INELASTIC DEFORMATION RESPONSE OF SDOF SYSTEMS SUBJECTED TO EARTHQUAKES

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SUMMARY

Performance-based seismic design requires reliable methods to predict earthquake demands on structures, and particularly inelastic deformations, to ensure that specific damage-based criteria are met. Several methods based on the response of equivalent linear single-degree-of-freedom (SDOF) systems have been proposed. These methods do not offer advantages over the traditional Newmark-Hall (NH) procedure, and have been shown to be inaccurate. In this study, the NH method is revised, considering the inelastic response of elastoplastic, bilinear, and stiffness-degrading systems with 5% damping subjected to two sets of earthquake motions. One set is an ensemble of 51 records in the Circumpacific Belt, and the other is a group of 44 records in California. A statistical analysis of the response data provides factors for constructing inelastic spectra. Such factors show that the "equal-displacement" and "equal-energy" rules to relate elastic and inelastic responses are unconservative for high ductilities in the acceleration- and velocity-sensitive regions of the spectrum. It is also shown that, on the average, the effect of the type of force-deformation relationship of nonlinear systems is not significant, and responses can be conservatively predicted using the simple elastoplastic model.

RESUMEN

El diseño sísmico enfocado al desempeño requiere métodos confiables para predecir las demandas de los terremotos, particularmente en cuanto a deformaciones inelásticas, para asegurar que se satisfagan niveles de daño prescritos. Para estimar la respuesta de estructuras de varios grados de libertad se han propuesto varios métodos basados en la respuesta de sistemas elásticos equivalentes. Dichos métodos no ofrecen ventajas sobre el procedimiento tradicional de Newmark-Hall (NH), e incluso se ha comprobado su imprecisión. En este estudio se revisa el método de NH considerando la respuesta de sistemas elastoplásticos, bilineales y con degradación de rigidez, con 5% de amortiguamiento, sometidos a dos grupos de terremotos. El primero comprende 51 registros en el Anillo del Pacífico, y él otro 44 registros en California. Del análisis estadístico de los resultados se obtienen factores para construir espectros inelásticos. Estos factores muestran que las reglas simples de preservación de energía y deformación que relacionan la respuesta inelástica con la elástica, no son conservadoras en las regiones de aceleración y velocidad del espectro. Se demuestra también que el efecto de la relación fuerza deformación no es significativo y las respuestas pueden conservadoramente predecirse utilizando el simple modelo elastoplástico.

INTRODUCTION

This paper summarizes the findings of a recently published study (Riddell and García, Ref. 1) where the reader is referred for the complete contents.

After the great deal of damage caused by earthquakes in the last 10 or 15 years, there seems to be agreement that changes in the seismic design process are necessary to build structures with predictable seismic performance. For this purpose, advanced techniques will have to be incorporated in future design procedures. Verification of the structure by means of nonlinear 3-D response-history analysis is possible today, however, significant improvement and standardization of this procedure, and the software required, are necessary before general use by the profession. A simplified nonlinear analysis procedure is the push-over method, in which the structure is subjected to a monotonically increasing lateral load of prescribed pattern; the structure progressively degrades, as structural members sequentially plastify, until it reaches a limit state or collapse condition. The incremental static analysis permits to determine the global force-displacement relationship of the building, or *capacity curve* (ATC, Ref. 2), in terms of the total lateral force (base shear) and the lateral deflection of the roof.

The capacity curve is then converted to a *capacity diagram* or force-deformation relationship of a simplified SDOF model of the multi-degree building. In order to determine compliance with a given performance level, the displacement response of the building due to a given earthquake demand must be determined. The ATC-40 document proposes approximate methods to estimate the nonlinear response of SDOF systems on the basis of the response of equivalently damped linear systems. Chopra and Goel (Ref. 3) have pointed out several deficiencies of the ATC-40 procedures, and have shown that deformations can be significantly underestimated.

In the traditional NH inelastic spectrum (Newmark and Hall, Ref. 4; Riddell and Newmark, Ref. 5), the response of a nonlinear SDOF system is directly read without iterations. In turn, the spectral ordinates are associated to known degrees of uncertainty, depending on the probability of exceedance of the factors selected to construct the spectrum. The earthquake demand (seismic hazard) is simply represented by the peak ground motion parameters: acceleration A, velocity V, and displacement D. Inelastic spectra can be constructed for a variety of situations and conditions, the only limitation being the quality and quantity of the ground motion data available to derive the factors for constructing the spectrum. Summaries of the statistical procedure to obtain such factors have been presented by Riddell (Ref. 6) and Riddell et al. (Ref. 1).

A simple SDOF system was used in the study, with force-deformation relationship given by three nonlinear models: elastoplastic, bilinear, and stiffness degrading. These models cover a broad range of structural behavior; they are intended to represent over all generic behavior, rather than specific characteristics of individual systems (Riddell and Newmark, Refs. 5, 7). A damping factor of five percent of critical was used. The Circumpacific Belt and California groups of earthquake records used are listed, and their characteristics commented, in Riddell et al. (Ref. 1).

RESPONSE CALCULATIONS AND STATISTICAL ANALYSIS OF THE DATA

Responses were calculated for 250 different frequencies for each record, following an iterative procedure to obtain inelastic responses associated with desired target ductility values of 1 (elastic), 1.5, 2, 3, 5 and 10. The results were presented in the form of tripartite logarithmic plots, as the spectra shown in Figure 1. In this case the abscissa is frequency ($f=\omega/2\pi$), but period ($T=1/f$) may be simultaneously read as well since a plot with T as abscissa is a mirror image of the former. This type of plot, named Inelastic Yield Spectrum (IYS), or Constant Ductility Spectrum, features in the displacement axis the yield deformation u_y necessary to limit the maximum deformation u_{max} of the system so that the target ductility μ is not exceeded. The acceleration axis, at 90^o clockwise from the displacement axis, features the quantity $\omega^2 u_y$, which multiplied by the mass of the system gives the yield strength R_y. In the tripartite logarithmic plot the spectral quantities in the displacement, velocity, and acceleration axes are interrelated; indeed, denoting them by S_d, S_v and S_a respectively the relationship $S_a=\omega S_v=\omega^2 S_d$ holds.

The purpose of the statistical analysis is to determine factors for constructing demand spectra when estimates can be made of the possible peak ground motion parameters for future earthquakes affecting a site. The parameters A, V and D control the response over three regions of the spectrum and provide a better basis for characterizing design spectra than using only one. In summary, the statistical analysis consists in determining factors ψ_μ which, applied to the ground motion estimates p_g, give the spectral ordinates S_μ for each of the three characteristic regions of the spectrum, i.e., $S_\mu=\psi_\mu p_g$, where p_g represents A, V or D depending on the spectral region under consideration. Alternatively, the inelastic spectrum S_μ can be obtained by deamplifying the elastic spectrum S_e, so that $S_\mu=\phi_\mu S_e$, where the deamplification factor ϕ_μ obviously corresponds to the ratio $\psi_\mu/\psi_{\mu=1}$ and S_e corresponds to the particular case of $S_{\mu=1}$.

Figure 2 shows the average spectra normalized to peak ground acceleration, displacement and velocity, from top to bottom respectively, for the two groups of records considered in the study. The average spectra feature segments that present approximately constant response amplification with respect to the corresponding peak ground motion parameters, thus making it possible to identify regions of spectral acceleration amplification, spectral velocity amplification, and spectral displacement amplification. Henceforth these spectral regions will be simply referred to as acceleration, velocity and displacement regions. Incidentally, in Figure 2, the similarity of the average spectra for both groups of records is apparent (note that this comparison is to be made in the spectral region associated with the normalization parameter of the spectra). Frequency-band statistics are computed within the determined spectral regions for the ensemble of normalized spectra; the mean values correspond to the aforementioned ψ_μ factors, the standard deviation is designated by σ_μ, and the coefficient of variation by $\Omega_\mu=\sigma_\mu/\psi_\mu$.

The calculated frequency-band statistics for the Circumpacific Belt and California groups, and for different force-deformation relationships, are presented in several tables in Riddell et al. (Ref. 1), one of them is included here as Table 1. It was found that the factors for elastoplastic systems were very similar to those of bilinear and stiffness-degrading systems. Indeed, comparing average spectra, it was found that differences occur mainly for intermediate frequencies and large ductilities, and, most importantly, use of the elasto-plastic idealization provides essentially always a conservative estimate of the average response.

Simple approximations for the ϕ_μ factors have been widely used after first introduced by Veletsos and Newmark (Ref. 8). They are based in the well-known "equal-displacement" and "equal-energy" rules that relate elastic and inelastic responses, which lead to the ratios 1/μ for the displacement and velocity regions, and $1/\sqrt{2\mu-1}$ for the acceleration region. The study showed that the old rules were unconservative (underestimate inelastic displacements) in the velocity region for systems with response ductility larger than 3, and in the acceleration region for μ>2. The following new rules, that present better fit to the computed ϕ_μ factors are recommended: $\phi_{\mu=}\mu^{-1.08}$ in the displacement region, ϕ_μ=(1.9μ–0.9)$^{-0.7}$ in the velocity region and ϕ_μ=(4.2μ–3.2)$^{-1/3}$ in the acceleration region. Nevertheless, in the displacement region, the 1/μ ratio can still be used since it is conservative, approximate enough, and attractive for its simplicity. It was also shown that the ϕ_μ factors for the Circumpacific Belt and California groups were very similar, suggesting that these factors have certain generality that possibly makes them applicable regardless of the tectonic environment.

DEMAND SPECTRA AND ESTIMATION OF INELASTIC DEFORMATIONS

The earthquake demand, or intensity of the ground shaking for the site under consideration, needs to be specified in terms of A, V, and D, the peak ground motion parameters. These parameters may have been determined by a specific seismic hazard analysis, or may be consistent with a code design spectrum, or may have been specified for a particular facility with special design requirements. The parameters must take into account specific conditions such as near-field effects, nearness to active faults, and site geology. The parameters may also be associated with various levels of the earthquake hazard: serviceability earthquake, design earthquake, and maximum earthquake (ATC, Ref. 2). Discussion of the criteria for specification of the seismic hazard is beyond the scope of this paper.

The construction of demand spectra is illustrated in Figure 3, using the factors given in Table 1. A, V, and D are drawn in a tripartite logarithmic plot, and the segments JK, KL and LM of the elastic spectrum are determined by amplifying the ground motion parameters by the $\psi_{\mu=1}$ factors corresponding to the three spectral regions. The limiting frequencies f_I, f_J, f_M, and f_N need to be set for each case. In this study, the following values were found appropriate for the data considered: f_I=0.05, f_J=0.15, f_M=10, and f_N=30. The elastic design spectrum is completed with the transition lines IJ and MN. To construct inelastic spectra, factors ψ_μ for the desired ductility factor μ are applied to the ground motion maxima to determine segments J'K', K'L', and L'M' (conversely, the elastic design spectrum may be deamplified by the given factors ϕ_μ). Point I' is determined by dividing the elastic ordinate at I by μ. Point N' may be conservatively taken coincident with point N; however, based on actual response spectra, the factor $\mu^{-\eta}$ may be used to pass from N to N', with η=0.11, 0.13, and 0.15 for elastoplastic, stiffness degrading, and bilinear systems respectively. When the ordinate of point L' results lower than the design ground acceleration A, L' may be joined directly to N'.

If a greater degree of conservatism is desired, factors associated with smaller probabilities of exceedance can be used. In other words, one is interested in *p*–percentile $\psi_{p\mu}$ factors, so that the probability that the response amplification will not exceed $\psi_{p\mu}$ is p. Assuming normal distribution, the percentile amplification factors are computed as $\psi_{p\mu} = \psi_\mu + \delta_p\sigma_\mu$, where the coefficient δ_p, which indicates the deviation from the mean, can be obtained from tables of standard normal probability. For instance, δ_p is equal to 0, 1, and 2 for p equal to 0.5, 0.841, and 0.977 respectively, and the associated $\psi_{p\mu}$ factors correspond to the 50–percentile, 84.1–percentile and 97.7–percentile values. Use of 84.1–percentile factors, i.e., 0.159 probability of exceedance are recommended.

EXAMPLES

The examples presented in Section 8.3 of the ATC-40 report (ATC, Ref. 2) are solved next using the method and data presented herein. The example building is a seven-story reinforced concrete frame located in seismic zone 4 in California. Its fundamental period of vibration is 0.88 seconds. The capacity curve obtained from a push-over analysis of the building is shown in Figure 4a. The corresponding capacity diagram –representing the first mode response of the building– is the curve ABCD shown in Figure 4b (the term "capacity diagram" is used here instead of "capacity spectrum" employed in ATC-40 since the latter is considered inappropriate). Figures 4a and 4b are adapted from the ATC-40 report. The conversion from one curve to the other is done by means of the following formula (Chopra, Ref. 9; ATC, Ref. 2): $R/m=V/(\alpha_1 W)$, $\alpha_1=L_1^2/M_1$, $L_1=\phi_1^T\mathbf{m}r$, $M_1=\phi_1^T\mathbf{m}\phi_1$, $F_1= L_1/M_1$, $u=\Delta_r/(F_1\phi_{1,roof})$, where R is the resistance function of the equivalent SDOF system, V is the base shear of the building, W is the total weight of the building, α_1 is the effective modal mass associated with the fundamental mode shape ϕ_1 (which can be interpreted as the part of the total mass responding to the earthquake in the first mode), M_1 is the generalized mass corresponding to ϕ_1, $\mathbf{m}$ is the mass matrix, r is the displacement transformation vector, F_1 is the modal amplitude or participation factor associated with ϕ_1, $\phi_{1,roof}$ is the component of ϕ_1 corresponding to the top story, Δ_r is the roof displacement, and u is the displacement of the equivalent SDOF system.

The demand earthquake considered in the ATC-40 example is represented by the elastic design spectrum shown in Figure 4c. Two sets of seismic coefficients (ICBO, Ref. 10) were used to illustrate the effect of different soil profiles: C_A=0.4 and C_V=0.4, and C_A=0.44 and C_V=0.64, the latter the softer. These two cases will be dealt with in Examples 1 and 2 respectively. C_A represents the effective peak acceleration or design ground acceleration, i.e., is equivalent to A in this paper. C_V is the ordinate of the 5% damped elastic design spectrum at T=1 sec. If the amplification factor ψ_μ=1.74 for μ=1 for the velocity region (Table 1) is assumed, the relation $\omega_s C_V=\omega_s\psi_\mu V=2.5C_A$ holds at $T=T_s$ (Fig. 4c), with $T_s= C_V/2.5C_A$. For the first soil type A= C_A=0.4g, T_s= 0.4 sec and $\omega_s=2\pi/T_s$=15.7 rad/sec, then V=1g/(15.7·1.74)=35.9 cm/sec. For the softer soil profile A=C_A=0.44g, T_s=0.582 sec, ω_s=10.8 rad/sec, and V=1.1g/(10.8·1.74)=57.4 cm/sec.

Example 1:

a) First, the same elastic spectrum considered in ATC-40 will be used. The corresponding ground motion, A=0.4g and V= 35.9 cm/sec, is plotted in Figure 11d. The spectral regions of acceleration amplification (LM in Figure 3) and velocity amplification (KL in Figure 3) correspond in this case to 2.5A=1g and ψ_μV=1.74·35.9=62.4 cm/sec as shown in Figure 4d.
b) A bilinear model will be used. The model parameters must be selected to fit the capacity diagram (Figure 4b) up to the expected maximum response u_{max}, so that the total area under the bilinear

representation is equal to the area under the capacity diagram, i.e. equal energy is associated with both. In this case, a bilinear model with yield point Y with coordinates (0.342, 2.75) is chosen (Figure 4b).

c) The chosen model has an elastic frequency $\omega = \sqrt{k/m} = \sqrt{R_y/u_y m} = \sqrt{0.342g/2.75}$ =6.93 rad/sec, or T=2π/ω=0.907 sec (very close to the actual fundamental period of the building), or f=1.1 cps. The model is represented in Figure 4d by point Y with tripartite coordinates: (f=1.1, u_y=2.75"=6.98 cm, ωu_y=6.93·6.98=48.3 cm/sec).

d) The system is in the velocity region, and its ordinate features a reduction ϕ_μ=48.3/62.4 =0.774 with respect to the elastic spectrum. Using the relationship $\phi_\mu=(1.9\mu-0.9)^{-0.7}$ a ductility response μ=1.23 is obtained.

e) The maximum displacement of the equivalent system is $u_{max}=\mu u_y$=1.23·2.75=3.38"=8.6 cm. This result is in very good agreement with the maximum displacement of 3.4" calculated in the ATC-40 report. The observation can be made however that the inelastic spectrum procedure is considerably simpler than the ATC-40 method.

f) The last step of the solution is to go back to estimate the displacement of the roof of the building. This is done using the above relation between u and Δ_r. According to the ATC-40 report calculations $F_1\phi_{1,roof}$=1.31, therefore Δ_r=1.31u_{max} =4.43"=11.3 cm.

g) In order to have an indication of the uncertainty underlying estimated deformations and structural performance, it is recommended to consider a demand spectrum associated to a 0.159 probability of exceedance (response amplification associated to mean plus one standard deviation probability level). It is worth noting that for this verification the ground motion parameters that define the seismic hazard do not change, but the earthquake demand does change as a result of other ground motion characteristics that influence the response, like the power, frequency content, and the duration of motion. Then, amplification factors $\psi_\mu + \sigma_\mu$ shall be used. In this case, factors of 2.84 and 2.39 are obtained from Table 1 for the acceleration and velocity regions respectively. The ordinates of the demand spectrum become 2.84A=2.84·0.4g=1.136g and 2.39V=2.39·35.9=85.8 cm/sec, as shown by the dashed line K"L"M" in Figure 4d.

h) The system is in the velocity region and features a reduction ϕ_μ=48.3/85.8=0.563 from the elastic ordinate K"L". Using the relationship $\phi_\mu=(1.9\mu-0.9)^{-0.7}$ a ductility response μ=1.67 is obtained. The maximum displacement of the system, for a probability of exceedance of 0.16, is $u_{max}=\mu u_y$=1.67·2.75=4.6"=11.67 cm. Note that this displacement is 36% larger than that obtained in item (e) above.

i) Similarly to item (f) above, the roof displacement is Δ_r=1.31·11.67=15.3 cm. Finally it is of interest to note that the example building is rather robust (3000 kips capacity for a total weight of 10540 kips); therefore, it experiences a mild inelastic response (low ductility) for the earthquake demands considered.

Example 2:

a) This example will be solved analytically without the aid of a figure. The ground motion in this case is A=0.44g and V=57.4 cm/sec, as calculated above. The elastic spectrum ordinates are 2.5A=1.1g in the acceleration region and ψ_μV=1.74·57.4=99.9 cm/sec in velocity region.

b) The system is represented by the same point determined above for Example 1 (point Y), with tripartite coordinates: (f=1.1, u_y=2.75"=6.98 cm, ωu_y=6.93·6.98=48.3 cm/sec).

c) The system is in the velocity region and presents a reduction ϕ_μ=48.3/99.9=0.484 with respect to the elastic spectrum. Then, the corresponding ductility is μ=1.96.
d) The maximum displacement of the equivalent system is $u_{max}=\mu u_y$=1.96·2.75=5.4"=13.7 cm. In this case, the various ATC-40 procedures give maximum displacements from 5.5" to 6".
e) The ordinates of the mean plus one standard deviation spectrum are in this case 2.84A=1.25g in the acceleration region and 2.39V=2.39·57.4=137 cm/sec in velocity region. The system is in the velocity region and presents a reduction factor ϕ_μ=48.3/137=0.3526. The corresponding ductility response is μ=2.81, and the maximum displacement of the equivalent system is $u_{max}=\mu u_y$=2.81·2.75=7.72"=19.6 cm, i.e., 43% larger than the displacement obtained for the spectrum shown in Figure 4c.

CONCLUDING REMARKS

Performance-based seismic design requires explicit assessment of earthquake demands on structures, particularly inelastic deformations, to ensure they are within acceptable limits. Since methods for nonlinear response history analysis of multi-degree-of-freedom buildings have not reached yet a stage of development to permit generalized use, simple approaches based on the response of single-degree-of-freedom (SDOF) systems have been proposed. The inelastic response of SDOF systems subjected to earthquakes can be directly and reliably estimated by means of the traditional Newmark-Hall procedure. This method may be considered to have a number of advantages, as pointed out by Riddell et al. (Ref. 1).

New factors for constructing demand spectra in the Newmark-Hall format were obtained for two large ensembles of earthquake records: Circumpacific Belt and California. With regard to specific issues addressed in this study, the following conclusions were drawn: a) the effect of the type of force-deformation relationship on the average response of nonlinear systems is not significant, and responses can be conservatively predicted using the simple elastoplastic model, b) the well known "equal-displacement" and "equal-energy" rules to relate elastic and inelastic responses are, on the average, unconservative for systems with moderate to large ductility. Improved rules are recommended. An example shows that inelastic deformations of SDOF systems can be directly obtained from a demand spectrum without consideration of the sequence of equivalent linear systems required by the ATC-40 procedures.

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Table 1: Factors for constructing elastic and inelastic demand spectra for elastoplastic systems with 5% damping. Californian records.

Spectral Region	**Ductility μ**	ψ_μ	σ_μ	**COV**	ϕ_μ
Displacement	1.0	1.705	0.647	0.380	1.000
	1.5	1.051	0.387	0.368	0.617
	2.0	0.753	0.268	0.356	0.441
	3.0	0.490	0.188	0.383	0.288
	5.0	0.293	0.120	0.407	0.172
	10.0	0.142	0.060	0.421	0.084
Velocity	1.0	1.738	0.650	0.374	1.000
	1.5	1.078	0.341	0.317	0.620
	2.0	0.803	0.247	0.307	0.462
	3.0	0.551	0.170	0.308	0.317
	5.0	0.368	0.113	0.306	0.212
	10.0	0.222	0.066	0.295	0.128
Acceleration	1.0	2.159	0.682	0.316	1.000
	1.5	1.490	0.394	0.265	0.690
	2.0	1.227	0.279	0.227	0.568
	3.0	0.997	0.191	0.191	0.462
	5.0	0.813	0.139	0.171	0.377
	10.0	0.643	0.119	0.185	0.298

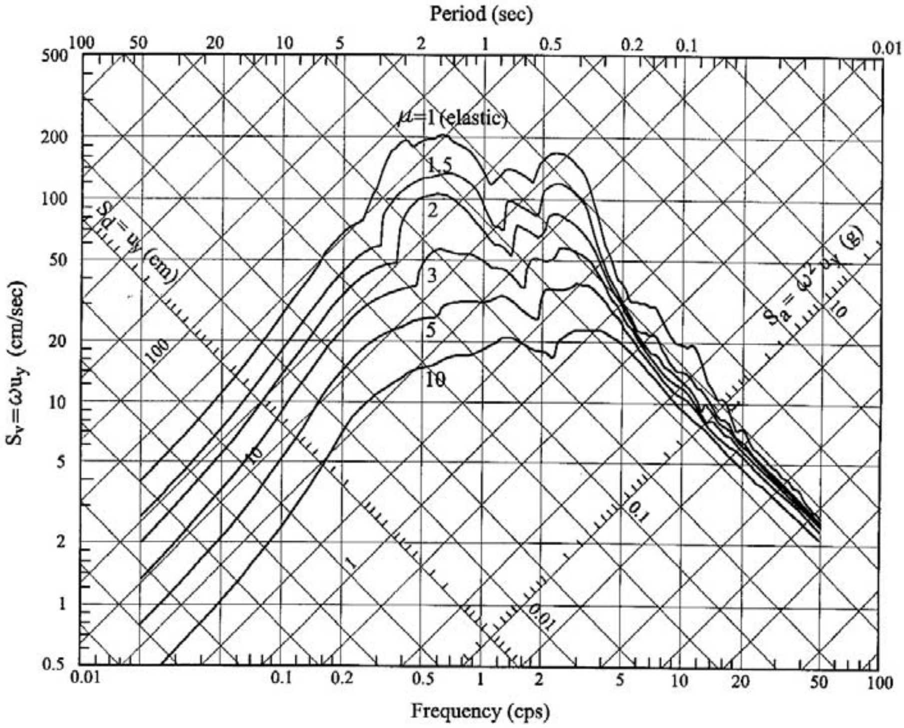

Figure 1. Inelastic Yield Spectra for stiffness-degrading systems with 5% damping subjected to the Sylmar, Northridge record, component N00E, January 17, 1994.

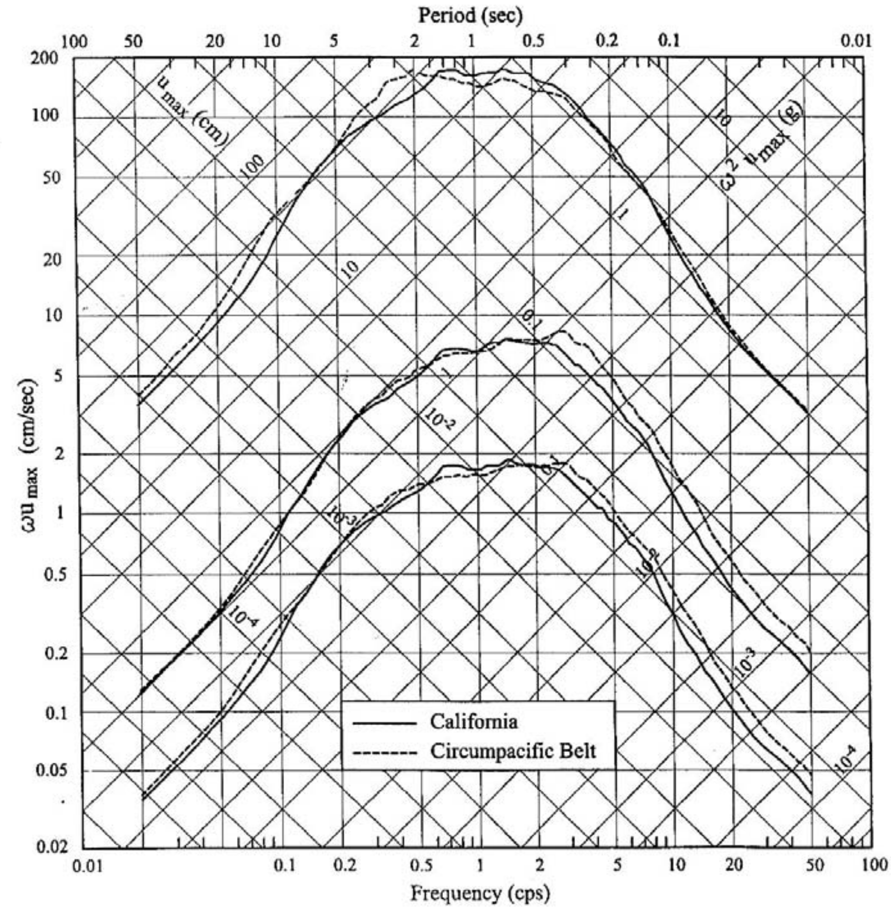

Figure 2. Average spectra normalized to peak ground acceleration, displacement, and velocity (from top to bottom), for elastic systems with 5% damping

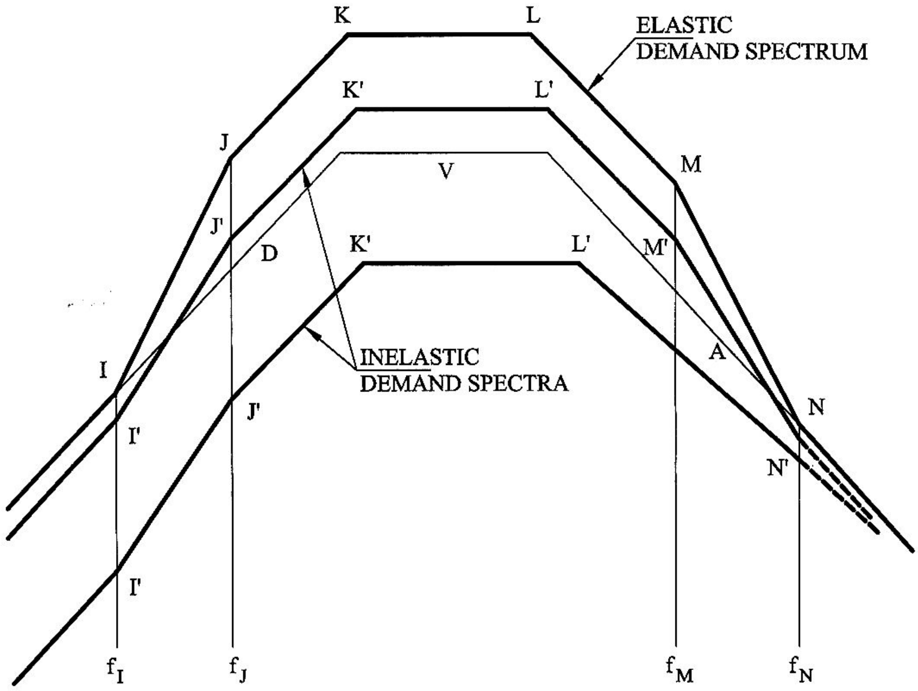

Figure 3. Construction of demand spectra

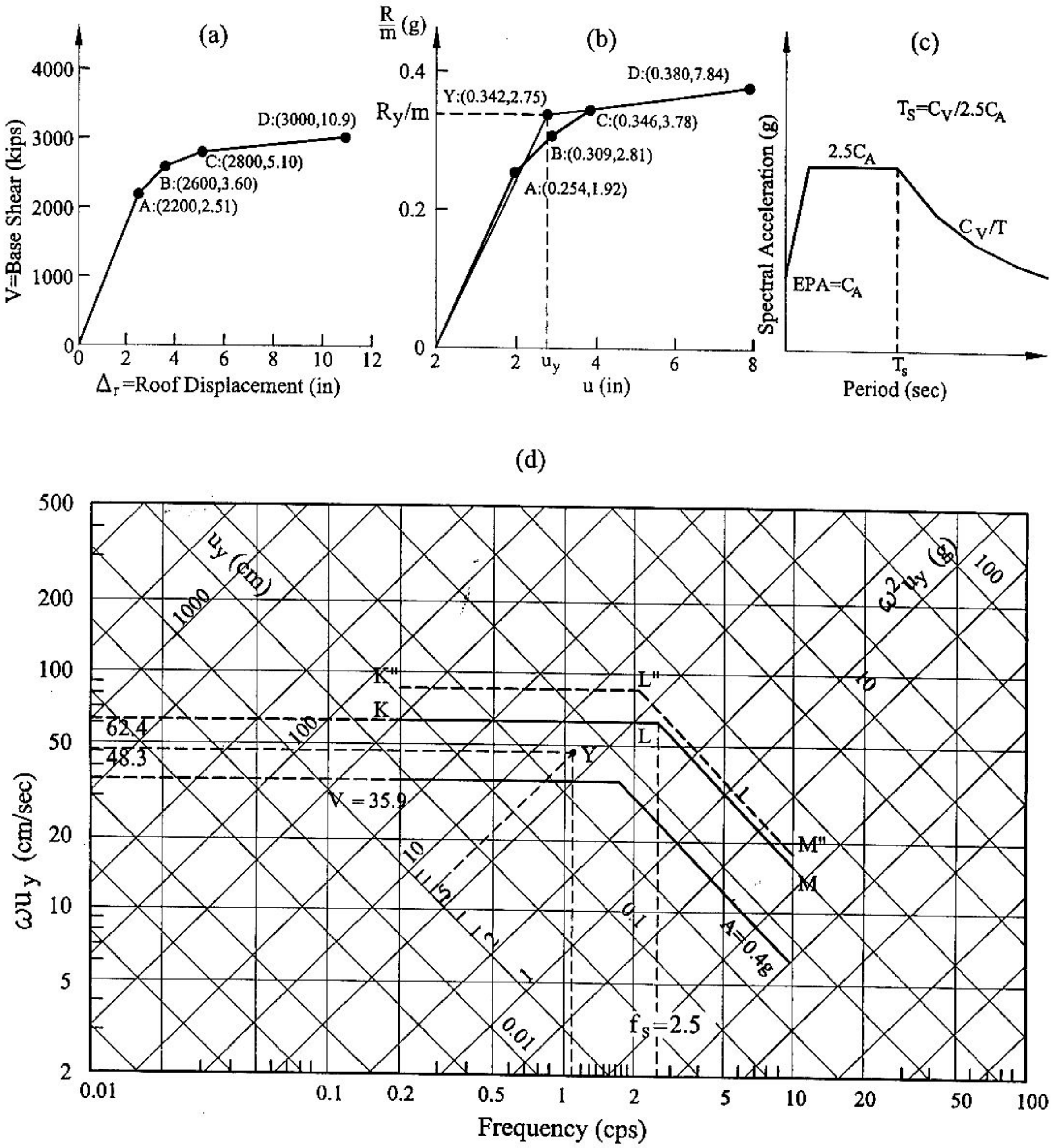

Figure 4. Example 1: a) Push-over curve, b) capacity diagram, c) demand earthquake, d)demand spectra