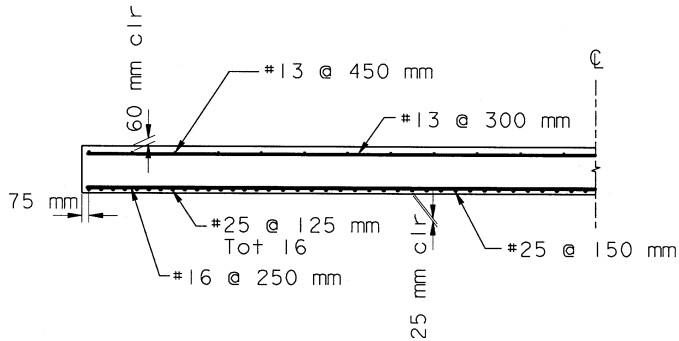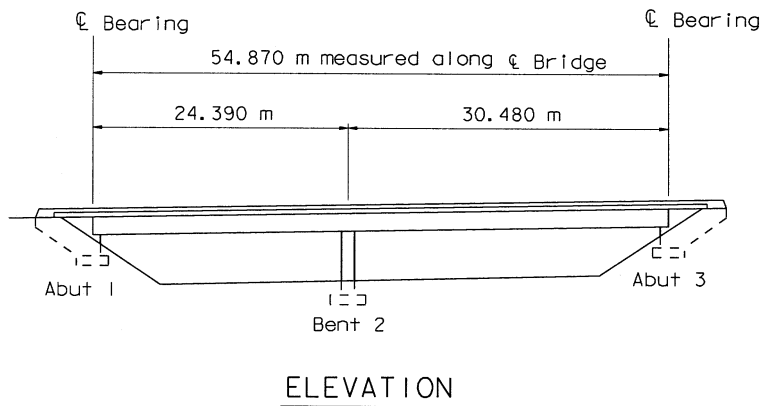
FIGURE 9.11 Slab reinforcement detail.

FIGURE 9.12 Two-span reinforced box girder bridge.

9.5.2 Box-Girder Bridge Design

Given

A two-span continuous cast-in-place reinforced concrete box girder bridge, with span length of 24 390 mm (L_1) and 30 480 mm (L_2), is shown in Figure 9.12. The total superstructure width (W) is 10 800 mm, and the roadway width (W_R) is 9730 mm with 75 mm (d_W) thick of future wearing surface.

The material properties are assumed as follows: Density of wearing surface ρ_w = 2250 kg/m³; concrete density ρ_c = 2400 kg/m³; concrete strength f'_c = 28 MPa, E_c = 26 750 MPa; reinforcement f_y = 420 MPa, E_s = 200 000 MPa.

Requirements

Design flexural and shear reinforcements for an exterior girder based on AASHTO-LRFD (1994) Limit State Strength I, Service I (cracks and deflection), and Fatigue Limit States.

Solution

    1. Determine Typical Section (see Figure 9.13)

       a. *Section dimensions*:

          Try the following dimensions:

          Overall Structural Thickness, h = <u>1680 mm</u> (Table 9.4)

          Effective length, s = 2900 – 205 = 2695 mm

          Design depth (*deck slab*),
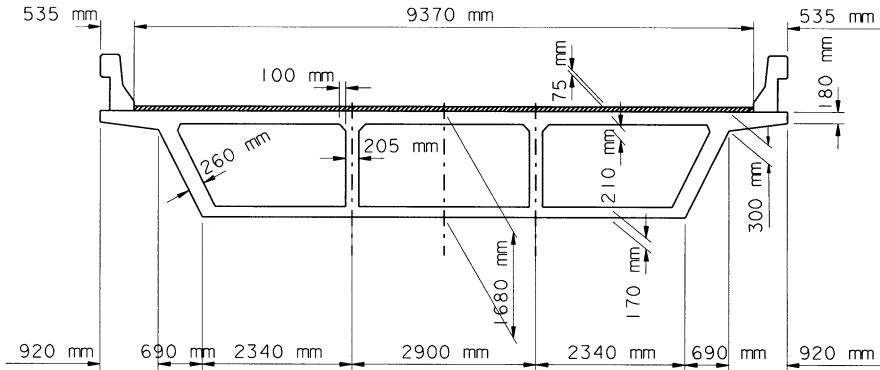
FIGURE 9.13   Typical section.
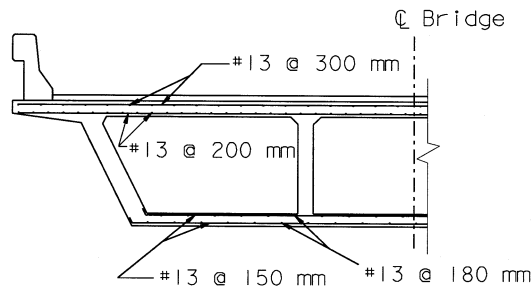
FIGURE 9.14   Slab reinforcement.

$$t_{\text{top}} = \underline{210 \text{ mm}}$$

$$> \frac{1}{20}(2900 - 205 - 100 \cdot 2) = 124.8 \text{ mm} \quad (\text{AASHTO } 5.14.1.3)$$

$$\frac{s}{t_{top}} = \frac{2695}{210} = 12.8 < 18 \quad (\text{AASHTO } 9.7.2.4)$$

Bottom flange depth,

$$t_{\text{bot}} = \underline{170 \text{ mm}} > 140 \text{ mm} \quad (\text{AASHTO } 5.14.1.3)$$

$$> \frac{1}{16}(2900 - 205 - 100 \cdot 2) = 156 \text{ mm} \quad (\text{AASHTO } 5.14.1.3)$$

Web thickness, $b_w = \underline{205 \text{ mm}} > 200$ mm for ease of construction (AASHTO 5.14.1.3)

b. *Deck slab reinforcement*:

The detail slab design procedure is covered in Chapter 15 of this handbook. The slab design for this example, using the empirical method, is shown in Figure 9.14.

2. **Calculate Design Loads**

The controlling load case is assumed to be Strength Limit State I.

a. *Permanent load*:

It is assumed that the self-weight of the box girder and the future wearing surface are equally distributed to each girder. The weight of the barrier rails is, however, distributed to the exterior girders only.

Dead load of box girder = (0.000 023 57)(4 938 600) = 116.4 N/mm

Dead load of the concrete barriers = 5.65(2) = 11.3 N/mm

Dead load of the future wearing surface = (0.0000221)(729 750) = 16.12 N/mm

b. *Live loads*:

i. Vehicle live loads:

A standard design truck (AASHTO 3.6.1.2.2), a standard design tandem (AASHTO 3.6.1.2.3), and the design lane load (AASHTO 3.6.1.2.4) are used to compute the extreme force effects.

ii. Multiple presence factors (AASHTO 3.6.1.1.2 and AASHTO Table 3.6.1.1.2-1):

No. of traffic lanes = INT (9730/3600) = 2 lanes

The multiple presence factor, m = 1.0

iii. Dynamic load allowance (AASHTO 3.6.2.1 and AASHTO Table 3.6.2.1-1):

IM = 15% for Fatigue and Fracture Limit State

IM = 33% for Other Limit States

c. *Load modifiers*:

For Strength Limit State:

$$\eta_D = 0.95; \quad \eta_R = 0.95; \quad \eta_I = 1.05; \quad \text{and} \quad \eta = \eta_D\eta_R\eta_I = 0.95 \text{ (AASHTO 1.3.2)}$$

For Service Limit State:

$$\eta_D = 1.0; \quad \eta_R = 1.0; \quad \eta_I = 1.0; \quad \text{and} \quad \eta = \eta_D\eta_R\eta_I = 1.0 \text{ (AASHTO 1.3.2)}$$

d. *Load factors*:

$$\gamma_{DC} = 0.9 \sim 1.25; \quad \gamma_{DW} = 0.65 \sim 1.50; \quad \gamma_{LL} = 1.75$$

e. *Distribution factors for live-load moment and shear* (AASHTO 4.6.2.2.1):

i. Moment distribution factor for exterior girders:

For Span 1 and Span 2:

$$W_e = \frac{2900}{2} + 1211 = 2661 \text{ mm} \ < \ S = 2900 \text{ mm}$$

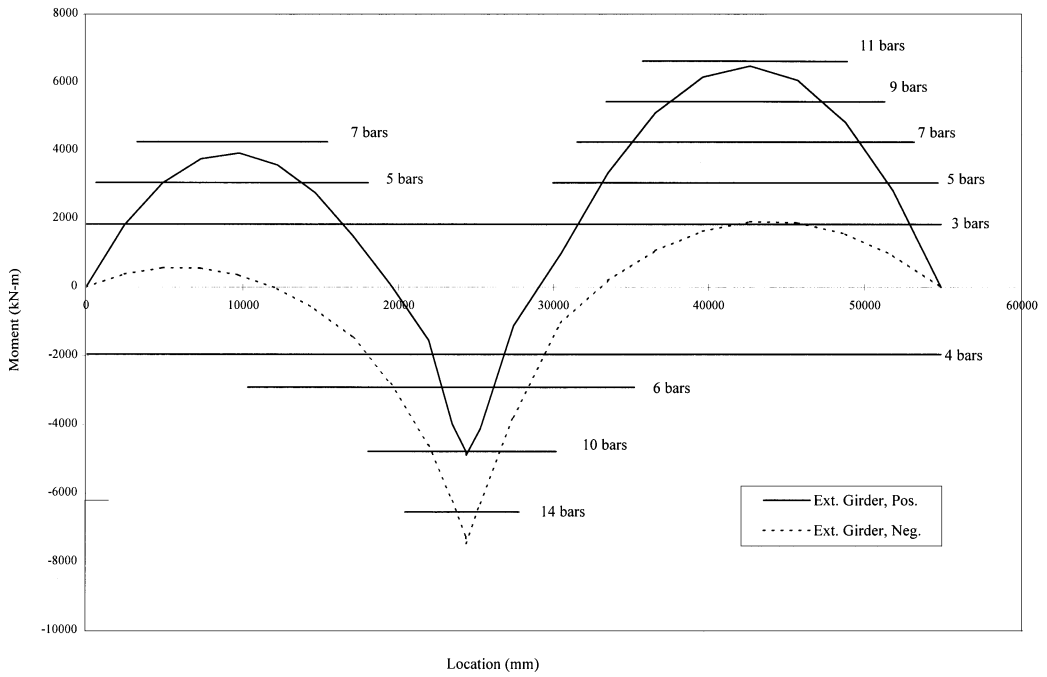$$g_m^E = \frac{W_e}{4300} = \frac{2661}{4300} = 0.619$$

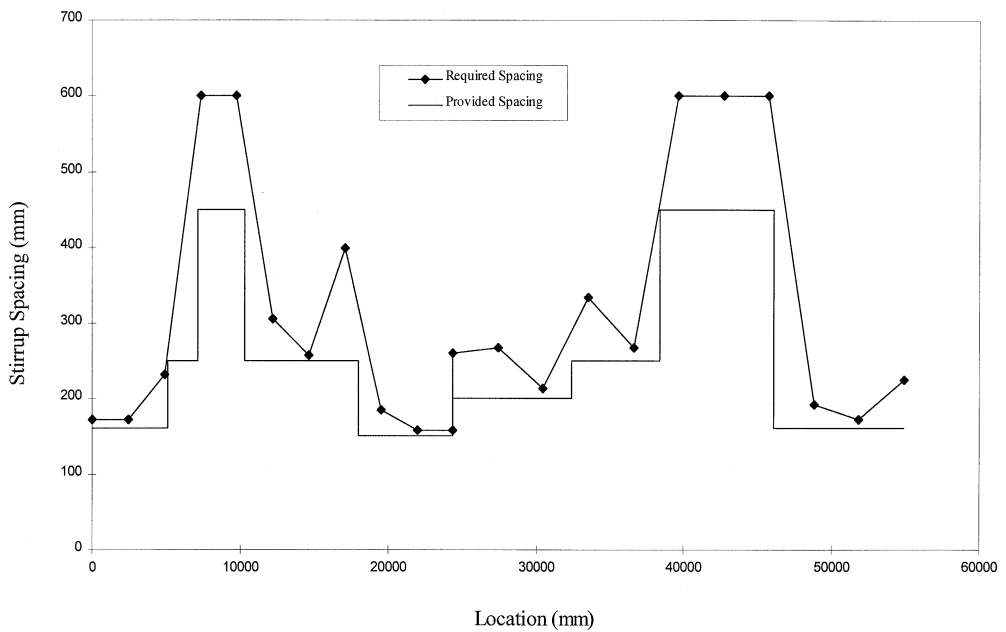**FIGURE 9.15** Design moment envelope and provided moment capacity with reinforcement cut-off.

FIGURE 9.16 Shear reinforcement spacing for the exterior girder.

TABLE 9.8   Moment Envelope Summary for Exterior Girder at Every ¹⁄₁₀ of Span Length of Span 1 and Span 2

| | | One Design Lane Load | | One Truck | | Train | Live Load Envelope | | Exterior Girder | | | | Factored Moment Envelope (kN-m) Exterior Girder | |
| | Distance | | | | | | | | | | | | | |
Span	(mm)	Positive	Negative	Positive	Negative	Negative	Positive	Negative	DC	DW	LL (Pos.)	LL (Neg.)	Positive	Negative
0.0 L_1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0.1 L_1	2439	216	−55	568	−93	−93	971	−179	607	70	601	−111	1819	378
0.2 L_1	4878	377	−110	955	−187	−187	1647	−359	1005	116	1020	−222	3053	561
0.3 L_1	7317	482	−165	1203	−281	−281	2082	−539	1196	137	1289	−333	3758	553
0.4 L_1	9756	533	−220	1303	−375	−375	2266	−719	1178	135	1403	−445	3923	350
0.5 L_1	12195	528	−275	1278	−468	−469	2228	−897	951	109	1379	−556	3577	−43
0.6 L_1	14634	467	−331	1157	−562	−563	2006	−1078	517	59	1242	−668	2762	−632
0.7 L_1	17073	352	−386	912	−656	−657	1565	−1258	−126	−15	969	−779	1494	−1466
0.8 L_1	19512	181	−441	570	−750	−751	939	−1439	−976	−113	581	−890	62	−2800
0.9 L_1	21951	38	−580	220	−844	−1055	331	−1785	−2035	−234	205	−1105	−1545	−4587
0.96 L_1	23414	−216	−748	29	−903	−1396	−177	−2344	−2810	−324	−110	−1451	−3981	−6211
1.0 L_1	24390	−326	−878	0	−938	−1616	−326	−2725	−3303	−380	−202	−1686	−4799	−7267
0.0 L_2	24390	−281	−904	0	−1059	−1643	−281	−2780	−3400	−392	−174	−1721	−4885	−7457
0.03 L_2	25304	−273	−754	0	−868	−1433	−273	−2394	−2835	−327	−169	−1482	−4112	−6295
0.1 L_2	27438	41	−466	224	−527	−960	339	−1569	−1597	−184	210	−971	−1130	−3772
0.2 L_2	30486	202	−234	640	−468	−468	1053	−856	−119	−14	652	−530	974	−1042
0.3 L_2	33534	470	−196	1072	−410	−410	1896	−741	1035	119	1173	−459	3349	195
0.4 L_2	36582	662	−168	1403	−351	−351	2528	−635	1862	214	1565	−393	5118	1071
0.5 L_2	39630	768	−140	1591	−293	−293	2884	−530	2365	272	1785	−328	6163	1645
0.6 L_2	42678	787	−112	1640	−234	−234	2968	−423	2543	292	1837	−262	6490	1919
0.7 L_2	45726	720	−84	1532	−176	−176	2758	−318	2395	275	1707	−197	6073	1890
0.8 L_2	48774	566	−56	1210	−117	−117	2175	−212	1922	221	1347	−131	4835	1562
0.9 L_2	51822	326	−28	708	−59	−59	1268	−106	1123	129	785	−66	2822	930
1.0 L_2	54870	0	0	0	0	0	0	0	0	0	0	0	0	0

TABLE 9.9 Shear Envelope Summary at Every 1/10 of Span 1 and Span 2

Span	Distance (mm)	One Design Lane Load Positive	One Design Lane Load Negative	One Truck Positive	One Truck Negative	Live Load Envelope Positive	Live Load Envelope Negative	Exterior Girder DC	Exterior Girder DW	Exterior Girder LL (Pos.)	Exterior Girder LL (Neg.)	Factored Shear Envelope Exterior Girder Positive	Factored Shear Envelope Exterior Girder Negative
0.0 L_1	0	100	−23	272	−38	462	−74	292	34	353	−56	981	176
0.1 L_1	2439	77	−23	233	−38	387	−74	206	24	296	−56	770	97
0.2 L_1	4878	55	−23	195	−38	314	−74	120	14	240	−56	562	18
0.3 L_1	7317	32	−23	159	−70	243	−116	36	4	186	−89	358	−115
0.4 L_1	9756	9	−23	124	−105	174	−163	−50	−6	133	−124	175	−274
0.5 L_1	12195	−13	−36	92	−143	109	−226	−136	−16	84	−173	14	−471
0.6 L_1	14634	−23	−59	64	−178	62	−396	−221	−26	48	−226	−126	−675
0.7 L_1	17073	−23	−81	41	−212	32	−363	−306	−35	24	−278	−243	−875
0.8 L_1	19512	−23	−104	23	−243	8	−427	−391	−45	6	−327	−353	−1072
0.9 L_1	21951	−23	−127	8	−271	−12	−487	−477	−55	−9	−373	−660	−1264
1.0 L_1	24390	−23	−149	0	−295	−23	−541	−563	−65	−16	−414	−787	−1449
0.0 L_2	24390	171	9	299	0	569	9	645	74	442	6	1606	882
0.1 L_2	27438	143	9	277	−3	511	5	539	62	383	4	1365	734
0.2 L_2	30486	115	9	250	−15	448	−11	432	50	335	−8	1141	386
0.3 L_2	33534	86	9	220	−38	379	−42	325	38	284	−31	910	249
0.4 L_2	36582	58	9	186	−59	305	−69	218	25	229	−52	675	115
0.5 L_2	39630	30	9	149	−87	228	−107	112	13	171	−80	435	−30
0.6 L_2	42678	9	−8	111	−119	157	−166	5	1	117	−125	202	−202
0.7 L_2	45726	9	−36	75	−154	109	−241	−102	−12	81	−180	41	−437
0.8 L_2	48774	9	−65	48	−192	73	−320	−209	−24	55	−240	−103	−681
0.9 L_2	51822	9	−93	20	−231	36	−400	−316	−36	27	−300	−248	−925
1.0 L_2	54870	9	−121	19	−272	34	−483	−422	−49	26	−362	−348	−1171

ii. Shear distribution factor for exterior girders:

Design Lane	Span 1	Span 2
One design lane loaded	$g_v^E = \dfrac{0.5(1015+2815)}{\left(\dfrac{\sqrt{5}}{2}\right)(2884)} = 0.594$	$g_v^E = \dfrac{0.5(1015+2815)}{\left(\dfrac{\sqrt{5}}{2}\right)(2884)} = 0.594$
Two or more design lanes loaded	$d_e = 1066 - 535 = 531 \ < \ 1500$	$d_e = 1066 - 535 = 531 \ < \ 1500$
	$e = 0.64 + \dfrac{531}{3800} = 0.78$	$e = 0.64 + \dfrac{531}{3800} = 0.78$
	$g_v^E = 0.78\left(\dfrac{2900}{2200}\right)^{0.9}\left(\dfrac{1680}{24,385}\right)^{0.1}$	$g_v^E = 0.78\left(\dfrac{2900}{2200}\right)^{0.9}\left(\dfrac{1680}{30,480}\right)^{0.1}$
	$= 0.765$	$= 0.749$
Govern	0.765	0.749

f. *Factored moment envelope and shear envelope*:
The moment and shear envelopes for the exterior girder, unfactored and factored based on Strength Limit State I, are listed in Tables 9.8 and 9.9. Figures 9.15 and 9.16 show the envelope diagram for moments and shears based on Strength Limit State I, respectively.

3. **Flexural Design**
    a. *Determine the effective flange width* (Section 9.4.6):
        i. Effective compression flange for positive moments:
        Span 1:
        For interior girder,

$$b_{top}^I = \text{the smallest of} \begin{cases} \dfrac{1}{4}L_{1,\text{eff}} = \dfrac{1}{4}(0.65)(24,390) = 3963 \text{ mm} \\[2ex] 12t_{top} + b_w = 12(210) + (205) = \underline{2725 \text{ mm}} \ \ \textit{governs} \\[2ex] \textit{the average spacing of adjacent beams} = 2900 \text{ mm} \end{cases}$$

For exterior girder,

$$b_{top}^E = \dfrac{1}{2}b_{top}^I + \text{the smallest of}$$

$$\begin{cases} \dfrac{1}{8}L_{1,\text{eff}} = \dfrac{1}{8}(0.65)(24,390) = 1982 \text{ mm} \\[2ex] 6t_{top} + \dfrac{1}{2}b_w = (6)(210) + \dfrac{1}{2}(291) = 1405 \text{ mm} \\[2ex] \textit{the width of the overhang} = 920 + \dfrac{291}{2} = \underline{1065 \text{ mm}} \ \ \textit{governs} \end{cases}$$

$$= \dfrac{1}{2}(2724) + 1065$$

$$= 2427 \text{ mm}$$

Span 2: The effective flange widths for Span 2 turns out to be the same as those in Span 1.

ii. Effective compression flange for negative moments:

Span 1:

For interior girder,

$$b_{bot}^{I} = \text{the smallest of} \begin{cases} \dfrac{1}{4}L_{eff} = \dfrac{1}{4}[(0.5)(24\ 390) + (0.25)(30\ 480) = 4954 \text{ mm} \\[2ex] 12t_{bot} + b_{w} = 12(170) + (205) = \underline{2245 \text{ mm}} \ \textit{governs} \\[2ex] \textit{the average spacing of adjacent beams} = 2900 \text{ mm} \end{cases}$$

For exterior girder,

$$b_{bot}^{E} = \frac{1}{2}b_{bot}^{I} + \text{the smallest of}$$

$$\begin{cases} \dfrac{1}{8}L_{eff} = \dfrac{1}{8}[(0.5)(24,390) + (0.25)(30,480) = 2477 \text{ mm} \\[2ex] 6t_{bot} + \dfrac{1}{2}b_{w} = (6)(170) + \dfrac{1}{2}(291) = 1166 \text{ mm} \\[2ex] \text{the width of the overhang} = 0 + \dfrac{291}{2} = \underline{146 \text{ mm}} \ \textit{governs} \end{cases}$$

$$= \frac{1}{2}(2245) + 146$$

$$= 1268 \text{ mm}$$

Span 2: The effective flange widths are the same as those in Span 1.

b. *Required flexural reinforcement*:

The required reinforcements are calculated using Eqs. (9.16) and (9.17), neglecting the compression steel

The minimum reinforcement required, based on Eq. (9.20), is

$$\rho_{min} \geq 0.03\frac{f_c'}{f_y} = 0.03\left(\frac{28}{420}\right) = 0.002$$

$$A_g(\text{Exterior girder}) = 1\ 103\ 530 \text{ mm}^2$$

$$A_{s_{min}}(\text{Exterior girder}) = (0.002)(1\ 103\ 530) = 2207 \text{ mm}^2$$

Use $A_{s_{min}} = \underline{2500 \text{ mm}^2}$

The required and provided reinforcements for sections located at ¹⁄₁₀ of each span interval and the face of the bent cap are listed in Table 9.10.

TABLE 9.10   Section Reinforcement Design for Exterior Girder

		Positive Moment					Negative Moment				
Section	Distance from Abut. 1 (mm)	M_u (kN-m)	A_s Required (mm²)	No. of Reinf. Bars Use #36	A_s (provided) (mm²)	ϕM_n (provided) (kN-m)	M_u (kN-m)	A_s Required (mm²)	No. of Reinf. Bars Use #32	A_s (Provided) (mm²)	ϕM_n (Provided) (kN-m)
0.0 L_1	0	0	0	3	3018	1841	0	0	4	3276	1954
0.1 L_1	2439	1819	2979	3	3018	1841	0	0	4	3276	1954
0.2 L_1	4878	3053	5020	5	5030	3055	0	0	4	3276	1954
0.3 L_1	7317	3758	6194	7	7042	4257	0	0	4	3276	1954
0.4 L_1	9756	3923	6469	7	7042	4257	0	0	4	3276	1954
0.5 L_1	12195	3577	5892	7	7042	4257	43	72	6	4914	2910
0.6 L_1	14634	2762	4537	5	5030	3055	632	1048	6	4914	2910
0.7 L_1	17073	1494	2444	3	3018	1841	1466	2448	6	4914	2910
0.8 L_1	19512	62	101	3	3018	1841	2800	4724	6	4914	2910
0.9 L_1	21951	0	0	3	3018	1841	4587	7848	10	8190	4780
0.96 L_1	23414	0	0	3	3018	1841	6211	10767	14	11466	6544
0.03 L_2	25304	0	0	3	3018	1841	6295	10920	14	11466	6544
0.1 L_2	27438	0	0	3	3018	1841	3772	6412	10	8190	4780
0.2 L_2	30486	974	1590	3	3018	1841	1042	1735	6	4914	2910
0.3 L_2	33534	3349	5512	7	7042	4257	0	0	6	4914	2910
0.4 L_2	36582	5118	8475	9	9054	5449	0	0	4	3276	1954
0.5 L_2	39630	6163	10243	11	11066	6629	0	0	4	3276	1954
0.6 L_2	42678	6490	10799	11	11066	6629	0	0	4	3276	1954
0.7 L_2	45726	6073	10091	11	11066	6629	0	0	4	3276	1954
0.8 L_2	48774	4835	7999	9	9054	5449	0	0	4	3276	1954
0.9 L_2	51822	2822	4637	5	5030	3055	0	0	4	3276	1954
1.0 L_2	54870	0	0	3	3018	1841	0	0	4	3276	1954

c. *Reinforcement layout*:
   i. Reinforcement cutoff (Section 9.4.7):
      • The extended length at cutoff for positive moment reinforcement, No. 36, is

$$\text{the largest of} \begin{cases} \text{Effective depth of the section } = \underline{1625 \text{ mm}} \quad governs \\ 15\, d_b = 537 \text{ mm} \\ 0.05 \text{ of span length } = 0.05\,(24\ 390) = 1220 \text{ mm} \end{cases}$$

      From , the stagger lengths for No. 36 and No. 32 bars are

$$l_d \text{ of No. 36 bars } = 1600 \text{ mm}$$
$$l_d \text{ of No. 32 bars } = 1300 \text{ mm}$$

      • The extended length at cutoff for negative moment reinforcement, No. 32, is

$$\text{the largest of} \begin{cases} \text{Effective depth of the section } = \underline{1601 \text{ mm}} \quad governs \\ 15\, d_b = 485 \text{ mm} \\ 0.05 \text{ of span length } = 0.05\,(30\ 480) = 1524 \text{ mm} \end{cases}$$

      • Negative moment reinforcements, in addition to the above requirement for bar cutoff, have to satisfy Eq. (9.43). The extended length beyond the inflection point has to be the largest of the following:

$$\begin{cases} d = \underline{1601 \text{ mm}} \quad governs \text{ for Span 1} \\ 12 d_b = 387.6 \text{ mm} \\ 0.0625 \times (\text{clear span length}) = (0.0625)(24\ 390) = 1524 \text{ mm} \\ \quad \text{or} \qquad\qquad\qquad\qquad = (0.0625)(30\ 480) = \underline{1905 \text{ mm}} \; governs \text{ for Span 2} \end{cases}$$

   ii. Reinforcement distribution (Section 9.4.2):

$$\frac{1}{10}(\text{average adjacent span length}) = \frac{1}{10}(30\ 480 + 24\ 385) = 2743 \text{ mm}$$

$$b_{\text{top}}^E = 2427 \text{ mm} < 2743 \text{ mm}$$

   All tensile reinforcements should be distributed within the effective tension flange width.

   iii. Side reinforcements in the web, Eq. (9.6)

$$A_{sk} \geq 0.001(d_e - 760) = 0.001(1625 - 760) = 0.865 \text{ mm}^2/\text{mm of height}$$

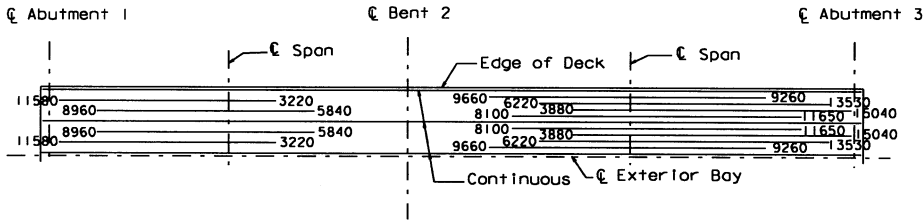$$A_{sk} \leq \frac{A_s}{1200} = \frac{13,462}{1200} = 11.21 \text{ mm}^2/\text{mm of height}$$

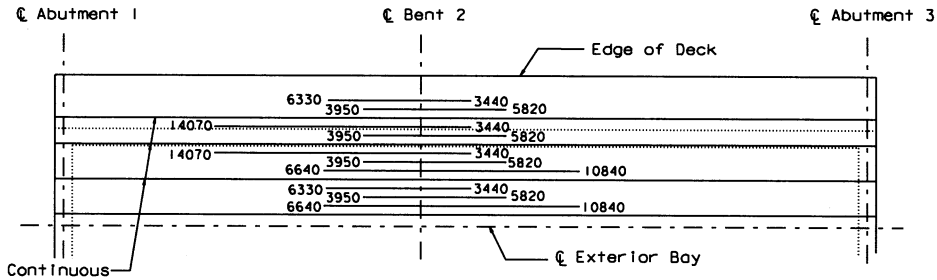**FIGURE 9.17** Bottom slab reinforcement of exterior girder.

FIGURE 9.18 Top deck reinforcement of exterior girder.

$$A_{sk} = 0.865(250) = 216 \text{ mm}^2 \quad \underline{\text{Use No. 19 at 250 mm on each side face of the web}}$$

The reinforcement layout for bottom slab and top deck of exterior girder are shown in Figure 9.17 and 9.18, respectively. The numbers next to the reinforcing bars indicate the bar length extending beyond either the centerline of support or span.

4. **Shear Design**

From Table 9.9, it is apparent that the maximum shear demand is located at the critical section near Bent 2 in Span 2.

a. *Determine the critical section near Bent 2 in Span 2*:

$$A_s = 11{,}466 \text{ mm}^2, \; b = 1268 \text{ mm}$$

$$a = \frac{A_s f_y}{0.85 f_c' b} = \frac{(11{,}466)(420)}{0.85(28)(1268)} = 160 \text{ mm}$$

$$d_v = \text{ the largest of} \begin{cases} d_e - \dfrac{a}{2} = 1601 - \dfrac{160}{2} = \underline{1521 \text{ mm}} \; governs \\[2mm] 0.9d_e = 0.9(1601) = 1441 \text{ mm} \\[2mm] 0.72h = 0.72(1680) = 1210 \text{ mm} \end{cases}$$

The critical section is at a distance of d_v from the face of the support, i.e., distance between centerline of Bent 2 and the critical section = 600 + 1521 = 2121 mm = $0.07L_2$.

b. *At the above section, find M_u and V_u, using interpolation from Tables 9.8 and 9.9*:

$$M_u = 3772 + (6295 - 3772)(0.03/0.07) = 4853 \text{ kN·m}$$

$$V_u = 1365 + (1606 - 1365)(0.03/0.1) = 1437 \text{ kN}$$

$$v = \frac{V_u}{\phi_v b_v d_v} = \frac{1437 \cdot (1000)}{(0.9)(291)(1521)} = 3.61 \text{ MPa}$$

$$\frac{v}{f_c'} = \frac{3.61}{28} = 0.129 < 0.25 \quad \text{O.K.}$$

c. *Determine θ and β, and required shear reinforcement spacing*:
    Try $\theta = 37.5°$, cot $\theta = 1.303$, $A_s = 11\ 466$ mm², $E_c = 200$ GPa, from Eq. (9.24).

$$\varepsilon_x = \frac{\dfrac{4\ 853\ 000}{1521} + 0.5(1437)(1.303)}{200(11\ 466)} = 1.80 \times 10^{-3}$$

From Figure 9.6, we obtain $\theta = 37.5°$, which agrees with the assumption.
Use $\theta = 37.5°$, $\beta = 1.4$, from Eq. (9.25)

$$V_s = \frac{1437}{0.9} - 0.083(1.4)\sqrt{28}(291)(1521) \times 10^{-3} = 1325 \text{ kN}$$

Use No. 16 rebars, $A_v = 199(2) = 398$ mm², from Eq. (9.26)

$$\text{Required spacing, } s \le \frac{(398)(420)(1521)}{1325 \times 10^3}(1.303) = 250 \text{ mm}$$

d. *Determine the maximum spacing required*:
    Note that $V_u = 1437$ kN $> 0.1\ f_c' b_v' d_v = 0.1(28)(291)(1521) \times 10^{-3} = 1239$ kN
    From Eqs. (9.27) and (9.29):

$$s_{\max} = \text{ the smallest of} \begin{cases} \dfrac{(398)(420)}{0.083\sqrt{28}(291)} = 1307 \text{ mm} \\[3mm] 0.4(1521) = 608 \text{ mm} \\[3mm] \underline{300 \text{ mm}} \quad \text{governs} \end{cases}$$

Use $s = 250$ mm < 300 mm OK.

e. *Check the adequacy of the longitudinal reinforcements, using Eq. (9.30)*:

$$A_s f_y = (11\ 466)(420) = 4\ 815\ 720 \text{ N}$$

$$\frac{M_u}{d_v \phi_f} + \left(\frac{V_u}{\phi_v} - 0.5 V_s\right)\cot\theta = \frac{4853 \times 10^6}{(1521)(0.9)} + \left(\frac{1437 \times 10^3}{0.9} - 0.5(1325 \times 10^3)\right)(1.303)$$

$$= 4\ 762\ 401 \text{ N} < A_s f_y \quad \text{O.K.}$$

Using the above procedure, the shear reinforcements, i.e., stirrups in the web, for each section can be obtained. Figure 9.16 shows the shear reinforcements required and provided in the exterior girder for both spans.

6. Crack Control Check (Section 9.4.2)

For illustration purpose, we select the section located at midspan of Span 1 in this example, i.e. at $0.5\,L_1$

a. *Check if the section is cracked*:

Service load moment, $M_{pos} = (1.0)(M_{DC} + M_{DW} + M_{LL+IM})$

$$= (1.0)(951 + 109 + 1379)$$
$$= 2439 \text{ kN-m}$$

Modulus of rupture $f_r = 0.63\sqrt{f_c'} = 0.63\sqrt{28} = 3.33$ MPa, $0.8f_r = 2.66$ MPa

$b_{top} = 2427$ mm, $b_{bot} = 1268$ mm, obtain

$I_g = 4.162 \times 10^{11}$ mm^4 and $\bar{y} = 655$ mm,

where $\bar{y}$ is the distance from the most compressed concrete fiber to the neutral axis

$$S = \frac{I_g}{(d - \bar{y})} = \frac{4.162 \times 10^{11}}{(1680 - 655)} = 4.06 \times 10^8 \text{ mm}^3$$

$$f_c = \frac{M_{pos}}{S} = \frac{2439 \times 10^6}{4.06 \times 10^8} = 6.01 \text{ MPa} > 0.8f_r = 2.66 \text{ MPa}$$

The section is cracked.

b. *Calculate tensile stress of the reinforcement*:

Assuming the neutral axis is located in the web, thus applying Eqs. (9.31) through (9.34) with $A_s = 7042$ mm², $A_s' = 0$, and $\beta_1 = 0.85$, solve for x

$$x = 239 \text{ mm} > h_f = b_{top} = 210 \text{ mm} \quad \text{O.K.}$$

From Eq. (9.35), obtain

$$I_{cr} = \frac{1}{3}(2427)(239)^3 - \frac{1}{3}(2427 - 291)(239 - 210)^3 + 7(7042)(1625 - 239)^2$$

$$= 1.057 \times 10^{11} \text{ mm}^4$$

and from Eq. (9.38), the tensile stress in the longitudinal reinforcement is

$$f_s = \frac{7(2439 \times 10^6)(1625 - 239)}{1.057 \times 10^{11}} = 224 \text{ MPa}$$

c. *The allowable stress can be obtained using Eq. (9.5), with Z = 30 000 for moderate exposure and d_c = 50 mm*

$$f_{sa} = \frac{Z}{(d_c A)^{1/3}} = \frac{30,000}{\left((50)\dfrac{(50 \cdot 2 \cdot 1268)}{7} \right)^{1/3}} = 310 \text{ MPa} > 0.6f_y = 252 \text{ MPa}$$

$\underline{\text{Use } f_{sa} = 252 \text{ MPa} > f_s = 223 \text{ MPa}} \qquad \text{O.K.}$

The other sections can be checked following the same procedure described above.

7. Check Deflection Limit

Based on the Service Limit State, we can compute the I_e for sections at $\frac{1}{10}$ of the span length interval. For illustration, let the section be at $0.4L_2$

Deflection distribution factor = (no. of design lanes)/(no. of supporting beams) = 2/4 = 0.5
Note that b_{top} = 2424 mm, t_{top} = 210 mm, b_w = 291 mm, h = 1680 mm, d = 1625 mm, b_{bot} = 1268 mm, t_{bot} = 170 mm, and neglecting compression steel

$$A_g = (2427)(210) + (1680 - 210 - 170)(291) + (1268)(170) = 1\ 103\ 530 \text{ mm}^2$$

$$y_t = \left[\frac{(509\ 670)\left(1680 - \dfrac{210}{2}\right) + (378\ 300)\left(170 + \dfrac{1300}{2}\right) + (215\ 560)\left(\dfrac{170}{2}\right)}{1\ 103\ 530} \right] = 1025 \text{ mm}$$

$$I_g = \frac{1}{12}(2427)(210)^3 + (509\ 670)(550)^2 + \frac{1}{12}(291)(1300)^3 + (378\ 300)(205)^2$$

$$+ \frac{1}{12}(1268)(170)^3 + (215\ 560)(940)^2$$

$$= 4.16 \times 10^{11} \text{ mm}^4$$

$$M_{cr} = f_r \frac{I_g}{y_t} = (3.33) \frac{4.16 \times 10^{11}}{1025} = 1.35 \times 10^9 \text{ N} - \text{mm}$$

Use Eqs. (9.31) throuth (9.35) to solve for x and I_{cr}, with A_s = 9054 mm² and $A'_s = 0$, we obtain

$$x = 272 \text{ mm}, \quad I_{cr} = 1.32 \times 10^{11} \text{ mm}^4$$

From Table 9.8:

$$M_a = 1862 + 214 + (0.5)(1565) = 2859 \text{ kN-m}$$

$$\frac{M_{cr}}{M_a} = \frac{1.35 \times 10^9}{2.86 \times 10^9} = 0.47$$

$$I_e = \left(\frac{M_{cr}}{M_a}\right)^3 I_g + \left[1 - \left(\frac{M_{cr}}{M_a}\right)^3\right] I_{cr} = (0.47)^3(4.16 \times 10^{11}) + \left[1 - (0.47)^3\right](1.32 \times 10^{11}) = 1.61 \times 10^{11} \text{ mm}^4$$

The above computation can be repeated to obtain I_e for other sections. It is assumed that the maximum deflection occurs where the maximum flexural moment is. To be conservative, the minimum I_e is used to calculate the deflection.

$$\Delta_{max} = \begin{cases} 19 \text{ mm} & \text{truck load} \\ 13 \text{ mm} & \text{lane} + 25\% \text{ of truck load} \end{cases} < \frac{L_2}{800} = \frac{30\ 480}{800} = 38 \text{ mm} \quad \text{O.K.}$$

8. **Check Fatigue Limit State**

   For illustration purpose, check the bottom reinforcements for the section at $0.7L_1$. For positive moment at this section, A_s = 4024 mm², A'_s = 4095 mm², d = 1625 mm, and d' = 79 mm. Note that the maximum positive moment due to the assigned truck is 757 kN-m, while the largest negative moment 598 kN-m.

$$M_{max} \text{ due to fatigue load} = 0.75(0.619)(757)(1 + 0.15) = 404 \text{ kN-m}$$

Use Eqs. (9.31) through (9.35) and (9.38) to obtain the maximum tensile stress in the main bottom reinforcements as

$$f_{max} = 64 \text{ MPa}$$

The negative moment at this section is

$$M_{min} \text{ due to fatigue load} = 0.75(0.619)(-598)(1 + 0.15) = -319 \text{ kN-m}$$

Using Eqs. (9.31) through (9.35) and (9.38), with $A_s = 4095 \text{ mm}^2$, $A_s' = 4024 \text{ mm}^2$, $d = 1601 \text{ mm}$, and $d' = 55 \text{ mm}$, we obtain the maximum compressive stress in the main bottom reinforcements as

$$f_{min} = -7.0 \text{ MPa}$$

Thus, the stress range for fatigue

$$f_{max} - f_{min} = 64 - (-7.0) = 71 \text{ MPa}$$

From Eq. (9.9), allowable stress range

$$f_r = 145 - 0.33(-7.0) + 55(0.3) = 164 \text{ MPa} > 71 \text{ MPa} \qquad \text{OK}$$

Other sections can be checked in the same fashion described above.

9. Summary

The purpose of the above example is mainly to illustrate the design procedure for flexural and shear reinforcement for the girder. It should be noted that, in reality, the controlling load case may not be the Strength Limit State; therefore, all the load cases specified in the AASHTO should be investigated for a complete design. It should also be noted that the interior girder design can be achieved by following the similar procedures described herein.

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