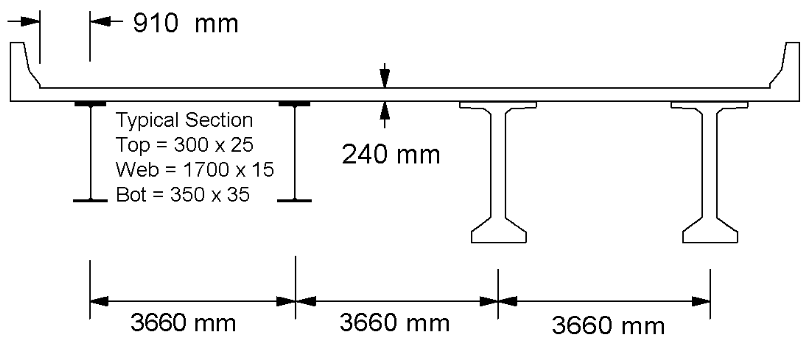
LECTURE 7 - SAMPLE CALCULATIONS FOR LOAD DISTRIBUTION

7.1 MULTI-GIRDER BRIDGE

CROSS-SECTION FOR STEEL OR PRESTRESSED
CONCRETE BRIDGE



AASHTO PRESTRESSED CONCRETE TYPE I-BEAM (28/96)

A = 8.2387E+5 mm^2 I = 6.3341E+11 mm^4 t_s = 240 mm

L = 43 000 mm s = 3660 mm n = 1.414

$$e_g = NA_{YT} + \frac{t_s}{2} \qquad \text{(S4.6.2.2.1)}$$

$$e_g = 1215 + \frac{240}{2} = 1335 \ mm$$

$$K_g = n\left(I + Ae_g^2\right) \qquad \text{(S4.6.2.2.1-1)}$$

$$K_g = 1.414\,(6.3341E11 + 8.2387E5(1335^2)) = 2.9723E12$$

$$\left(\frac{K_g}{Lt_s^3}\right)^{0.1} = \left(\frac{2.9723E12}{43\ 000(240)^3}\right)^{0.1} = 1.175$$

Table S4.6.2.2.2b-1

Moment in interior beam with two or more design lanes loaded

$$D_M = 0.075 + \left(\frac{s}{2900}\right)^{0.6}\left(\frac{s}{L}\right)^{0.2}\left(\frac{K_g}{Lt_s^3}\right)^{0.1}$$

$$D_M = 0.075 + \left(\frac{3660}{2900}\right)^{0.6}\left(\frac{3660}{43\ 000}\right)^{0.2}(1.175) = 0.900$$

Table S4.6.2.2.3a-1

Shear in interior beam with two or more design lanes loaded

$$D_v = 0.20 + \frac{s}{3600} - \left(\frac{s}{10\ 700}\right)^{2.0}$$

$$D_v = 0.20 + \frac{3660}{3600} - \left(\frac{3660}{10\ 700}\right)^{2.0} = 1.099$$

STEEL PLATE GIRDER

A = 4.525E+4 mm^2 I = 2.0557E+10 mm^4 t_s = 240 mm

L = 43000 mm   s = 3660 mm   n = 7

$$e_g = NA_{YT} + \frac{t_s}{2}$$

$$e_g = 967 + \frac{240}{2} = 1087\ mm$$

$$K_g = n\left(I + Ae_g^2\right)$$

$$K_g = 7\ (2.0557E10 + 4.525E4(1087^2)) = 5.1816E11$$

$$\left(\frac{K_g}{Lt_s^3}\right)^{0.1} = \left(\frac{5.1816E11}{43\ 000(240)^3}\right)^{0.1} = 0.986$$

Note that, with the same spacing and deck thickness, this term is significantly smaller for the less stiff steel girder than for the concrete girder. This will be reflected in a lower distribution factor for the steel girder.

Table S4.6.2.2.2b-1

Moment in interior beam with two or more design lanes loaded

$$D_M = 0.075 + \left(\frac{s}{2900}\right)^{0.6}\left(\frac{s}{L}\right)^{0.2}\left(\frac{K_g}{Lt_s^3}\right)^{0.1}$$

$$D_M = 0.075 + \left(\frac{3660}{2900}\right)^{0.6}\left(\frac{3660}{43\ 000}\right)^{0.2}(0.986) = 0.768$$

Table S4.6.2.2.3a-1

Shear in interior beam with two or more design lanes

$$D_v = 0.20 + \frac{s}{3600} - \left(\frac{s}{10\,700}\right)^{2.0}$$

$$D_v = 0.20 + \frac{3660}{3600} - \left(\frac{3660}{10\,700}\right)^{2.0} = 1.099$$

Table S4.6.2.2.2b-1

Moment in interior beam with one design lane loaded

$$D_M = 0.06 + \left(\frac{s}{4300}\right)^{0.4}\left(\frac{s}{L}\right)^{0.3}\left(\frac{K_g}{Lt_s^3}\right)^{0.1}$$

$$D_M = 0.06 + \left(\frac{3660}{4300}\right)^{0.4}\left(\frac{3660}{43\,000}\right)^{0.3}(0.986) = 0.501$$

For single-lane loading to be used for fatigue design, remove the multiple presence factor = 1.20

$$D_M = \frac{0.501}{1.2} = 0.418$$

Table S4.6.2.2.3a-1

Shear in interior beam with one design lane loaded

$$D_v = 0.36 + \frac{s}{7600}$$

$$D_v = 0.36 + \frac{3660}{7600} = 0.842 \qquad \text{(Strength)}$$

$$D_v = \frac{0.842}{1.2} = 0.701 \qquad \text{(Fatigue)}$$

STEEL PLATE GIRDER - EXTERIOR BEAM
(PRESTRESSED CONCRETE BEAM SIMILAR)

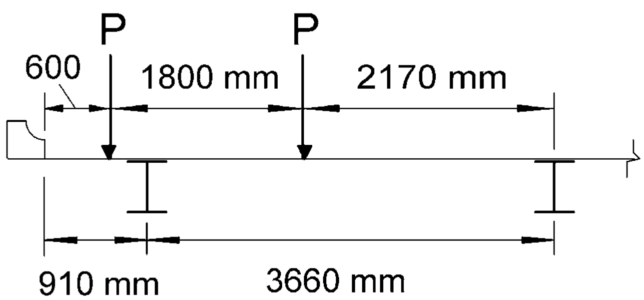


Table S4.6.2.2.2d-1

Moment in Exterior Girder with One Lane Loaded: Use Lever Rule

$$D_M = \frac{2170 + 1800 + 2170}{3660} = 1.678 \ \textit{Wheels} = 0.839 \ \textit{Lanes}$$

(Fatigue)

$$D_M = 0.839 \ x \ 1.20 = 1.0068$$ (Strength)

Moment in Exterior Girder with Two or More Lanes Loaded

$$e = 0.77 + \frac{d_e}{2800}$$

$$e = 0.77 + \frac{910}{2800} = 1.095$$

$$D_M = eD_{M \ interior}$$

$$D_M = 1.095(0.768) = 0.842$$

Table S4.6.2.2.3b-1

Shear in Exterior Girder with One Lane Loaded

Simple Beam Distribution (Lever Rule) Same as Moment

Shear in Exterior Girder with Two or More Lanes Loaded

$$e = 0.6 + \frac{d_e}{3000}$$

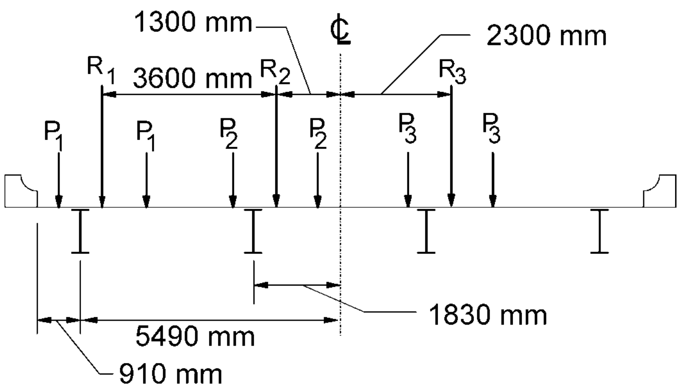
$$e = 0.6 + \frac{910}{3000} = 0.903$$

$$D_v = eD_{v\,interior}$$

$$D_v = 0.903(1.099) = 0.993$$

For girders with rigid cross-frames, an additional check needs to be performed on the exterior girders (S4.6.2.2.2d and S4.6.2.2.3b). The results from this check are applicable to both moment and shear distribution factors. The additional check on the exterior beam will be shown only for the steel girder cross-section. It would be the same for the concrete cross-section, if rigidly connected by cross-frames, because the geometry is identical in this example.

Additional Check for Rigidly Connected Girders



Multiple Presence Factors:

$M_1 = 1.20$ $M_2 = 1.00$ $M_3 = 0.85$ (Table S3.6.1.1.2-1)

One Lane Loaded

$$R = \frac{N_L}{N_B} + \frac{X_{ext}\Sigma_e}{\Sigma x^2} \qquad \text{(SC4.6.2.2.2d-1)}$$

$$R = \frac{1}{4} + \frac{5490(4900)}{(2(5490^2 + 1830^2))} = 0.652 \qquad \text{(Fatigue)}$$

$$D_{M1} = M_1 R = 1.20(.652) = 0.782 \qquad \text{(Strength)}$$

Two Lanes Loaded

$$R = \frac{N_L}{N_B} + \frac{X_{ext}\Sigma_e}{\Sigma x^2} \qquad \text{(SC4.6.2.2.2d-1)}$$

$$R = \frac{2}{4} + \frac{5490(4900 + 1300)}{2(5490^2 + 1830^2)} = 1.008$$

$$D_{M2} = M_2 R = 1.0(1.008) = 1.008$$

Three Lanes Loaded

$$R = \frac{N_L}{N_B} + \frac{\Sigma_{ext}\Sigma_e}{\Sigma x^2} \qquad \text{(SC4.6.2.2.2d-1)}$$

$$R = \frac{3}{4} + \frac{5490(4900 + 1300 - 2300)}{2(5490^2 + 1830^2)} = 1.070$$

$$D_{M3} = M_3 R = 0.85(1.070) = 0.909$$

D_{M2} = 1.008 controls for strength design

Summary of Live load distribution factor for the cross-section with steel girders

Strength limit state:

	Load case	Moment interior beams	Moment exterior beams	Shear interior beams	Shear exterior beam
Distribution factors from Tables in Article S4.6.2.2.2	Multiple lanes loaded	0.768	0.842	1.099	0.993
	Single lane loaded	0.501	1.0068	0.842	1.0068
Additional check for girders with rigid cross-frames	Multiple lanes loaded	N/A	1.008	N/A	1.008
	Single lane loaded	N/A	0.782	N/A	0.782
Design value		0.768	1.008	1.099	1.008*

* It is allowed to have a design live load distribution factor for the exterior girder smaller than that for the interior girder. However, the total factored load (DL + LL + ...etc.) for exterior girder should not be less than that of interior girders except when it is not possible to widen the bridge in the future.

Fatigue limit state:

	Load case	Moment interior beams	Moment exterior beams	Shear interior beams	Shear exterior beam
Distribution factors from Tables in Article S4.6.2.2.2	Multiple lanes loaded	N/A	N/A	N/A	N/A
	Single lane loaded	0.418	0.839	0.701	0.839
Additional check for girders with rigid cross-frames	Multiple lanes loaded	N/A	N/A	N/A	N/A
	Single lane loaded	N/A	0.652	N/A	0.652
Design value		0.418	0.839	0.701	0.839

For this example, the exterior beam has a live load distribution factor for moment which is 31% higher than an interior beam. If the spacing of the beams was increased by 400 mm, the exterior beam would carry about 16% more live load. The exterior beam must not have less capacity than an interior beam. Whether it has more total capacity will also depend on the magnitude of the dead load moment.

The application of the distribution factors, calculated above, to the live load shear and moment envelopes previously calculated two-span girder (43 m - 43 m) is illustrated below. The distribution factors used are those for the interior steel girder tabulated above. The first table below repeats the undistributed, unfactored shears and moments per lane calculated previously for one span. The dynamic load allowance has been included. The second table shows the applicable factors and the extended factored and distributed shears and moments per girder.

	M O M E N T - KN.m				S H E A R			
DIST	+FATG	+STREN	-FATG	-STREN	+FATG	+STREN	-FATG	-STREN
0	0	0	0	0	315	573	-34	-66
4.3	1160	2147	-145	-282	270	482	-34	-68
8.6	1941	3670	-290	-564	226	397	-46	-107
12.9	2430	4600	-435	-846	183	318	-80	-172
17.2	2591	5010	-579	-1128	144	247	-125	-240
21.5	2524	4924	-724	-1410	107	183	-168	-311
25.8	2292	4389	-869	-1692	73	127	-209	-383
30.1	1802	3404	-1014	-1974	44	80	-248	-456
34.4	1111	2038	-1159	-2458	24	41	-283	-528
38.7	396	695	-1304	-3127	10	16	-315	-598
43	0	0	-1448	-5064	0	0	-342	-666

	MOMENT - KN.m				SHEAR - KN			
DIST	+FATG	+STREN	-FATG	-STREN	+FATG	+STREN	-FATG	-STREN
L. F.	0.75	1.75	0.75	1.75	0.75	1.75	0.75	1.75
D. F.	0.418	0.768	0.418	0.768	0.701	1.099	0.701	1.099
0	0	0	0	0	166	1101	-18	-126
4.3	364	2886	-46	-379	142	926	-18	-131
8.6	609	4932	-91	-758	119	763	-24	-205
12.9	762	6182	-136	-1137	96	612	-42	-330
17.2	812	6733	-182	-1516	75	475	-65	-462
21.5	791	6618	-227	-1895	56	352	-88	-598
25.8	719	5899	-272	-2274	38	245	-110	-737
30.1	565	4575	-318	-2653	23	153	-130	-876
34.4	348	2739	-363	-3304	12	78	-149	-1015
38.7	124	934	-409	-4203	5	30	-165	-1150
43	0	0	-454	-6806	0	0	-180	-1280

The following figures, all applicable to the case of two or more lanes loaded, show the application of the load factors and the distribution factors calculated above to the shears and moments calculated previously for the two-span unit with 43 m spans. The vertical axis is the ratio of results obtained with the LRFD Specification to the corresponding result obtained using the 15th edition of the Standard Specifications.

Considering Figure 1, the impact of the combined truck and lane loads of the HL93 loading in the LRFD Specification, compared to the 15th edition, is clearly indicated by the bars indicating the raw data. Most of these bars at the various ten points show approximately 1.6 times the moment produced by the 15th edition. Once the results are factored by the load factor 1.75 in the LRFD Specification and 2.17 in the 15th edition, the results are somewhat closer with typical ratios of about 1.3. Use of the more refined distribution factor for the example cited brings the total factored and distributed moments, including the dynamic load allowance, to approximately the same level in the two specifications. As will be shown below, this close comparison is very dependent on the individual structure.

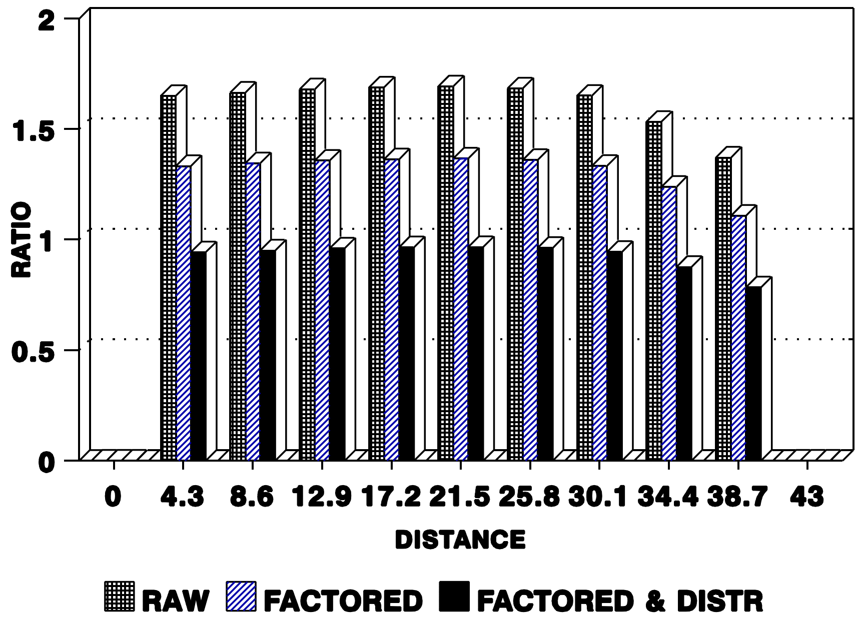


Figure 1 - LRFD/15th Edition - Steel Positive LL Moment

Figure 2 shows the same results for negative moment, with the spike in raw data at the 8/10 point indicating the place where the two trucks in the HL93 loading began to be applied for negative moment between the points of contraflexure.

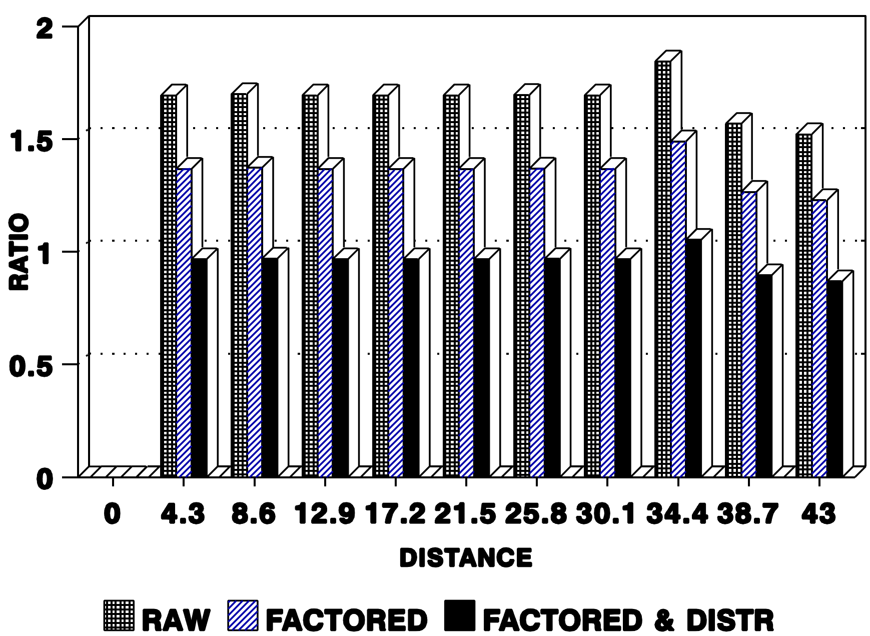


Figure 2 - LRFD/15th Edition - Steel Negative LL Moment

Figures 3 and 4 present information for the steel example similar to that shown in Figures 1 and 2, except that they apply to shear rather than bending moment. The distribution factor for shear is relatively larger than that for bending moment in the example shown. The effect of this can be clearly seen in the bar charts. In the case of shear in this example, there is almost no difference between the ratios of the distributed results obtained in the new and the old specifications once the load factor is applied, i.e., the distribution factors were essentially the same. Thus, there is very little difference between two of each of the three bar clusters.

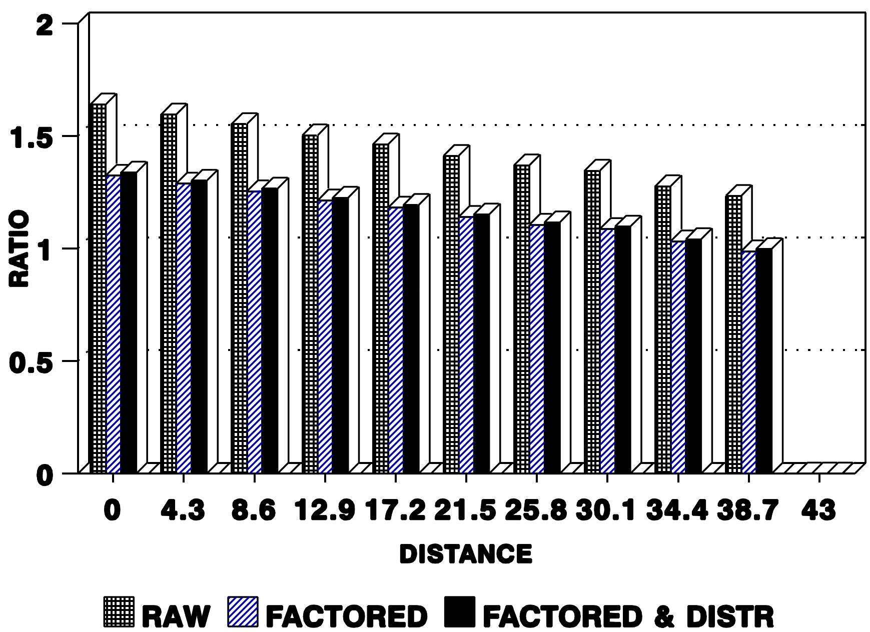


Figure 3 - LRFD/15th Edition - Steel Positive LL Shear

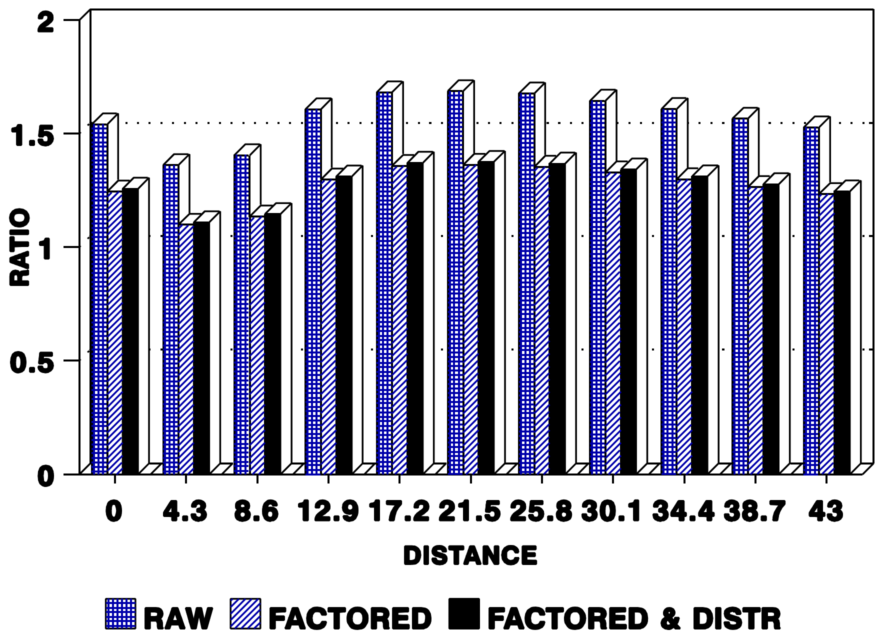


Figure 4 - LRFD/15th Edition - Steel Negative LL Shear

Figure 5 shows comparable results obtained for the prestress concrete beam example. Considering the strength limit state, the load factors 1.75 for the LRFD Specification and 2.17 for the 15th edition are again applied to raw data, which is identical for both the steel and the concrete design in this example. However, once the distribution factor is applied, there is a clear difference between the results for the steel and prestress concrete girders in the case under consideration. This difference is not obtained using the 15th edition, but would be observed, at least qualitatively, using a refined analysis such as a grid analogy or a finite element analysis. The difference is solely derived from the higher distribution factor for the prestressed concrete girder.

Figure 5 can also be used to draw conclusions about the service limit state for tension in prestress concrete beams. In this case, a load factor of 0.8 is used. As it happens, this is almost exactly the ratio between the load factors and the LRFD Specification and the 15th edition for the strength limit states. Thus, the relative position of the raw and factored data, given as ratios in Figure 5, is the same for both the strength and service limit states for the prestressed concrete beam example cited herein. After applying the load factors, the results obtained for the criteria of tension in the bottom of a prestressed concrete beam should be quite similar for the LRFD Specification and the 15th edition.

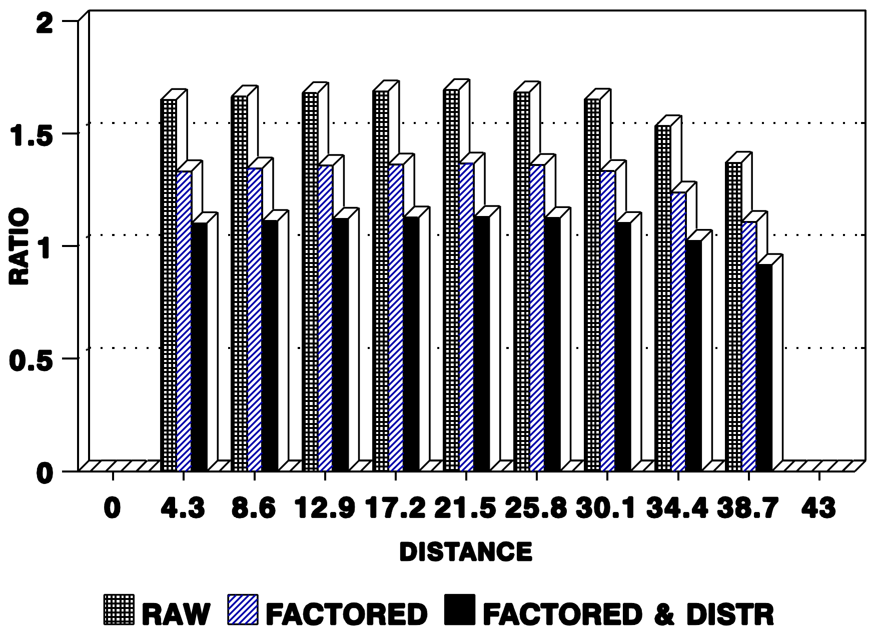


Figure 5 - LRFD/15th Edition - Concrete Positive LL Moment

7.2 LIVE LOAD DISTRIBUTION FACTOR FOR A TRUSS

Considering the four-lane deck truss shown in Figure 6, the maximum load transmitted to the left truss from any traffic lane is produced when the loads on the lane under consideration are positioned in their extreme left position.

Considering loads on Lane #1 (see Figure 7):

F_1 = 13.1/13.0 = 1.0077

where:

F_i: the fraction of the total load on the i^{th} traffic lane transmitted to the left truss

Considering loads on Lane #2:

F_2 = 9.5/13.0 = 0.7308

Considering loads on Lane #3:

F_3 = 5.9/13.0 = 0.4538

Considering loads on Lane #4:

F_4 = 2.3/13.0 = 0.1769

Live Load Distribution Factors

a. Case of a single traffic lane loaded

Maximum load in the left truss caused by loads on a single lane is produced by loading Lane #1. Multiple presence factor for a single-lane loading is 1.2 (Article S3.6.1.1.2).

Distribution factor = 1.0077 x 1.2 = 1.2092

b. Case of two traffic lanes loaded

Maximum load in the left truss caused by loads on two lanes is produced by loading Lane #1 and Lane #2. Multiple presence factor for a two-lane loading is 1.0 (Article S3.6.1.1.2).

Distribution factor = (1.0077 + 0.7308) x 1.0 = 1.7385

c. Case of three traffic lanes loaded

Maximum load in the left truss caused by loads on three lanes is produced by loading Lanes #1 through 3. Multiple presence factor for a three-lane loading is 0.85 (Article S3.6.1.1.2).

Distribution factor = (1.0077 + 0.7308 + 0.4538) x 0.85 = 1.8635

d. Case of four traffic lanes loaded

In this case, all lanes are loaded and the loads in each lane are positioned in their extreme left position. Multiple presence factor for a four-lane loading is 0.65 (Article S3.6.1.1.2).

Distribution factor = (1.0077 + 0.7308 + 0.4538 + 0.1769) x 0.65 = 1.54

The distribution factor to be used in the analysis of the truss is the maximum value obtained from Cases a through d = 1.8635

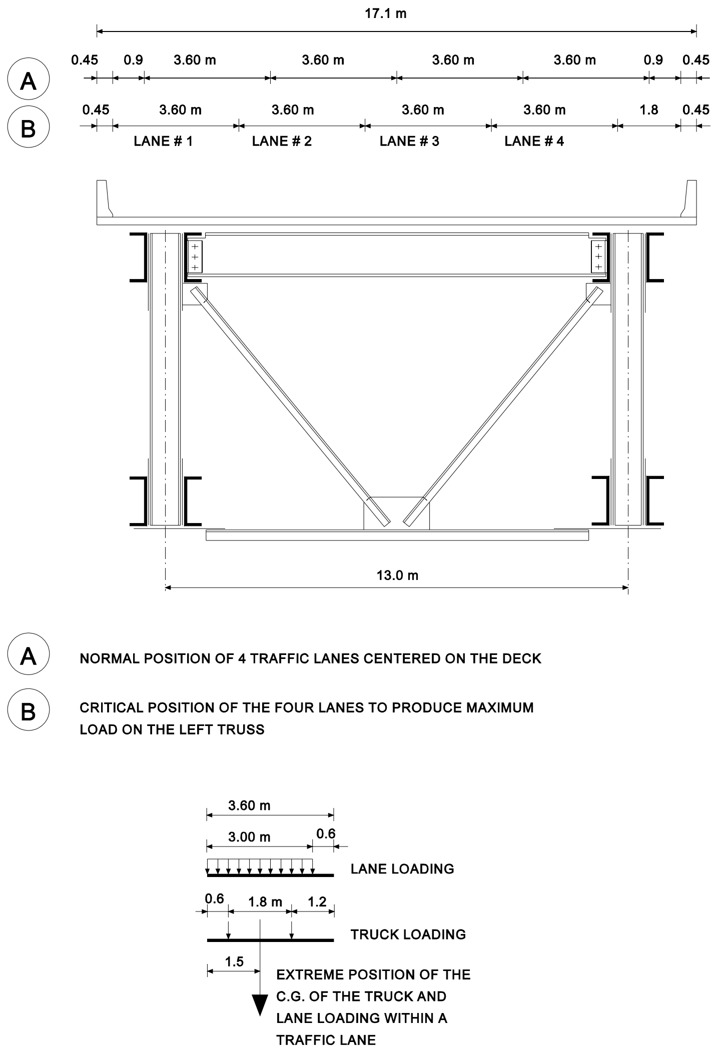


Figure 6 - Geometry of Example Problem

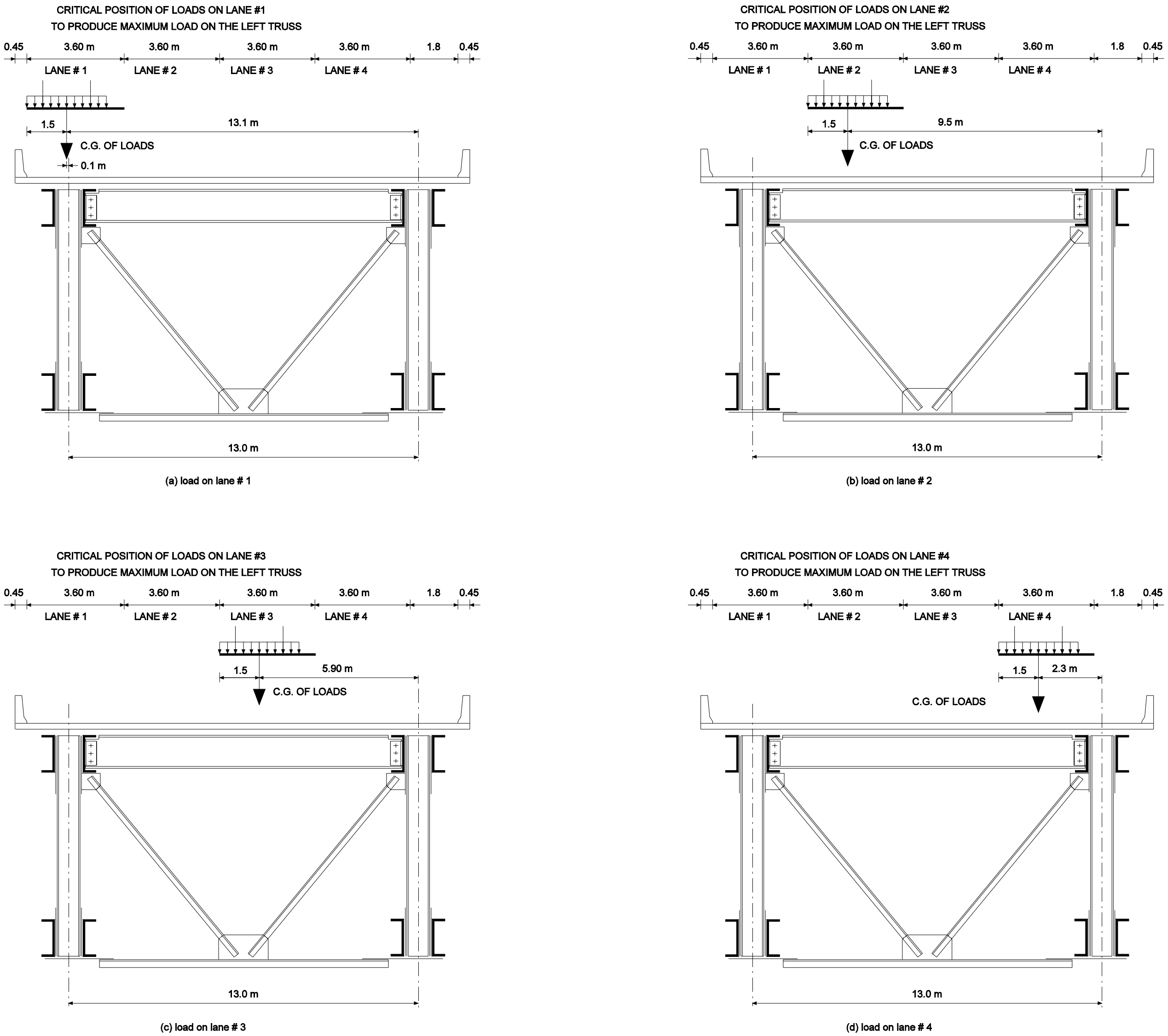


Figure 7 - Critical Positioning of Load for each Lane

LECTURE 8 - ANALYSIS II

8.1 OBJECTIVE OF THE LESSON

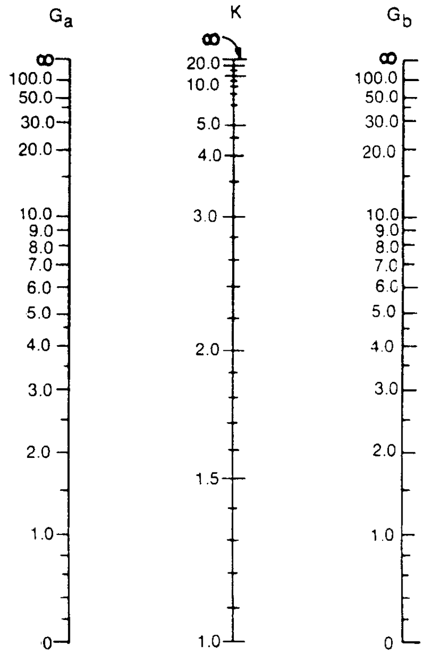
The objective of this lesson is to acquaint the student with:

- effective length provisions,
- effective flange width provisions,
- the requirements for earthquake design of bridges, and
- the analysis methods for earthquake effects.

8.2 EFFECTIVE LENGTH FACTOR

The effective length factor, K, is used to adjust the physical length of a column to account for the boundary conditions at the ends of the column when those boundary conditions are not pinned. LRFD Specification Article 4.6.2.5 permits the continued use of K = 0.75 for bolted and welded end connections for trusses, and K = 0.875 for pin-connected trusses. Similarly, there are effective length factors, suggested in the commentary, for other idealized end conditions.

Clearly the actual effective length of a column, or perhaps more precisely its buckling load, as part of an assemblage of beams and columns is far more complicated than can be conveyed by the simple idealizations. In order to more accurately determine the buckling strength of a member, in a framework, it is possible to do a rigorous non-linear analysis using computer programs which take second order effects into account. This will generally only be worthwhile when relatively large members are involved. Simple approximation for the effect of a framework on the boundary conditions of an individual column are given by the so called "alignment chart" reproduced in Figure 8.2-1.



SIDESWAY PERMITTED

FOR COLUMN ENDS SUPPORTED BY BUT NOT RIGIDLY CONNECTED TO A FOOTING OR FOUNDATION, G IS THEORETICALLY EQUAL TO INFINITY, BUT UNLESS ACTUALLY DESIGNED AS A TRUE FRICTIONLESS PIN, MAY BE TAKEN EQUAL TO 10 FOR PRACTICAL DESIGN. IF THE COLUMN END IS RIGIDLY ATTACHED TO A PROPERLY DESIGNED FOOTING, G MAY BE TAKEN EQUAL TO 1.0. SMALLER VALUES MAY BE TAKEN IF JUSTIFIED BY ANALYSIS.

IN COMPUTING EFFECTIVE LENGTH FACTORS FOR MONOLITHIC CONNECTIONS, IT IS IMPORTANT TO PROPERLY EVALUATE THE DEGREE OF FIXITY IN THE FOUNDATION. THE FOLLOWING VALUES CAN BE USED:

G_a	
1.5	FOOTING ANCHORED ON ROCK
3.0	FOOTING NOT ANCHORED ON ROCK
5.0	FOOTING ON SOIL
1.0	FOOTING ON MULTIPLE ROWS OF END BEARING PILES

Figure 8.2-1 - Alignment Chart

The alignment chart contains a factor "G" which is the sum of the I/L ratios for the columns that are joined divided by the sum of the ratios for the beams or girders. The alignment chart contains implicitly the assumption that all of the columns and the framework buckled simultaneously.

The alignment chart also contains the assumption that the mid-points of the supporting members are pinned end and free to translate. A variety of references are provided in the commentary to Article S4.6.2.5 to provide information on how to alter the alignment chart results for nonstandard boundary conditions. It is also possible to adjust the stiffness ratio factor, "G", in order to account for the softening of the columns as they approach a buckling load. This effectively lowers the I/L ratio for the columns, thereby decreasing the value of "G" and lowering the effective length factor.

8.3 EFFECTIVE FLANGE WIDTH

The articles on effective flange width provide information for the following cases:

- The general situation, as typified by a steel or concrete girder, or the deck slab
- Segmental box girders and other single-cell construction
- The ribs of orthotropic decks
- The width of an orthotropic deck, participating with the main girder web
- The distribution of a normal force into a deck

The provisions of this article are basically those contained in the 1993 Standard Specification for Highway Bridges, the AASHTO Guide Specification for Segmental Box Girder Construction, and literature on orthotropic plate and box girder design.

The provisions for segmental box girder design and for orthotropic plate, acting with the webs of main beams, contained provisions for the variation of effective flange width along the span of the girder.

8.4 OVERVIEW OF EARTHQUAKE EFFECTS

8.4.1 Background Information on the Development of the Seismic Specifications

Prior to 1971, the AASHTO Specifications for seismic design of bridges were based in part on the lateral force requirements for buildings developed by the Structural Engineering Association of California. The 1971 San Fernando earthquake was a major turning point in the development of seismic design criteria for bridges, and several developments thereafter led to the current Specifications.

- In 1973, the California Department of Transportation introduced seismic design criteria for bridges, which included the relationship of the site to active faults, the seismic response of the soils at the site and the dynamic response characteristics of the bridge.
- In 1975, AASHTO adopted Interim Specifications which were a slightly modified version of the 1973 CALTRANS provisions, and made them applicable to all regions of the United States.

- In 1979, a "Workshop on Earthquake Resistance of Highway Bridges" was conducted by the Applied Technology Council (ATC) in San Diego. The workshop considered current state-of-the-art and practice, problem areas in seismic design, and current research efforts and findings. Its objective was to facilitate the development of new and improved seismic design standards for highway bridges.

- In 1981, the Applied Technology Council published "Seismic Design Guidelines for Highway Bridges" (Report ATC-6) as a state-of-the-art document on practices for the seismic design of bridges.

- In 1983, AASHTO adopted the ATC-6 Report as an approved alternate Guide Specification for Seismic Design of Highway Bridges, which in 1991 became Standard Specifications for Seismic Design of Highway Bridges and was included as Supplement A.

The current seismic specifications are based on Supplement A of the 1992 Standard Specifications for Highway Bridges. As research work in this area continues, future updates of these Specifications may be expected.

8.4.2 General Provisions

8.4.2.1 OBJECTIVE AND PRINCIPLES

The main objective of the specifications for earthquake design of bridges is to establish design and detailing provisions to minimize the susceptibility of bridges to damage from earthquakes. Some degree of damage caused by earthquakes is allowed, but the bridge should have a low probability of collapse.

The principles used to develop the seismic specifications are:

- Design earthquake motions and forces are realistic and based on a low probability of being exceeded during the normal life expectancy of a bridge, i.e, about 10% probability in 50 years.

- Small to moderate earthquakes should be resisted within the elastic range without significant damage.

- Large earthquakes should not cause collapse of all or part of the bridge.

- Where possible, earthquake damage should be readily detectable and accessible for inspection and repair.

8.4.2.2 APPLICABILITY

The provisions apply to bridges of conventional slab, beam girder, box girder and truss superstructure construction with spans not exceeding 150 m. For other types of construction or bridges with spans exceeding 150 m, appropriate provisions shall be specified and/or approved by the Owner.

Unless otherwise specified by the Owner, the provisions need not be applied to the following types of structures, unless they cross an active fault:

- Buried Structures
- Box Culverts

The potential for soil liquefaction and slope movements needs to be considered.

8.4.2.3 PRELIMINARY PLANNING AND DESIGN

It is important to consider the seismic hazard as early as possible in the planning process of a new bridge. The following steps should be considered:

- Factors such as the seismicity of the site, the proximity to an active fault and the soil conditions must be taken into account when selecting the type of bridge and the materials used. In areas close to faults or on unstable soil conditions, a structure type that can allow for larger deformations should be preferred.
- Redundancy and ductility are characteristics that can significantly enhance the seismic bridge performance, and should be included as criteria early in the preliminary design stage.
- Simplicity, symmetry and uniformity along the spans are desirable characteristics.
- Locations where damage is expected to occur should be easy to inspect and repair.

8.4.2.4 FLOW CHART FOR SEISMIC DESIGN

The flow chart in Figure 8.4.2.4-1 summarizes the steps involved in the design of bridge components for earthquake loads.

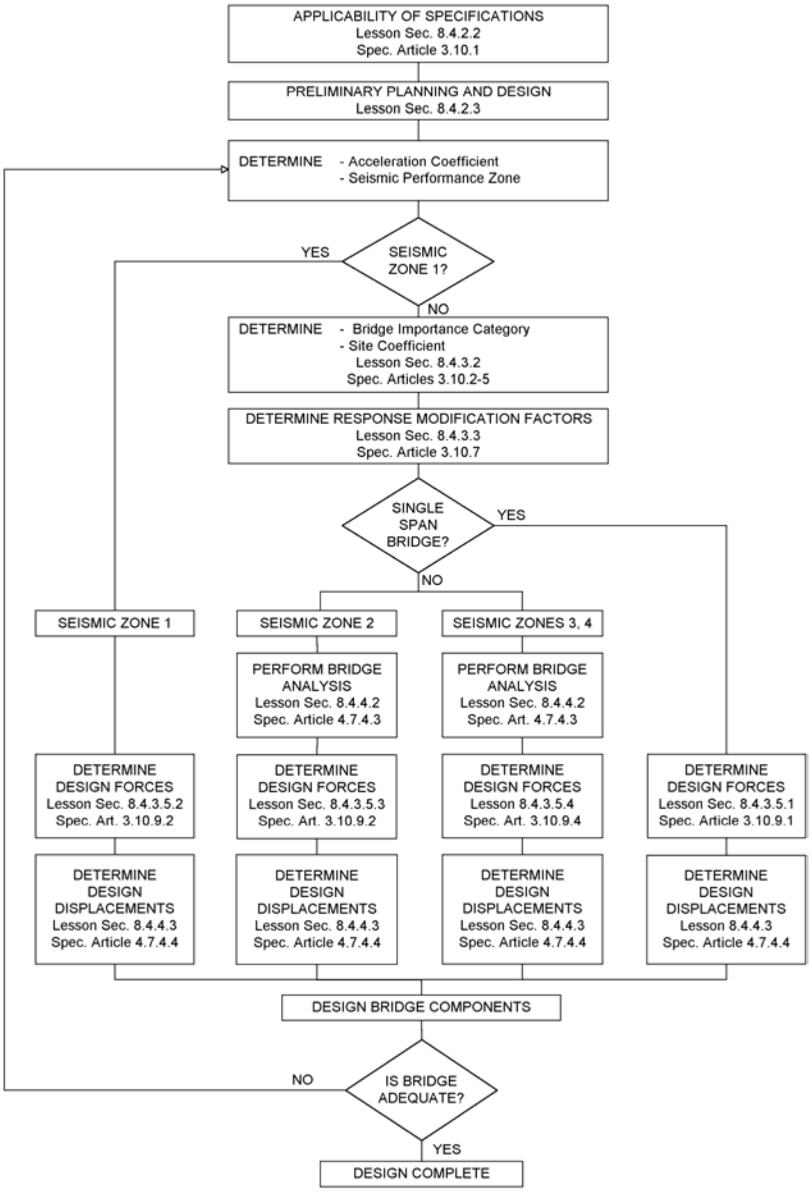


Figure 8.4.2.4-1 - Design Procedure Flow Chart

8.4.3 Earthquake Design Loads (Article 3.10)

8.4.3.1 ELASTIC SEISMIC RESPONSE COEFFICIENT

Earthquake design loads are horizontal forces given by the product between the elastic seismic response coefficient, C_{sm}, defined in Equation 8.4.3.1-1 below, and the equivalent weight of the superstructure.

$$C_{sm} = \frac{1.2AS}{T_m^{2/3}} \leq 2.5A \qquad (8.4.3.1\text{-}1)$$

where:

T_m = period of vibration in the m^{th} mode (SEC)

A = acceleration coefficient (see Section 8.4.3.2.1)

S = site coefficient (see Section 8.4.3.2.4)

The elastic response coefficient may be normalized with respect to the acceleration coefficient and the result plotted against the period of vibration, for different soil profiles based on 5% damping, as shown in Figure 8.4.3.1-1 below:

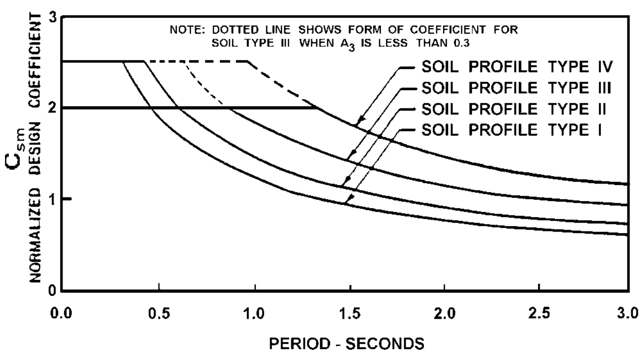


Figure 8.4.3.1-1 - Seismic Response Coefficients for Various Soil Profiles, Normalized with Respect to Acceleration Coefficient "A"

Since an earthquake may excite several modes of vibration in a bridge, the elastic response coefficient needs to be found for each relevant mode.

The equivalent weight, which is used to calculate the design forces corresponding to the values determined for the elastic response coefficient C_{sm}, is a function of the actual weight and bridge configuration. For simple spans, the equivalent weight may be determined using the guidelines in Section 8.4.3.5.2. When the analysis methods of earthquake effects, described in Section 8.4.4, are used, the equivalent weight is automatically included.

The only exceptions to the use of Equation 8.4.3.1-1, for determining the elastic response coefficient, are in the following three cases:

- Bridges on Soil Profiles III or IV, discussed in Section 8.4.3.2.4, and in areas where the coefficient "A" of Section 8.4.3.2.1 is not less than 0.30, i.e., A > 0.30, in this case, C_{sm} need not exceed 2.0A, i.e., C_{sm} # 2.0A.

- Bridges on Soil Profiles III or IV, and for modes other than the fundamental mode which have periods less than 0.3 SEC, T_m < 0.3 SEC, in this case, the coefficient C_{sm} should be calculated from the following equation:

$$C_{sm} = A(0.8 + 4.0\,T_m) \qquad (8.4.3.1\text{-}2)$$

- The period of vibration for any mode exceeds 4.0 SEC, (T_m > 4.0 SEC for all relevant values of "m"), in this case, the coefficient C_{sm} is calculated from Equation 3 below:

$$C_{sm} = 3ASTm^{0.75} \qquad (8.4.3.1\text{-}3)$$

8.4.3.2 FACTORS AFFECTING SEISMIC LOADS

The main factors that have to be accounted for in the calculation of earthquake design forces on bridges are reviewed in this section.

8.4.3.2.1 Acceleration Coefficient

Contour Lines are used for determining the acceleration coefficient "A" used in Equation 8.4.3.1-1. The acceleration coefficient contour maps, provided in the Specifications as Figures S3.10.2-1, S3.10.2-2 and S3.10.2-3, were prepared by the U. S. Geological Survey for different areas of the United States. (A copy of these maps is included in Appendix A.)

The following guidelines for using the contour lines are provided:

- The numbers given on contour maps are expressed in percent - the corresponding numerical values of the acceleration coefficient "A" are obtained by dividing contour values by 100.

- Local maxima and minima are given by the contour maps inside the highest and lowest contour line, respectively, for a particular region.

- Linear interpolation shall be used for sites between contour lines or between a contour line and a local maximum or minimum.

The seismic hazard at a site is reflected by the value of the contour line at that site. The contour maps rely on a uniform risk model of seismic hazard that assumes that there is a probability of 90% that the acceleration coefficient given by the map at a given location will not be exceeded during a 50-year period. Statistical analysis shows that an event with a 90% non-exceedance during a 50-year interval has a return period of about 475 years. Thus, the design earthquake is defined as an earthquake with a return period of about 475 years. Some jurisdictions use a maximum probable earthquake, which is defined as an earthquake with a return period of about 2,500 years, sometimes more.

Special studies to determine site- and structure-specific acceleration coefficients shall be performed by a qualified professional if any one of the following conditions exist:

- The site is located close to an active fault.
- Long duration earthquakes are expected in the region.
- The importance of the bridge is such that a longer exposure period (and, therefore, return period) should be considered.

The effect of soil conditions at the site are considered in Article 8.4.3.2.2.

8.4.3.2.2 Seismic Zones

Four different levels of seismic zones are defined according to the intensity ranges of the corresponding acceleration coefficients, as shown in Table 8.4.3.2.2-1 below.

Table 8.4.3.2.2-1 - Seismic Zones

Acceleration Coefficient	Seismic Zone
A # 0.09	1
0.09 < A # 0.19	2
0.19 < A # 0.29	3
0.29 < A	4

The reason for defining different seismic zones is to permit different requirements for method of analysis, minimum support lengths, column design details and foundation and abutment design procedures, so that such requirements are compatible with the corresponding variations in seismic risk across the country. This table eliminated the need for "Seismic Performance Categories" in the Standard Specification, Division 1A. In the LRFD Specification, the level of analysis is tied to the importance and the geometry of the bridge.

8.4.3.2.3 Bridge Importance Categories

The requirements for seismic design depend on the importance of the bridge. The bridge Owner or those having jurisdiction must first classify the bridge into one of the following three categories:

- Critical Bridges: Bridges that must remain open to all traffic after the "design earthquake" and be usable by emergency vehicles and for security/defense purposes immediately after a "maximum probable earthquake", which has a return period of 2,500 years.

- Essential Bridges: These bridges should be open to emergency vehicles and for security/defense purposes immediately after the "design earthquake", which has a return period of 475 years.

- Other Bridges

8.4.3.2.4 Site Effects

8.4.3.2.4a Site Coefficient

The effect of the soil conditions at the site of a bridge on structural response to earthquakes is accounted for through the use of the site coefficient, S. The site coefficient is included in Equation 8.4.3.1-1 for the calculation of the elastic seismic response coefficient, C_{sm}. It is used to modify the acceleration coefficient.

The site coefficient may take one of four different numerical values, depending on the soil profile type at the site of the bridge, as shown in Table 8.4.3.2.4a-1 below:

Table 8.4.3.2.4a-1 - Site Coefficients

Site Coefficient	Soil Profile Type			
	I	II	III	IV
S	1.0	1.2	1.5	2.0

In locations where the soil profile type cannot be determined because of insufficient knowledge on the soil properties, or it does not fit any of the four types described below, the site coefficient for Soil Profile Type II shall be used.

8.4.3.2.4b Soil Profile Types

Four different types of soil profiles are used to represent the different subsurface conditions that can exist at the site of a bridge. The subsurface conditions associated with each soil profile were selected on the basis of a statistical study of spectral shapes developed on such soils close to seismic source zones in past earthquakes. The soil conditions corresponding to each soil profile type are described in Table 8.4.3.2.4b-1.

It can be seen that softer soils have higher site coefficients due to the increased amplification of the ground motion as it travels through the soft material. This phenomenon is similar to resonance. Generally, the greater the difference between the speed of the earthquake shock wave in the rock compared to the soil, the greater the magnification. This characteristic helps to explain the pockets of damage in soft soil or fill areas observed in the Loma Prieta and Mexico City earthquakes.

Table 8.4.3.2.4b - Classification of Soil Types

SOIL PROFILE TYPE	CHARACTERISTICS	SITE COEFFICIENT
I	• Rock of any description, either shale-like or crystalline in nature, or • Stiff soils where the soil depth is less than 60 m, and the soil types over-lying the rock are stable deposits of sand, gravel or stiff clays, or • Soils that may be characterized by a shear wave velocity that is greater than 750 m/SEC.	1.0
II	• Stiff cohesive or deep cohesionless soils where the soil depth exceeds 60 m and the soil types over-lying the rock are stable deposits of sands, gravel, or stiff clays.	1.2
III	• Soft to medium-stiff clays and sands characterized by 9 m or more of soft to medium-stiff clays with or without intervening layers of sand or other cohesionless soils.	1.5
IV	• Soft clays or silts greater than 12 m in depth, or • Loose natural deposits or man-made, non-engineered fill, or • Materials that may be characterized by a shear wave velocity that is smaller than 150 m/SEC.	2.0

8.4.3.3 RESPONSE MODIFICATION FACTORS

8.4.3.3.1 General

Response Modification Factors, denoted as R-factors, are used to achieve more economical earthquake design of bridges by recognizing that properly designed and detailed columns and piers will deform inelastically when the seismic forces exceed the elastic load limits. Consequently, the elastically computed effects of seismic design forces are reduced by dividing them by the appropriate values of the response modification factor.

8.4.3.3.2 Values

The values of the response modification factors for connections are smaller than those used for substructure members in order to preserve the integrity of the bridge under extreme seismic loads.

The response modification factors for main members of the substructure, such as piers, reinforced concrete pile bents, single columns, steel or composite steel-concrete pile bents and multiple column bents are shown in Table 8.4.3.3.2-1 below, as a function of the Importance Category of the bridge.

Table 8.4.3.3.2-1 - Response Modification Factors -Substructures

Substructure	Importance Category		
	Critical	Essential	Other
Wall-type piers - larger dimension	1.5	1.5	2.0
Reinforced concrete pile bents • vertical piles only • with batter piles	 1.5 1.5	 2.0 1.5	 3.0 2.0
Single columns	1.5	2.0	3.0
Steel or composite steel and concrete pile bents • vertical pile only • with batter piles	 1.5 1.5	 3.5 2.0	 5.0 3.0
Multiple column bents	1.5	3.5	5.0

The response modification factors specified for connections are shown in Table 8.4.3.3.2-2 below. They do not depend on the Importance Category of the bridge.

Table 8.4.3.3.2-2 - Response Modification Factors - Connections

Connection	All Importance Categories
Superstructure to abutment	0.8
Expansion joints within a span of the superstructure	0.8
Columns, piers, or pile bents to cap beam or superstructure	1.0
Columns or piers to foundations	1.0

Four different categories of connections are included; superstructure to abutment, expansion joints within a span, substructure to cap beam or superstructure, and columns or piers to foundations. For connections of substructure to abutment and expansion joints within a span the application of the R-factors results in an actual magnification of the seismic force effects.

As an alternative to the use of the R-factors for connections, monolithic joints between structural members and/or structures, such as column to footing connections, may be designed to transmit the maximum force effects that can be developed by the inelastic hinging of the column or multi-column bent they connect as specified in Section 8.4.3.5.4. In general, forces determined on the basis of plastic hinging will be less than those based on the R-factors for connections, resulting in a more economical design.

8.4.3.3.3 Application

Since seismic loads may act in any lateral direction, the appropriate R-factor shall be used for both orthogonal axes of the structure, which usually are the longitudinal and transverse axes of the bridge. In the case of a curved bridge, the longitudinal axis may be the chord joining the two abutments.

Wall-type piers may be treated as wide columns in the strong direction, provided the appropriate R-factor in this direction is used. They may be analyzed as a single column in the weak direction, if all the provisions for columns, as specified in LRFD S5, are satisfied.

8.4.3.4 COMBINATION OF SEISMIC FORCE EFFECTS

A combination of orthogonal seismic forces is used to account for the directional uncertainty of earthquake motions and the simultaneous occurrences of earthquake forces in two perpendicular horizontal directions. The elastic seismic force effects resulting from analyses in the two perpendicular directions shall be combined to form the following two load cases:

- Load Case I: 100% of the absolute value of the force effects in one of the above two perpendicular directions combined with 30% of the absolute value of the force effects in the second perpendicular direction.

- Load Case II: In this case, the above percentages are reversed as compared to Load Case I, i.e., 100% of the absolute value of the force effects in the second perpendicular direction are combined with 30% of the absolute value of the force effects in the first perpendicular direction.

8.4.3.5 CALCULATION OF DESIGN FORCES

8.4.3.5.1 General

Minimum design forces are distributed to bearings, and hence to substructure on the basis of their tributary permanent load. The tributary permanent load shall be determined as follows:

- For the longitudinal connection design force at the line of the fixed bearings of each uninterrupted segment of a superstructure, the total permanent load of the segment shall be used.

- For the transverse connection design force of a bearing that supports an uninterrupted segment or a simply supported span and provides restrain in the transverse direction, the permanent load reaction at that bearing shall be used.

Where a group load other than EXTREME EVENT I, specified in S3.4.1-1, governs the design of columns, the possibility that seismic forces transferred to the foundations may be larger than those calculated using the procedure specified above, due to possible overstrength of the columns, has to be considered.

The provisions for calculating the minimum connection force effect in seismic design of bridges depend on the seismic zones in which the bridge is located. Specifications established for each of the four seismic zones, defined in Section 8.4.3.2.2, are outlined in the following sections.

Regardless of seismic zone, for single span bridges, the minimum design connection force effect in the restrained direction between the superstructure and the substructure shall not be less than the product between the acceleration coefficient, described in Section 8.4.3.2.1, and the tributary permanent load, described in Section 8.4.3.5.2. Connections which transfer forces from the superstructure to the substructure include, but are not limited to, fixed bearings and shear keys. Earthquake augmented earth soils on walls of abutments should also be considered.

Seat widths at expansion bearings shall either comply with the minimum displacement requirements, described in Section 8.4.4.3, or longitudinal restrainers shall be provided, as described in Section 8.4.3.5.6.

8.4.3.5.2 Requirements for Seismic Zone 1

Seismic analysis is not generally required for bridges located in Zone 1. Consequently, default values for minimum design forces are specified instead of rigorous seismic analysis as described below.

Using an acceleration coefficient level of 0.025, the sites located in Zone 1 are split into the following two categories:

- Sites where the acceleration coefficient is less than 0.025, A < 0.025, and the soil profile is either Type I or Type II. For these sites, the horizontal design connection force in the restrained directions must not be smaller than 0.1 times the vertical reaction due to the tributary permanent load and the tributary live loads assumed to exist during an earthquake.

- Sites where the acceleration coefficient is larger than 0.025, i.e., 0.025 # A # 0.09. For these sites, the horizontal design connection force in the restrained directions must not be smaller than 0.2 times the vertical reaction due to the tributary permanent load and the tributary live loads assumed to exist during an earthquake.

During the development of the LRFD Specification, some states objected to the seat width requirements, because some details, notably shelf-type supports for beams and girders, such as that used with inverted T-cap piers were significantly affected by the increased force effects generated by increased width. This same phenomena is implicit in Division 1-A of the Standard Specifications. The division of Zone 1, at an acceleration coefficient of 0.025 for sites with favorable soil conditions, is intended to provide some relief to parts of the country with very low seismicity.

When elastomeric bearings are used to support continuous segments or simply supported spans, there are no restrained directions due to their flexibility. The bearings and the their connection to the masonry and sole plates must be designed to resist the horizontal seismic design forces transmitted through the bearings. For all bridges located in Seismic Zone 1, and for all single span bridges, the seismic shear forces transmitted through elastomeric bearings must <u>not</u> be smaller than the design connection force values specified above.

8.4.3.5.3 Seismic Zone 2

Bridges located in Seismic Zone 2 require analysis according to the minimum requirements described in Section 8.4.4. The calculation of seismic design forces must comply with the following provisions, depending on the type of component:

- All components, including pile bents and retaining walls, except foundations for these components, the seismic design forces are determined by dividing the elastic seismic forces on each component obtained according to Section 8.4.3.4 by the appropriate response modification factor, R of Section 8.4.3.3.

- Foundations - Seismic design forces for foundations, other than pile bents and retaining walls, are determined by dividing elastic seismic forces obtained according to Section 8.4.3.4 by half the response modification factor, R of Section 8.4.3.3, for the substructure component to which it is attached. The corresponding value of R/2 must not be smaller than 1.0, i.e., R/2>= 1.0.

8.4.3.5.4 Seismic Zones 3 and 4

Bridges located in Seismic Zones 3 and 4 require analysis according to the minimum requirements described in Section 8.4.3.5.4.

The design forces of each component are taken as the lesser of those determined using:

- the specified R-factors and the forces resulting from an inelastic hinging analysis; or

- on the elastic design forces, i.e., R=1.0,

for all components of a column, column bent and its foundation and connections.

In general, the design forces resulting from an R-factor and inelastic hinging analysis will be less than those from an elastic analysis. However, in the case of architecturally oversized column(s), the forces from an inelastic hinging analysis may exceed the elastic forces in which case the elastic forces may be used for that column, column bent and its connections and foundations.

Inelastic hinges are ascertained to form before any other failure due to overstress or instability in the structure and/or in the foundation. Inelastic hinges only are permitted at locations in columns where they can be readily inspected and/or repaired. Inelastic flexural resistance of substructure components is determined in accordance with the provisions of Sections 5 and 6.

In most cases, the maximum force effects on the foundation will be limited by the extreme horizontal force that a column is capable of developing. In these circumstances, the use of a lower force, lower than that specified in Article 3.10.9.4.2, is justified and should result in a more economic foundation design.

Superstructure and substructure components and their connections to columns are designed to resist a lateral shear force from the column determined from the inelastic flexural resistance of the column by multiplying the nominal resistance of concrete sections by 1.30 and that of steel sections by 1.25.

These shear forces, calculated on the basis of inelastic hinging, may be taken as the extreme seismic forces that the bridge is capable of developing.

8.4.3.5.5 Longitudinal Restrainers

Restrainers may be needed at various locations on the bridge structure to reduce relative movements of parts of the structure so that maximum design displacements are not exceeded during an earthquake. Friction alone is not considered to be an effective restrainer.

The external force used for design of restrainers is equal to the acceleration coefficient multiplied by the permanent load on the lighter of the two adjoining spans or parts of the structure.

Special provisions for restrainer design include:

- Sufficient slack must be allowed to ensure that the restrainer does not start to work before the design displacement is exceeded.

- When provided at columns or piers, the restrainer of each span may be attached to columns or piers instead of interconnecting adjacent spans.

8.4.3.5.6 Hold-Down Devices

The conditions that require the use of hold-down devices at supports and at hinges, and the appropriate uplift design force requirements are:

- Continuous structures where the vertical seismic force due to the longitudinal seismic load opposes and exceeds 50%, but is less than 100%, of the reaction due to permanent loads. In this case, the net uplift force for the design of the hold-down device is 10% of the reaction due to permanent loads that would be exerted if the span were simply supported.

- Cases where the vertical seismic forces cause a net uplift. In this cases, the design force of the hold-down device is the larger of either one of the two following forces:

- 120% of the difference between the vertical seismic force and the reaction due to permanent loads.

- 10% of the reaction due to permanent loads.

8.4.4 Analysis of Earthquake Loads (Specification Article 4.7.4)

8.4.4.1 MINIMUM ANALYSIS REQUIREMENTS

No seismic analysis is required in the following two cases:

- Bridges located in Seismic Zone 1 need not be analyzed for seismic loads, regardless of their importance and geometry. However, the minimum design connection force requirements of Section 8.4.3.5.2 and the minimum design displacement requirements of Section 8.4.4.3 must be satisfied.

- Single span bridges do not require seismic analysis, regardless of seismic zone and importance category. However, the minimum superstructure to abutments design connection force requirements of Section 8.4.3.5.1 and the minimum seat width requirements of Section 8.4.4.3 must be satisfied at each abutment.

The minimum requirements for seismic analysis of bridges depend on the type, geometry and importance category of the bridge, and the seismic zones. The seismic analysis methods required to be applied for multi-span bridges of different importance categories in Seismic Zones 2, 3 and 4 are summarized in Table 1 by using the following abbreviations for the various methods:

*	=	no seismic analysis required
SM/UL	=	single mode elastic method - either the single mode spectral method or the uniform load method
MM	=	multi-mode elastic method
TH	=	time history method

Specific provisions for applying these methods to seismic analysis of multi-span bridges are described in the following sections. Regardless of the method of analysis, the mass of the structure used to determine frequencies and mode shapes or time histories should be the nominal mass, i.e., not factored mass.

Table 8.4.4.1-1 - Minimum Analysis Requirements for Seismic Effects

Seismic Zone	Single-Span Bridges	Multi-Span Bridges					
		Other Bridges		Essential Bridges		Critical Bridges	
		regular	irregular	regular	irregular	regular	irregular
1	No seismic design required	*	*	*	*	*	*
2		SM/UL	SM/UL	SM/UL	MM	MM	MM
3		SM/UL	MM	MM	MM	MM	TH
4		SM/UL	MM	MM	MM	TH	TH

Regularity is a function of the number of spans and the distribution of weight and stiffness. Regular bridges have less than seven spans, no abrupt or unusual changes in weight, stiffness or geometry and no large changes in these parameters from span-to-span or support-to-support, abutments excluded. They are defined in Table 8.4.4.1-2. Any bridge not satisfying the requirements of Table 8.4.4.1-2 is considered to be "irregular". A more rigorous analysis procedure may be used in lieu of the recommended minimum.

Table 8.4.4.1-2 - Regular Bridge Requirements

Parameter	Value				
Number of Spans	2	3	4	5	6
Maximum subtended angle for a curved bridge	90E	90E	90E	90E	90E
Maximum span length ratio from span to span	3	2	2	1.5	1.5
Maximum bent/pier stiffness ratio from span to span, excluding abutments	---	4	4	3	2

Curved bridges comprised of multiple simple-spans are considered to be “irregular” if the subtended angle in plan is greater than 20°. Such bridges mustl be analyzed by either the multimode elastic method or the time-history method.

A curved continuous-girder bridge may be analyzed as if it were straight, provided all of the following requirements are satisfied:

- The bridge is regular as defined in Table 8.4.4.1-2, except that for a two-span bridge the maximum span length ratio from span to span must not exceed 2;

- The subtended angle in plan is not greater than 90E; and

- The span lengths of the equivalent straight bridge are equal to the arc lengths of the curved bridge.

If these requirements are not satisfied, then curved continuous-girder bridges must be analyzed using the actual curved geometry.

8.4.4.2 ANALYSIS METHODS FOR MULTI-SPAN BRIDGES

8.4.4.2.1 Single Mode Elastic Methods of Analysis

Single Mode Spectral Method (SM)

This method is based on the fundamental mode of vibration of the bridge in either the longitudinal or transverse direction. The fundamental mode can be found by applying a uniform horizontal load to the structure and calculating the corresponding elastically-deformed shape. The natural period associated with the fundamental mode can be calculated by equating the corresponding maximum potential and kinetic energies. The amplitude of the deformed shape may be found from the elastic seismic response coefficient, C_{sm}, of Section 8.4.3.1, and the corresponding spectral displacement. This amplitude is used to determine seismic force effects.

This procedure is illustrated through the following example. Assume the bridge deck, shown in Figure 8.4.4.2.1-1, is subjected to given transverse and longitudinal loadings.

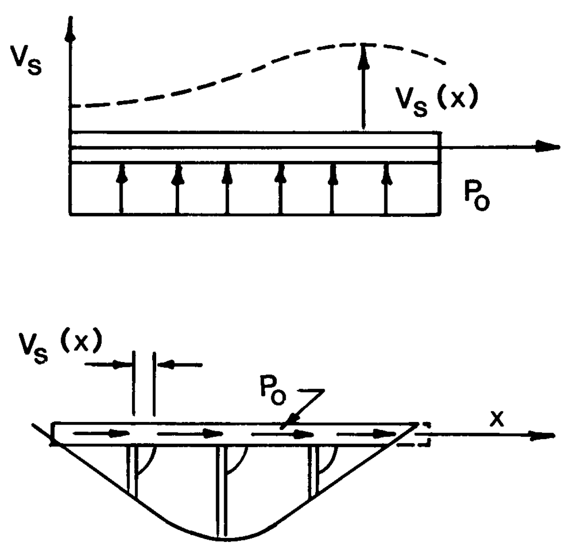


Figure 8.4.4.2.1-1 - Bridge Deck Subjected to Assumed Transverse and Longitudinal Loading

The seismic analysis, based on the single mode spectral method for the given deck, consists of the following main steps:

- Determine the elastic deflection curve, $V_s(x)$ (in mm), corresponding to a given uniform live load P_o (N/mm), arbitrarily set equal to 1.0.

- Determine the distribution of the dead load of the bridge superstructure and tributary substructure, W(x) (in N/mm).

- Calculate the factors α, β and γ from the following equations:

$$\alpha = \int V_s(x)dx \qquad (8.4.4.2.1\text{-}1)$$

$$\beta = \int W(x)V_s(x)dx \qquad (8.4.4.2.1\text{-}2)$$

$$\gamma = \int W(x)V_s^2(x)dx \qquad (8.4.4.2.1\text{-}3)$$

- Calculate the period of the fundamental vibration mode from the following equation:

$$T_m = \frac{2\pi}{31.623}\sqrt{\frac{\gamma}{P_o g \alpha}} \qquad (8.4.4.2.1\text{-}4)$$

where:

g = acceleration of gravity (in m/SEC^2)

- Use the resulting value of T_m to calculate the elastic response coefficient, C_{sm}, from Equation 8.4.3.1-1.

- Calculate the equivalent horizontal static loading caused by an earthquake from the equation below:

$$p_e(x) = \frac{\beta C_{sm}}{\gamma} W(x) V_s(x) \qquad (8.4.4.2.1\text{-}5)$$

where:

C_{sm} = the dimensionless elastic seismic response coefficient

$P_e(x)$ = the intensity of the equivalent static seismic loading applied on the bridge in the SM analysis method to represent the fundamental mode of vibration (in N/mm)

- Apply the seismic loading, $P_e(x)$, to the structure and determine the resulting member force effects.

Uniform Load Method (UL)

The uniform load method is based on the fundamental mode of vibration in either the longitudinal or transverse direction. The period of this mode of vibration is taken as that of an equivalent single mass-spring oscillator. The stiffness of this equivalent spring is calculated using the maximum displacement which occurs when an arbitrary uniform lateral load is applied to the bridge. The elastic seismic response coefficient, C_{sm}, specified in Article S3.10.6, is used to calculate the equivalent uniform seismic load from which seismic force effects are found.

The uniform load method, described in the following steps, may be used for both transverse and longitudinal earthquake motions. It is essentially an equivalent static method of analysis which uses a uniform lateral load to approximate the effect of seismic loads. The method is suitable for regular bridges that respond principally in their fundamental mode of vibration. Whereas all displacements and most member forces are calculated with good accuracy, the method is known to overestimate the transverse shears at the abutments by up to 100%. If such conservatism is undesirable, then the single mode spectral analysis method is recommended.

The following steps are involved in the uniform load method:

- Calculate the static displacements $v_s(x)$ due to an assumed uniform load p_o as shown in Figure 1. The uniform loading p_o is applied over the length of the bridge; it has units of force/unit length and may be arbitrarily set equal to 1.0. The static displacement $v_s(x)$ has units of length.

- Calculate the bridge lateral stiffness, K, and total weight, W, from the following expressions:

$$K = \frac{p_o L}{V_{s,MAX}} \qquad (8.4.4.2.1\text{-}6)$$

$$W = \int w(x)dx \qquad (8.4.4.2.1\text{-}7)$$

where:

L = total length of the bridge (mm)

v_{sMAX} = maximum value of $v_s(x)$ (mm)

and

w(x) = weight per unit length of the dead load of the bridge superstructure and tributary substructure (N/mm)

The weight should take into account structural elements and other relevant loads including, but not limited to, pier caps, abutments, columns and footings. Other loads, such as live loads, may be included. Generally, the inertia effects of live loads are not included in the analysis; however, the probability of a large live load being on the bridge during an earthquake should be considered when designing bridges with high live-to-dead load ratios which are located in metropolitan areas where traffic congestion is likely to occur.

- Calculate the period of the bridge, T_m, using the expression:

$$T_m = \frac{2\pi}{31.623}\sqrt{\frac{W}{gK}} \qquad (8.4.4.2.1\text{-}8)$$

where:

g = acceleration of gravity (m/SEC^2)

- Calculate the equivalent static earthquake loading p_e from the expression:

$$p_e = \frac{C_{sm} W}{L} \qquad (8.4.4.2.1\text{-}9)$$

where:

C_{sm} = the dimensionless elastic seismic response coefficient given by Equation S3.10.6.1-1

p_e = equivalent uniform static seismic loading per unit length of bridge applied to represent the primary mode of vibration (N/mm)

- Calculate the displacements and member forces for use in design either by applying p_e to the structure and performing a second static analysis or by scaling the results of the first step above, by the ratio p_e/p_o."

8.4.4.2.2 Multi-Mode Spectral Method

This method of seismic analysis must be applied for bridges in which each mode of vibration includes coupling between the displacements in more than one of the three coordinate directions. As a minimum, linear dynamic analysis must be performed by using a three-dimensional computational model to represent the actual bridge structure. The following guidelines apply to use of the multi-mode spectral method:

Application Guidelines

- The number of modes included in the analysis should be at least three times the number of spans in the physical model.

- The elastic seismic response spectrum specified in Section 8.4.3.1 must be used separately for each vibration mode included in the analysis.

- The forces and displacements in each member of the bridge may be estimated by using the complete quadratic combination method (CQC). Alternatively, the square root of the sum of the squares method (SRSS) may be used if the responses from the various modes are well separated and the CQC method is not available.

- For bridges with closely-spaced modes, whose natural frequencies are within 10% of each other, an alternative method of combining the modal effects, such as an "absolute sum", should be used.

8.4.4.2.3 Time-History Method

Unlike modal analysis methods (SM and MM) which rely on the frequency domain, in the time-history method, the analysis of seismic effects is performed in the time domain by numerical integration of the equations of motion governing the dynamic response of a bridge. The sensitivity of the resulting numerical solution to the size of the time step selected for the integration process must be addressed.

The time-history method can be applied either for elastic or inelastic analysis by following the guidelines below:

- In an elastic analysis, the response modification factors, specified in Section 8.4.3.3, are used.

- In an inelastic analysis, the response modification factors are taken as equal to 1.0 for all substructures and connections.

The earthquake loads are represented in the time-history method as time-dependent inputs of the ground acceleration. The time histories of such input shall be selected in consultation with the Owner, although a site-specific spectrum should always be preferred, if available. Otherwise, five spectrum compatible time histories shall be selected as input for the time-history method, such that they are the same as the spectrum used for modal methods, and modified for the appropriate soil profile.

The time-history method of seismic analysis is considered to be more rigorous than the modal methods if the bridge structure is properly modeled and a representative time- history of ground acceleration can be selected as input. Consequently, the time-history method is recommended for critical bridges and/or those that are geometrically complex or close to active earthquake faults.

8.4.4.3 MINIMUM DISPLACEMENT REQUIREMENTS

The design seat width at the expansion bearing under consideration is taken as the greater of the seat widths calculated by the two approaches given below:

- A seat width that accommodates the maximum displacement calculated in accordance with the provisions described in Section 8.4.4.2 above, and

- a seat width taken as a percentage of the empirical seat width, N, defined in Equation 8.4.4.3-1.

$$N = (200 + 0.00172L + 0.0067H)(1 + 0.000\,125\,S^2) \quad (8.4.4.3\text{-}1)$$

where:

N = minimum support length measured normal to the centerline of bearing (mm)

L = length of the bridge deck to the adjacent expansion joint, or to the end of the bridge deck. For hinges within a span, L shall be the sum of the distances to either side of the hinge. For single-span bridges, L equals the length of the bridge deck (mm)

H = for abutments, average height of columns supporting the bridge deck to the next expansion joint (mm); for columns and/or piers, H is the height of the column and/or pier (mm); for hinges within a span, H is the average height of the two adjacent columns or piers (mm); for single-span bridges, H = 0.0

S = angular skew of support measured from line normal to span (DEG)

The percentage of the empirical width, N, to be used to determine the required width of the bridge seat depends on the seismic zones as indicated in Section 8.4.3.2.2, the acceleration coefficient of Section 8.4.3.2.1 and the soil type of Section 8.4.3.2.4 at the site, as specified in Table 8.4.4.3-1 below.

Table 8.4.4.3-1 - Percentage N by Zone and Acceleration Coefficient

ZONE	ACCELERATION COEFFICIENT	SOIL TYPE	% N
1	< 0.025	I or II	$ 50
1	< 0.025	III or IV	100
1	> 0.025	All	100
2	All Applicable	All	100
3	All Applicable	All	150
4	All Applicable	All	150

If the bridge seat widths requirements, established in accordance with the above, are not satisfied, longitudinal restrainers complying with Section 8.4.3.5.5 shall be provided. Bearings restrained from longitudinal movement must be designed in compliance with Section 8.4.3.4.

APPENDIX A

Acceleration Coefficient Maps

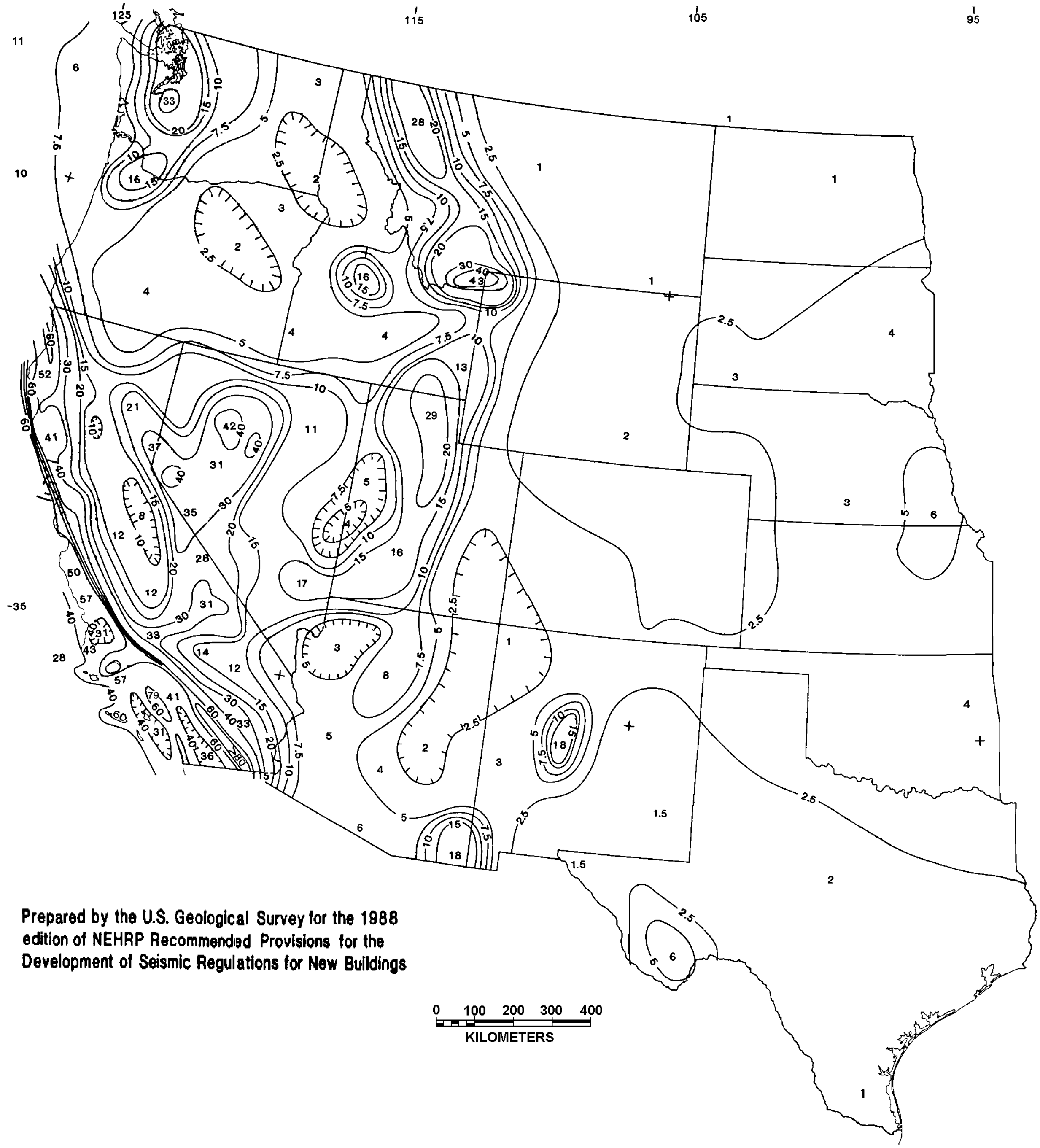


Figure A-1 - Acceleration Coefficient for Contiguous States Generally West of the 95th Longitude

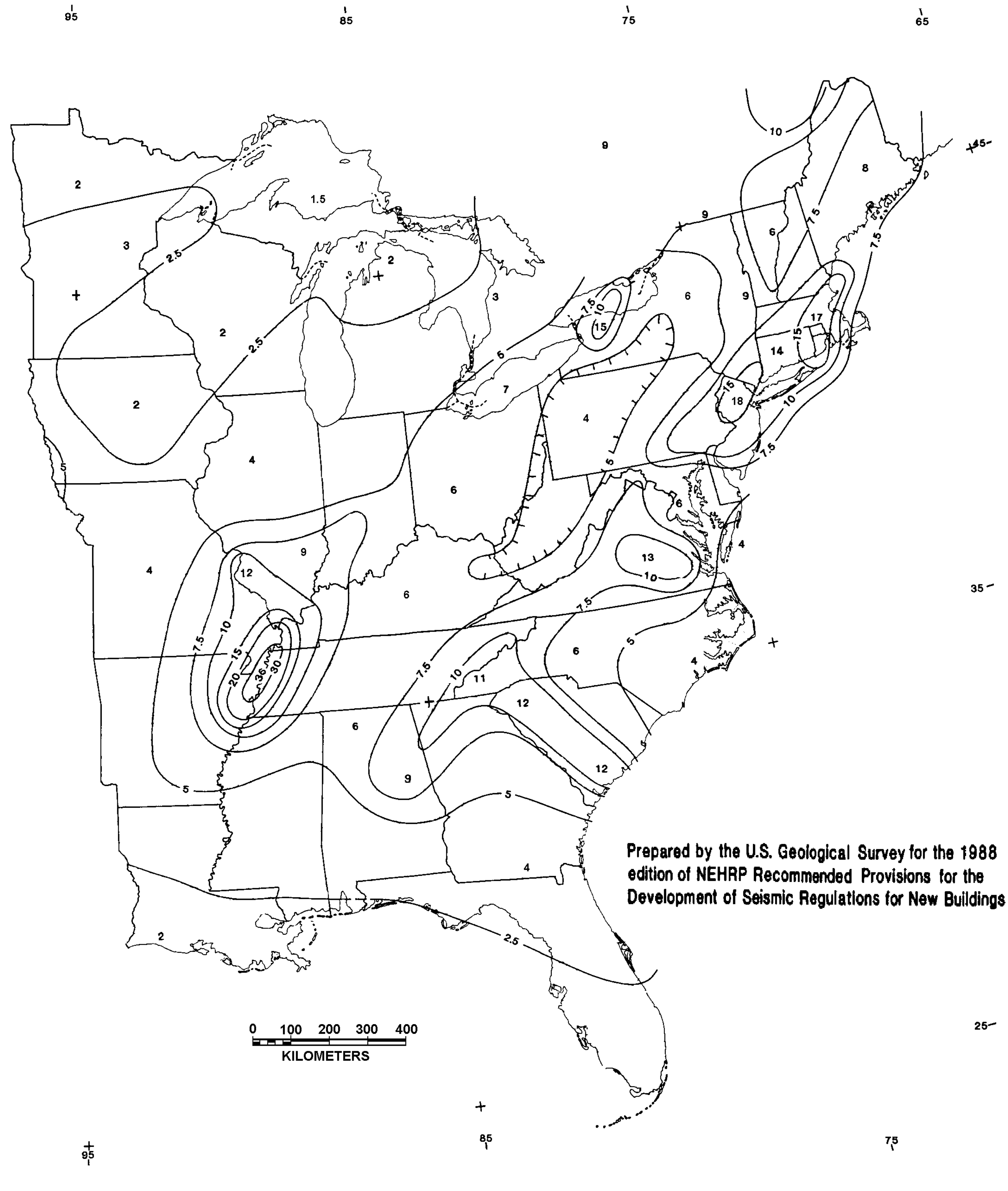


Figure A-2 - Acceleration Coefficient for Contiguous States Generally East of the 95th Longitude

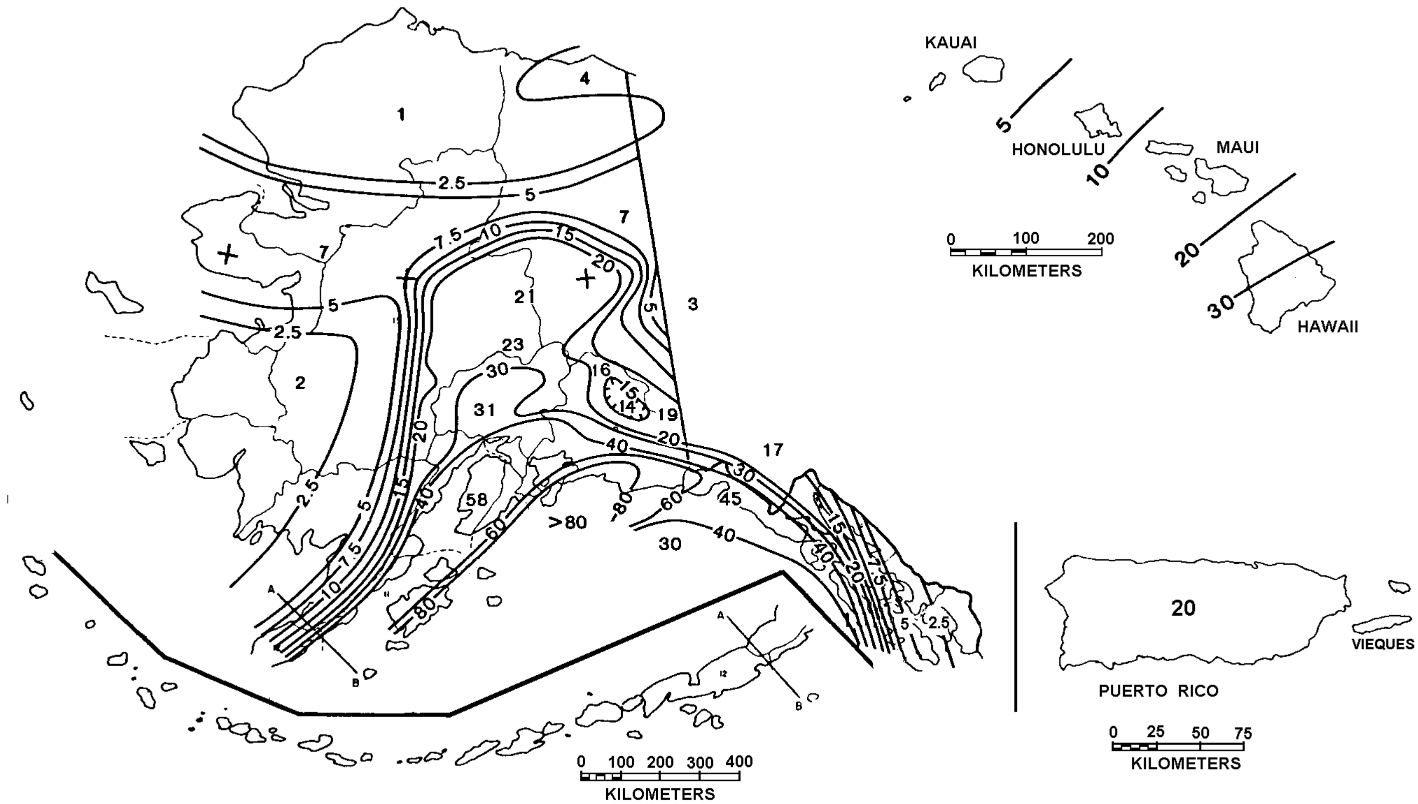


Figure A-3 - Acceleration Coefficient for Alaska, Hawaii and Puerto Rico

LECTURE 9 - CONCRETE STRUCTURES - I

9.1 OBJECTIVE

The objective of this lesson is to acquaint the student with the limit states which apply to concrete design, the basic provisions related to strut and tie models, prestress losses, flexural resistance, and shear and torsion.

9.2 INTRODUCTION

The major change in the design of concrete structures represented by the AASHTO LRFD Specifications for Highway Bridge Design is the combining of provisions for reinforced concrete and prestressed concrete into one section, Section 5 - Concrete Structures. Not only are the provisions for all types of concrete structures combined into one section, but also the various design procedures for members reinforced with conventional rebars, prestressing tendons or any combination thereof have been unified into one consistent procedure.

9.3 LIMIT STATES

Just as for all structural components, concrete members must be proportioned to satisfy the requirements at all appropriate limit states: service, fatigue, strength and extreme event limit states.

9.3.1 Service Limit State

For prestressed and partially prestressed concrete members, stresses and deformations should be investigated at each critical stage during stressing, handling, transportation and erection, as well as during the subsequent service life.

Compressive stresses in the concrete of prestressed and partially prestressed members must be checked against the compressive stress limits specified in Article S5.9.4 for Service Load Combination I.

Similarly, tensile stresses in the concrete of prestressed and partially prestressed members must be checked against the tensile stress limits specified in Article S5.9.4 for Service Load Combination III.

Service Load Combination I is also used to distribute tension reinforcement to control cracking through Equation 9.3.1-1.

$$f_{sa} = \frac{Z}{(d_c A)^{1/3}} \leq 0.6 f_y \qquad (9.3.1\text{-}1)$$

where:

d_c = depth of concrete measured from extreme tension fiber to center of bar or wire located closest thereto. The clear concrete cover used to calculate d_c shall be the smaller of the actual clear cover or 50 mm.

A = area of concrete having the same centroid as the principal tensile reinforcement and bounded by the surfaces of the cross-section and a straight line parallel to the neutral axis, divided by the number of bars or wires (mm^2). The maximum clear concrete cover used in calculating A shall not exceed 50 mm.

Z = crack width parameter (N/mm)

The quantity Z in Equation 9.3.1-1 is not to exceed 30 000 N/mm for members in moderate exposure conditions, 23 000 N/mm for members in severe exposure conditions and 17 500 N/mm for buried structures.

Service Load Combination II is not applicable for concrete members.

9.3.2 Fatigue Limit State

Fatigue is the accumulation of damage due to repetitively applied tensile stresses of all magnitudes. In order to prevent fatigue damage, the stress range in straight reinforcement resulting from the fatigue load combination, specified in Table S3.4.1-1, is limited to:

$$f_f = 145 - 0.33 f_{min} + 55 \left(\frac{r}{h} \right) \qquad (9.3.2\text{-}1)$$

where:

f_f = stress range (MPa)

f_{min} = the minimum live load stress, resulting from the fatigue load combination specified in Table S3.4.1-1, combined with the more severe stress from either the permanent loads or the permanent loads, shrinkage and creep-induced external loads, positive if tension, negative if compression (MPa) r/h = ratio of base radius to height of rolled-on transverse deformations; if the actual value is not known, 0.3 may be used

Similarly, the stress range in prestressing tendons is limited to:

- 125 MPa for radii of curvature in excess of 9000 mm, and
- 70 MPa for radii of curvature not exceeding 3600 mm.

A linear interpolation may be used for radii between 3600 and 9000 mm. Where the radius of curvature is less than shown, or metal-to-metal fretting caused by prestressing tendons rubbing on hold-downs or deviations is apt to be a consideration, then it will be necessary to consult the literature for more complete presentations which will allow the increased bending stress in the case of sharp curvature, or fretting, to be accounted for in the development of permissible fatigue stress ranges. Metal-to-metal fretting is not normally expected to be a concern in conventional pretensioned beams.

The live load stress represented by the load factor of 0.75 in the Fatigue Load Combination is an effective or weighted average live load stress, representing the average damage of all vehicles crossing the bridge during its design life. It does not represent the maximum live load. This average representation of live load cannot be used to judge if a section will ever experience fatigue or whether the section is cracked. The maximum live load stress must be used for these determinations.

The codewriters consider that twice the maximum tensile live load stress, due to the Fatigue Load Combination, represents the maximum expected live load tensile stress. In other words, a load factor of 2.0 x 0.75 or 1.5 is needed to produce the expected maximum fatigue stress.

Thus, the fatigue limit state must only be considered in regions where the compressive concrete stress under permanent loads and prestress is less than twice the maximum tensile live load stress due to the Fatigue Load Combination of Table S3.4.1-1, reproduced herein as Table 2.4.1.2-1. In other words, for fatigue to occur and the fatigue limit state to be applicable, the concrete member must experience a net tensile stress due to the maximum expected live load tensile stress.

This philosophy is identical as that applied to metal structures.

Similarly, cracked section properties are used for fatigue investigations where the stress, due to the summation of permanent loads, prestress and twice the maximum tensile live load stress, due to the Fatigue Load Combination, is tensile and greater than $0.25 / f_c'$. Otherwise, uncracked section properties may be used.

9.3.3 Strength Limit State

The strength limit state issues to be considered are ultimate strength and stability.

Unlike the Service Load Combinations, the Strength Load Combinations I through V are material independent and must all be applied to concrete structures.

The resistance factors, φ, to be applied for concrete structures, in conjunction with the load factors represented by the various Load Combinations, are summarized below for conventional construction. Other resistance factors are specified in Article S5.5.4.2.2 for segmental construction.

- for flexure and tension of reinforced concrete 0.90
- for flexure and tension of prestressed concrete 1.00
- for shear and torsion:
    - normal density concrete 0.90
    - low-density concrete 0.70
- for axial compression with spirals or ties, except as specified in Article S5.10.11.4.1b for Seismic Zones 3 and 4 at the extreme event limit state 0.75
- for bearing on concrete 0.70
- for compression in strut-and-tie models 0.70
- for compression in anchorage zones:
    - normal density concrete. 0.80
    - low-density concrete 0.65
- for tension in steel in anchorage zones 1.00

For compression members with flexure, the value of φ may be increased linearly to the value for flexure as the factored axial load resistance, φP_n, decreases from 0.10 $f'_c A_g$ to 0.

For partially prestressed components in flexure with or without tension, the values of φ may be taken as:

$$\varphi = 0.90 + 0.10\,(PPR) \qquad (9.3.3\text{-}1)$$

for which:

$$PPR = \frac{A_{ps} f_{py}}{A_{ps} f_{py} + A_s f_y} \qquad (9.3.3\text{-}2)$$

where:

PPR = partial prestress ratio

A_s = area of non-prestressed tension reinforcement (mm^2)

A_{ps} = area of prestressing steel (mm^2)

f_y = specified yield strength of reinforcing bars (MPa)

f_{py} = yield strength of prestressing steel (MPa)

Resistance factors are not applied to the development and splice lengths of reinforcement as specified in Article S5.11.

9.3.4 Extreme Event Limit State

The concrete structure, as a whole, and its individual components, must be proportioned to resist collapse due to extreme events, for example, earthquakes and vessel collision, as appropriate to its site and use.

9.4 FLEXURE

The various equations presented in the LRFD Specification for the calculation of nominal flexural resistance, M_r, are based upon the assumption that all of the reinforcing bars can be modeled as lumped together at a single point, and all of the prestressing tendons likewise lumped together.

9.4.1 Limits of Reinforcement

9.4.1.1 MAXIMUM REINFORCEMENT

For the determination of the maximum amount of prestressed and non-prestressed reinforcement, a new variable must be defined, d_e, the effective depth of the centroid of the tensile force in the tensile reinforcement (conventional rebar, prestressing tendons or any combination thereof) from the extreme compression fiber at the nominal resistance of the section. The variable, d_e, is defined in Equation 9.4.1.1-1, which is derived by summing moments equal to zero about the extreme compression fiber to locate the centroid of the tensile force.

$$d_e = \frac{A_{ps} f_{ps} d_p + A_s f_y d_s}{A_{ps} f_{ps} + A_s f_y} \qquad (9.4.1.1\text{-}1)$$

where:

d_e = the corresponding effective depth from the extreme compression fiber to the centroid of the tensile force in the tensile reinforcement (mm)

All of the provisions for maximum reinforcement can be unified in terms of c/d_e.

where:

c = the distance from the extreme compression fiber to the neutral axis (mm)

The maximum reinforcement for sections reinforced with conventional rebars, prestressing tendons or combinations thereof is such that the inequality of Equation 9.4.1.1-2 is satisfied.

$$\frac{c}{d_e} \# 0.42 \qquad (9.4.1.1\text{-}2)$$

If the inequality is satisfied, the section is considered under-reinforced. If the inequality is not satisfied, the section is over-reinforced. Over-reinforced reinforced concrete sections are not permitted. Over-reinforced prestressed or partially prestressed members are permitted only if demonstrated to be sufficiently ductile. Reinforced concrete sections are those with a partial prestressing ratio, defined by Equation 9.3.3-2, less than 50%.

The development of Equation 9.4.1.1-2, by Professor Antoine E. Naaman of the University of Michigan, is the result of extensive nonlinear analysis of ductility. His analysis demonstrated that the ductility index (the ratio of the sectional curvature at ultimate to that at yield, or the corresponding rotation at ultimate to that at yield) is a function of the ratio c/d_e and correspondingly the global reinforcing index. This expression for ductility has true physical meaning as the global reinforcing index is directly proportional to the net tensile force in the steel.

In the Standard Specifications in Article 8.16.3.1, the maximum reinforcement in ordinarily reinforced concrete sections is limited to 0.75 of the ratio of reinforcement that would produce balanced strain conditions for the section. Also, in the Standard Specifications in Article 9.18.1, the maximum reinforcement index for prestressed sections is limited to $0.36\beta_1$. It has been shown in Namaan (1992) that limiting the reinforcement ratio for reinforced concrete sections and limiting the reinforcing index for prestressed concrete sections amounts to limiting the ratio, c/d_e. Thusly, the determination of maximum amount of reinforcement has been unified.

9.4.1.2 MINIMUM REINFORCEMENT

The minimum reinforcement of any section of a flexural component, reinforced with conventional rebars, prestressing tendons or any combination thereof, is that adequate to develop a factored flexural resistance, M_r, at least equal to the lesser of:

- 1.2 times the cracking strength determined on the basis of elastic stress distribution and the modulus of rupture, f_r, of the concrete as specified in Article S5.4.2.6, or

- 1.33 times the factored moment required by the applicable strength load combinations specified in Table S3.4.1-1.

The provisions for shrinkage and temperature reinforcement of Article S5.10.8 must also be considered.

9.4.2 Stress in Prestressing Steel at Nominal Flexural Resistance

While it is generally assumed that the reinforcing steel yields at the ultimate, the stress in the prestressing steel, f_{ps}, is unknown and should be determined as indicated in Article S5.7.3.1.

The following approximations for average stress in both bonded and unbonded prestressing tendons at nominal flexural resistance are applicable for rectangular or flanged sections subjected to flexure about a single axis where the Whitney stress block of Article S5.7.2.2 is used.

In the case where a significant number of prestressing elements are on the compression side of the neutral axis, it is more appropriate to use a method based on the conditions of equilibrium and strain compatibility as indicated in Article S5.7.2.1.

9.4.2.1 COMPONENTS WITH BONDED TENDONS

The average stress in bonded prestressing tendons, f_{ps}, may be taken as given by Equation 9.4.2.1-1 where f_{pe} is not less than ½ f_{pu}.

$$f_{ps} = f_{pu}\left(1 - k\frac{c}{d_p}\right) \qquad (9.4.2.1\text{-}1)$$

For which:

$$k = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) \qquad (9.4.2.1\text{-}2)$$

For T-section behavior:

$$c = \frac{A_{ps}f_{pu} + A_s f_y - A'_s f'_y - 0.85\beta_1 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w + k A_{ps}\dfrac{f_{pu}}{d_p}} \qquad (9.4.2.1\text{-}3)$$

For rectangular section behavior:

$$c = \frac{A_{ps} f_{pu} + A_s f_y - A'_s f'_y}{0.85 f'_c \beta_1 b + k A_{ps} \dfrac{f_{pu}}{d_p}} \qquad (9.4.2.1\text{-}4)$$

where:

A_{ps}	=	area of prestressing steel (mm^2)
f_{pu}	=	specified tensile strength of prestressing steel (MPa)
f_{py}	=	yield strength of prestressing steel (MPa)
A_s	=	area of mild steel tension reinforcement (mm^2)
A'_s	=	area of compression reinforcement (mm^2)
f_y	=	yield strength of tension reinforcement (MPa)
f'_y	=	yield strength of compression reinforcement (MPa)
b	=	width of compression flange (mm)
b_w	=	width of web (mm)
h_f	=	depth of compression flange (mm)
d_p	=	distance from extreme compression fiber to the centroid of the prestressing tendons (mm)
c	=	distance between the neutral axis and the compressive face (mm)
β_1	=	stress block factor specified in Article S5.7.2.2

As an alternative for preliminary design (or taking a conservative design approach), the value of f_{ps} can be estimated as f_{py}.

9.4.2.2 COMPONENTS WITH UNBONDED TENDONS

The average stress in unbonded prestressing tendons, f_{ps}, may be taken as given by Equation 9.4.2.2-1.

$$f_{PS} = f_{pe} + 6300\left(\frac{d_p - c}{\ell_e}\right) \le f_{py} \qquad (9.4.2.2\text{-}1)$$

for which:

$$\ell_e = \frac{2\ell_i}{2 + N_s}$$

For T-section behavior:

$$c = \frac{A_{ps} f_{ps} + A_s f_y - A'_s f'_y - 0.85 \beta_1 f'_c (b - b_w) h_f}{0.85 f'_c \beta_1 b_w} \qquad (9.4.2.2\text{-}2)$$

For rectangular section behavior:

$$c = \frac{A_{ps} f_{ps} + A_s f_y - A'_s f'_y}{0.85 f'_c \beta_1 b} \qquad (9.4.2.2\text{-}3)$$

where:

d_p = distance from extreme compression fiber to the centroid of the prestressing tendons (mm)

f_{pe} = effective stress in prestressing steel at section under consideration after all losses (MPa)

ℓ_e = effective tendon length (mm)

ℓ_i = length of tendon between anchorages (mm)

N_s = number of support hinges crossed by the tendon between anchorages or discretely bonded points

c = distance between the neutral axis and the compressive face (mm)

The equation is equally applicable to biaxial flexure with axial load, as specified in Article S5.7.4.5, in addition to the general application indicated above.

As an estimate for preliminary design, f_{ps} may be estimated by:

$$f_{ps} = f_{pe} + 103 \text{ (MPa)} \qquad (9.4.2.2\text{-}4)$$

9.4.3 Flexural Resistance

The unified approach requires only one equation, Equation 9.4.3-1, to determine the nominal flexural resistance for flanged or rectangular sections reinforced with conventional rebars, prestressing tendons or any combination thereof. For flanged sections, the equation is directly applicable. For rectangular sections, b_w is taken as b, and the last term of the equation becomes zero.

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d_s - \frac{a}{2}\right) - A'_s f'_y\left(d'_s - \frac{a}{2}\right) + 0.85 f'_c (b - b_w)\,\beta_1 h_f\left(\frac{a}{2} - \frac{h_f}{2}\right) \quad (9.4.3\text{-}1)$$

where:

A_{ps} = area of prestressing steel (mm^2)

f_{ps} = average stress in prestressing steel at nominal bending resistance specified in Equation 9.4.2.1-1 (MPa)

d_p = distance from extreme compression fiber to the centroid of prestressing tendons (mm)

A_s = area of non-prestressed tension reinforcement (mm^2)

f_y = specified yield strength of non-prestressed reinforcing bars (MPa)

d_s = distance from extreme compression fiber to the centroid of non-prestressed tensile reinforcement (mm)

A'_s = area of compression reinforcement (mm^2)

f'_y = specified yield strength of compression reinforcement (MPa)

d'_s = distance from extreme compression fiber to the centroid of compression reinforcement (mm)

f'_c = specified compressive strength of concrete at 28 days, unless another age is specified (MPa)

b = width of the compression face of the member (mm)

b_w = web width or diameter of a circular section (mm)

β_1 = stress block factor specified in Article S5.7.2.2

h_f = compression flange depth of an I or T member (mm)

a = $c\beta_1$; depth of the equivalent stress block (mm)

Equation 9.4.3-1 is based upon the same assumptions inherent to the provisions of the Standard Specifications, except for the slight modification presented in Article SC5.7.3.2.2. This modification consists of the multiplication of the last term in the equation by β_1. The assumptions inherent to the equation are listed in Article S5.7.2, and include the assumptions that plane sections remain plane after loading, the maximum usable unconfined compressive concrete strain is equal to 0.003, the tensile strength of concrete is negligible and the Whitney stress block is appropriate.

For sections other than flanged or rectangular sections, these assumptions can be used to derive the nominal flexural resistance of the sections.

9.4.4 Crack Control

Cracking in the precompressed zone of partially prestressed members is permitted. In this case, the tensile stress in the reinforcement at service loads, f_{sa} is the change in stress after decompression.

9.5 STRUT-AND-TIE MODEL

Where conventional methods of strength of materials are not applicable because of non-linear strain fields, the strut-and-tie model provides a convenient way of approximating load paths and force effects in structures at the strength and extreme event limit states. In fact, the load paths may be visualized first and the geometry of concrete and steel selected to implement the visualized load paths.

Traditional sectional models are based upon the assumption that the reinforcement required at a particular section depends only upon the values of the factored section force effects, V_u, M_u, and T_u, and does not consider the mechanical interaction among these force effects as the strut-and-tie model. Further, the sectional models assume that the shear distribution remains uniform and the longitudinal strains vary linearly with the members depth.

Articles S5.6.3.2 through S5.6.3.6 provide provisions for modeling and proportioning concrete members using the strut-and-tie model.

9.5.1 Structural Modeling

Concrete members may be modeled as an assemblage of concrete compression struts and steel tension ties, interconnected at nodes to form a truss capable of carrying all of the loads applied to the members to its supports. Cracked reinforced concrete carries load principally by compressive stresses in the concrete and tensile stresses in the steel reinforcement. After significant cracking, at the strength or extreme-event limit states, the principal compressive stress trajectories in the concrete tend towards straight lines and thusly can be approximated by straight compressive struts. Tension ties model the principal steel reinforcement provided in the concrete member.

Where the struts and ties meet, the concrete is subjected to multi-directional stresses. These truss joints are represented by node regions. Because the struts and ties have significant transverse dimensions, the truss joints become node regions with finite dimensions.

Establishing the geometry of the truss usually involves trial and error. First, member sizes are assumed and the truss geometry established. Next the member forces are determined and the member sizes verified.

9.5.2 Proportioning Compressive Struts

9.5.2.1 STRENGTH OF STRUTS

The nominal resistance of a compressive strut is given by Equation 9.5.2-1.

$$P_n = f_{cu} A_{cs} + f_y A_{ss} \qquad (9.5.2.1\text{-}1)$$

where:

P_n = nominal resistance of a compressive strut (N)

A_{ss} = area of reinforcement in the strut (mm^2)

f_{cu} = limiting compressive stress as specified in Article 9.5.2.3 (MPa)

A_{cs} = effective cross-sectional area of strut as specified in Article 9.5.2.2 (mm^2)

This general equation is applicable to reinforced and unreinforced struts. With A_{ss} equal to zero for unreinforced struts, the equation reverts to Equation 9.5.2.1-2 for unreinforced struts:

$$P_n = f_{cu} A_{cs} \qquad (9.5.2.1\text{-}2)$$

9.5.2.2 EFFECTIVE CROSS-SECTIONAL AREA OF STRUTS

The effective cross-sectional area of the compressive strut, A_{cs}, is a function of the available concrete area and how the strut is anchored. A compressive strut can be anchored by a zone of reinforcement, a bearing, another strut or any combination thereof. Obviously, the compressive strut must be contained within the cross section of its concrete member. In addition, the strut must be completely within the bounds of the projection normal to the longitudinal axis of the strut of the bearing, strut and/or reinforcement zone which anchors it. The zone of reinforcement is considered to extend up to six bar diameters beyond the bars themselves. Examples of the effective cross-sectional areas of struts anchored by several of the various possibilities is shown in Figure 9.5.2.2-1.

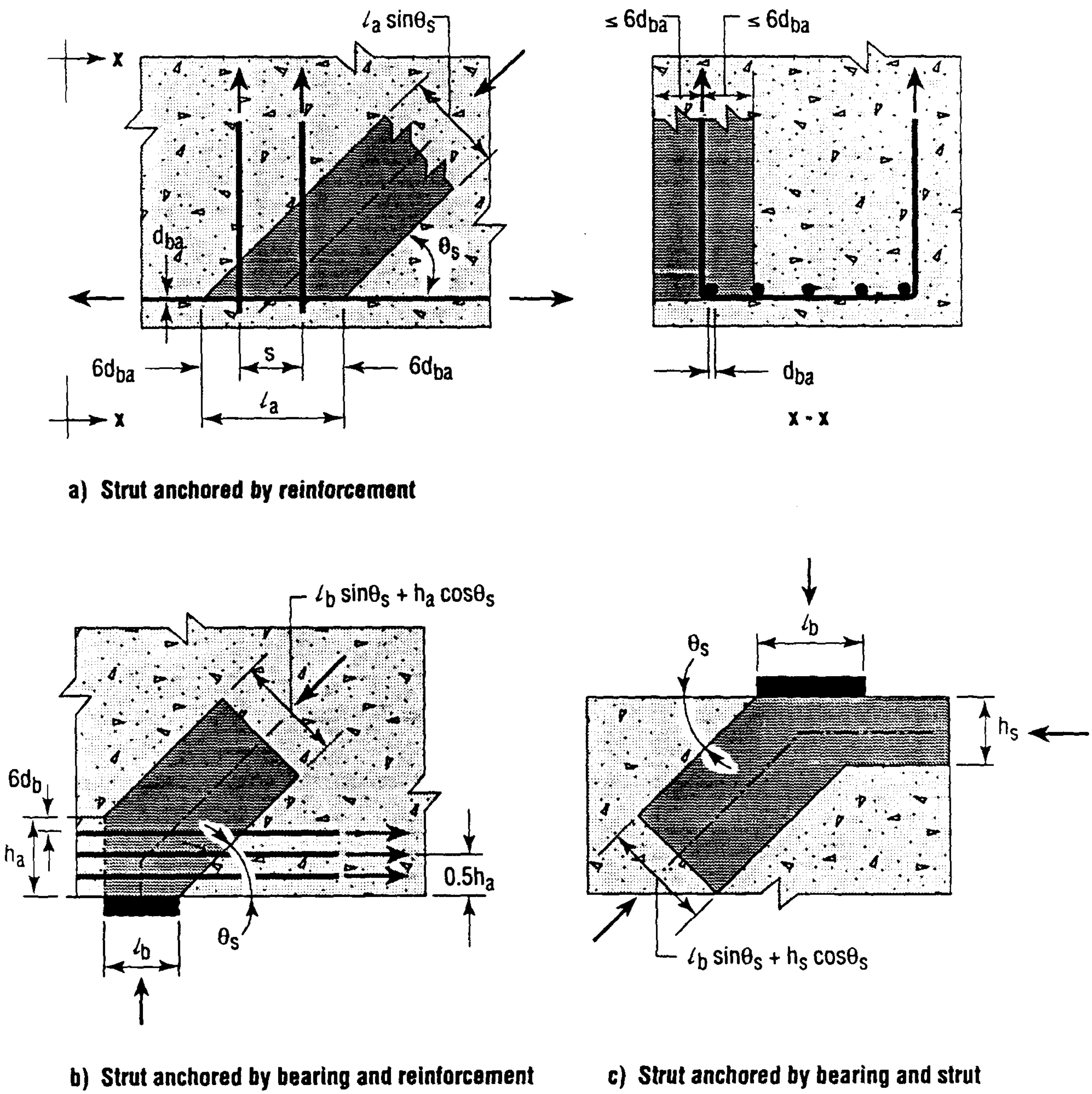


Figure 9.5.2.2-1 - Influence of Anchorage Conditions on Effective Cross-Sectional Area of Strut

9.5.2.3 LIMITING COMPRESSIVE STRESS IN STRUTS

The limiting compressive stress in the strut, f_{cu}, is given in Equation 9.5.2.3-1.

$$f_{cu} = \frac{f'_c}{0.8 + 170\,\varepsilon_1} \le 0.85\, f'_c \qquad (9.5.2.3\text{-}1)$$

for which:

$$\varepsilon_1 = \varepsilon_s + (\varepsilon_s + 0.002) \cot^2 \alpha_s \qquad (9.5.2.3\text{-}2)$$

where:

α_s = the smallest angle between the compressive strut and adjoining tension ties (DEG)

ε_s = the tensile strain in the concrete in the direction of the tension tie (mm/mm)

f'_c = specified compressive strength (MPa)

Concrete can resist a compressive stress of $0.85f'_c$ if it is not subjected to principal tensile strains greater than 0.002. Thus, the limiting compressive stress for struts which are not crossed by or joined to tension ties is $0.85f'_c$.

For compressive struts crossed by or joined to tension ties, the limiting compressive stress must be lower than $0.85f'_c$. Equation 9.5.2.3-1 estimates the limiting compressive stress for such struts based upon the assumptions that the principal compressive strain in the direction of the strut, ε_2, is equal to 0.002. For a tension ties consisting of reinforcing bars, the tensile strain in the concrete in the direction of the tension tie, ε_s, is taken as the tensile strain in the bars due to the factored loads. For a tie consisting of prestressing tendons, ε_s, is taken as zero until the precompression of the concrete is overcome. For strains beyond precompression, ε_s is equal to $(f_{ps} - f_{pe})/E_p$. As the angle between the strut and tie, Θ_s, decreases, ε_s increases and therefore f_{cu} decreases. No compressive stresses are allowed in a hypothetical strut superimposed over a tie (Θ_s equals zero, therefore, f_{cu} equals zero). If the tensile strain in the tie, ε_s, varies over the width of the strut, the value at the centerline of the strut may be used for simplicity.

9.5.3 Proportioning Tension Ties

9.5.3.1 STRENGTH OF TIES

The nominal resistance of a tension tie, P_n, is given in Equation 9.5.3.1-1.

$$P_n = f_y A_{st} + A_{ps} [f_{pe} + f_y] \qquad (9.5.3.1\text{-}1)$$

where:

A_{st} = total area of longitudinal mild steel reinforcement in the tie (mm^2)

A_{ps} = area of prestressing steel (mm^2)

f_y = yield strength of mild steel longitudinal reinforcement (MPa)

f_{pe} = stress in prestressing steel due to prestress after losses (MPa)

The two terms in this equation represent the contribution to the tie resistance of any reinforcing bars and/or prestressing tendons, respectively. The term relating to the prestressing tendons is intended to ensure that the prestressing steel does not yield, maintaining control over unlimited cracking. The second term acknowledges that the stress in the tendon will be increased by the strain required to crack the concrete. The increase in stress is arbitrarily limited to the increase in stress that the reinforcing bars will undergo at cracking. If there is no mild steel present, f_y may be taken as 415 MPa.

9.5.3.2 ANCHORAGE OF TIES

The reinforcement composing the tension tie must be anchored to the node regions by specified embedment lengths, hooks or mechanical anchorage devices, in accordance with the requirements for development of reinforcement as specified in Article S5.11. The nominal resistance of the tension tie must be developed at the interface of the node region and the tie.

9.5.4 Proportioning Node Regions

The limits of compressive concrete stresses in node regions are related to the degree of expected confinement in these regions provided by the concrete in compression. Unless confining reinforcement is provided, and its effectiveness is verified by analysis or experimentation, the concrete compressive stresses in the node regions shall not exceed the allowable values specified in Article S5.6.3.5 for different strut and tie geometries. Higher allowable compressive stresses correspond to greater degrees of expected confinement.

Nodes anchoring tension ties have lower specified stress limits based upon the detrimental effect of the tensile straining caused by the tension ties. If the tension ties consist of post-tensioned tendons and the stress in the concrete does not exceed f_{pc}, no tensile straining of the node occurs and the higher allowable compressive stress, that for nodes bounded only by struts and bearing areas, is appropriate.

The compressive stresses in the node region can be reduced by increasing the size of the bearing plates, or the dimensions of the compressive struts or tension ties.

The steel reinforcement in tension ties must be uniformly distributed over an effective area of concrete equal to at least the tension tie force divided by the specified concrete compressive stress.

In addition to the strength criteria for compressive struts and tension ties, the node regions must be designed for the anchorage requirements of Article S5.6.3.4.2.

The bearing stress on the node region from concentrated applied loads or reactions must satisfy the bearing provisions of Article S5.7.5.

9.5.5 Crack Control Reinforcement

Members or regions thereof designed by the strut-and-tie model, excluding slabs and footings, must contain an orthogonal grid of reinforcing bars near each face. The spacing of bars in these grids must not exceed 300 mm. This reinforcement controls the width of cracks, and ensures a minimum ductility for the member so that significant redistribution of internal stresses is possible if required.

The ratio of reinforcement area to gross concrete area must not be less than 0.003 in each direction. For thinner members, the crack control reinforcement will consist of two grids, one near each face. For thicker members, multiple girds throughout the thickness may be required to achieve a practical arrangement.

Crack control reinforcement located within a tension tie may be considered to be part of the tie reinforcement.

9.6 PRESTRESSING

9.6.1 Introduction

The LRFD Specification introduces the concept of partial prestressing to bridge engineering in the United States. The introduction of partial prestressing allowed the development of a unified theory of concrete structures with conventional reinforced and prestressed concrete members as boundary cases. Partial prestressing encompasses the following design solutions: (1) a concrete member reinforced with a combination of prestressed and non-prestressed reinforcement to simultaneously resist the specified force effects, (2) a prestressed concrete member designed to crack in tension under service load, and (3) a prestressed concrete member in which the effective prestress force in the prestressed reinforcement is purposely lower than the maximum allowable value. The provisions of Article S5.9, regarding prestressing, are based upon those of the Standard Specifications, ACI 343, ACI 318, and the Ontario Highway Bridge Design Code, but have been extended to accommodate partial prestressing.

In general, the LRFD Specification treats prestressing forces as a part of the member's resistance and not as a part of the member's loading. However, where design is totally governed by prestressing force, such as that of anchorages and similar components, the prestressing force is considered a load for which a load factor is specified in Article S3.4.3.

9.6.2 Stress Limitations for Prestressing Tendons

Tendons of high-strength steel bars or strands are generally used to prestress concrete members. Any material satisfying the strength, stiffness and ductility requirements of the LRFD Specification may also be used provided they also satisfy the intent of Article S5.4.1.

The tendon stress due to prestress or at any of the service limit states shall not exceed the values of Table 9.6.2-1, or those recommended by the manufacturer of the tendons or anchorages employed.

For post-tensioning, the short-term allowable of 0.90 f_{py} may be allowed for short periods of time prior to seating to offset seating and friction losses, provided that the other values in Table 1 are not exceeded.

Table 9.6.2-1 - Stress Limits for Prestressing Tendons

Condition	Tendon Type		
	Stress Relieved Strand and Plain High-Strength Bars	Low Relaxation Strand	Deformed High-Strength Bars
Pre-tensioning			
Immediately prior to transfer ($f_{pt} + \Delta f_{pES}$)	0.70 f_{pu}	0.75 f_{pu}	-
At service limit state after all losses (f_{pe})	0.80 f_{py}	0.80 f_{py}	0.80 f_{py}
Post-tensioning			
Prior to seating - short-term f_s may be allowed	0.90 f_{py}	0.90 f_{py}	0.90 f_{py}
At anchorages and couplers immediately after anchor set ($f_{pt} + \Delta f_{pES} + \Delta f_{pA}$)	0.70 f_{pu}	0.70 f_{pu}	0.70 f_{pu}
At end of the seating loss zone immediately after anchor set ($f_{pt} + \Delta f_{pES} + \Delta f_{pA}$)	0.70 f_{pu}	0.74 f_{pu}	0.70 f_{pu}
At service limit state after losses (f_{pe})	0.80 f_{py}	0.80 f_{py}	0.80 f_{py}

The tendon stress at any of the strength or extreme event limit states shall not exceed the tensile strengths of Table 9.6.2-2.

Table 9.6.2-2 - Properties of Prestressing Strand and Bar

Material	Grade or Type	Diameter in mm	Tensile Strength, f_{pu} (MPa)	Yield Strength, f_{py} (MPa)
Strand	1725 MPa (Grade 250) 1860 MPa (Grade 270)	6.35 to 15.24 9.53 to 15.24	1725 1860	85% of f_{pu}, except 90% of f_{pu} for low-relaxation strand
Bar	Type 1, Plain Type 2, Deformed	19 to 35 16 to 35	1035 1035	85% of f_{pu} 80% of f_{pu}

9.6.3 Stress Limitations for Concrete

In the LRFD Specification, stress limits for segmentally constructed bridges have been added to those for traditional non-segmental construction, which appear alone in the Standard Specifications. The values in the CUS edition of the LRFD Specification that can be compared to those of the Standard Specifications appear very different. The reason is that in the LRFD Specification the value of f_c' is specified as in ksi, while in the Standard Specifications the values are in psi.

For non-segmental construction, the substantive changes represented by the LRFD Specification result in both higher and lower limits than the Standard Specifications.

The limits on temporary compressive stresses before losses and, in areas other than the precompressed tensile zone, on tensile stresses below which auxiliary reinforcement is not required remain unchanged. The limit on temporary tensile stresses before losses where auxiliary reinforcement is provided has been reduced by 7%. The auxiliary reinforcement must resist 120% of the calculated tension, up from the 100% of the Standard Specifications.

The limit on compressive stress at service limit states after losses has been increased by 11% to 0.45f_c'. The limits on tensile stresses at the service limit states after losses remain unchanged.

9.6.4 Loss of Prestress

9.6.4.1 GENERAL

Loss of prestress can be characterized as that due to: (1) instantaneous loss and (2) time-dependent loss. Losses due to anchorage set, friction and elastic shortening are instantaneous. Losses due to creep, shrinkage and relaxation are time-dependent.

For pretensioned members, prestress losses due to: (1) elastic shortening, (2) shrinkage, (3) creep of concrete and (4) relaxation of steel must be considered. For members constructed and prestressed in a single stage, prestressing losses, relative to the stress immediately before transfer, may be taken as:

$$\Delta f_{pT} = \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2} \qquad (9.6.4.1\text{-}1)$$

where:

Δf_{pES} = loss due to elastic shortening (MPa)

Δf_{pSR} = loss due to shrinkage (MPa)

Δf_{pCR} = loss due to creep of concrete (MPa)

Δf_{pR2} = loss due to relaxation of steel after transfer (MPa)

Where the appropriate lump sum estimate of losses specified in S5.9.5.3 is used, that part of the loss due to relaxation which occurs before transfer in pretensioned members, Af_{pR1}, should be deducted from the total relaxation.

For post-tensioned members, prestress losses due to: (1) friction and (2) anchorage set must be considered in addition to the losses considered for pretensioned members. This is reflected in Equation 9.6.4.1-2.

$$\Delta f_{pT} = \Delta f_{pF} + \Delta f_{pA} + \Delta f_{pES} + \Delta f_{pSR} + \Delta f_{pCR} + \Delta f_{pR2} \qquad (9.6.4.1\text{-}2)$$

where:

Δf_{pT} = total loss (MPa)

Δf_{pF} = loss due to friction (MPa)

Δf_{pA} = loss due to anchorage set (MPa)

Accurate estimate of total prestress loss requires recognition that the time-dependent losses resulting from creep and relaxation are interdependent. If required, rigorous calculation of prestress losses should be made in accordance with a method supported by research data. However, for conventional construction, such refinement is seldom warranted or even possible at the design stage, since many of the factors are either unknown or beyond the designer's control. Thus, three methods of estimating time-dependent losses are provided in the LRFD Specification: (1) a simplified lump-sum estimate, (2) a refined itemized estimate, and (3) the background necessary to perform a rigorous time-step analysis.

A procedure for estimating the losses for partially prestressed members which is analogous to that for fully prestressed members is outlined in Article SC5.9.5.1.

9.6.4.2 INSTANTANEOUS LOSSES

9.6.4.2.1 Anchorage Set

Anchorage set loss is the result of movement of the tendon prior to seating of the wedges or the anchorage gripping device. The magnitude of the minimum set is prestressing-system dependent. The loss occurs prior to transfer and is the major source of difference between the jacking stress and the stress at transfer.

The magnitude of the anchorage set should be taken as the greater of that required to control stresses in the prestressing steel at transfer, or that recommended by the anchorage manufacturer.

A common value of anchorage set is 10 mm, although values as low as 1.6 mm are possible with some anchorage devices. For wedge-type anchors, the set may vary from 3 mm to 10 mm.

For short tendons, a small anchorage set is desirable and power wedge seating should be used. For long tendons, the effect of anchorage set on tendon force is insignificant and power seating is not necessary.

Due to friction, the loss of prestress due to anchorage set may affect only a part of the member.

9.6.4.2.2 Friction

The only friction loss possible in a pretensioned member is at hold-down devices for draping or harping tendons. The LRFD Specification specifies that these losses should be considered.

In post-tensioned members, losses due to friction between internal tendons and duct walls may be taken as given by Equation 9.6.4.2.2-1.

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-(Kx + \mu\alpha)}\right) \qquad (9.6.4.2.2\text{-}1)$$

Losses due to friction between the external tendon across a single deviator pipe may be taken as:

$$\Delta f_{pF} = f_{pj}\left(1 - e^{-\mu(\alpha + 0.04)}\right) \qquad (9.6.4.2.2\text{-}2)$$

where:

f_{pj} = stress in the prestressing steel at jacking (MPa)

x = length of a prestressing tendon from the jacking end to any point under consideration (mm)

K = wobble friction coefficient (mm^{-1})

μ = coefficient of friction (1/RAD)

α = sum of the absolute values of angular change of prestressing steel path from jacking end, or from the nearest jacking end if tensioning is done equally at both ends, to the point under investigation (RAD)

e = base of Napierian logarithms

These losses are a function of the jacking stress in the tendon, the tendon geometry and friction coefficients of the specified tendons. The friction coefficients, the wobble friction coefficient and the coefficient of friction, should be based upon experimental data for the specified tendons. In the absence of such data, estimated ranges of the values are given in Table 9.6.4.2.2-1.

The 0.04 radians in Equation 2 represents an inadvertent angle change. This angle change may vary depending on job specific tolerances on deviator pipe placement. The inadvertent angle change need not be considered for calculation of losses due to wedge seating movement. This additional loss seems due, in part, to the tolerances allowed in the placement of the deviator pipes. Small misalignments of the pipes can result in significantly increased angle changes of the tendons at the deviation points. The inadvertent angle change of 0.04 radians added to the theoretical angle change accounts for this effect based on typical deviator length of 915 mm and placement tolerance of ±9 mm. The 0.04 value is to be added to the theoretical value at each deviator. The value may vary with tolerances on pipe placement.

Table 9.6.4.2.2-1 - Friction Coefficients for Post-Tensioning Tendons

Type of Steel	Type of Duct	K	μ
Wire or strand	Rigid and semirigid galvanized metal sheathing	6.6×10^{-7}	0.15-0.25
	Polyethylene	6.6×10^{-7}	0.23
	Rigid steel pipe deviators for external tendons	6.6×10^{-7}	0.25
High strength bars	Galvanized metal sheathing	6.6×10^{-7}	0.30

9.6.4.2.3 Elastic Shortening

The loss in prestress due to elastic shortening in pretensioned members is taken as the concrete stress at the centroid of the prestressing steel at transfer, f_{cgp}, multiplied by the modular ratio of the prestressing steel to the concrete at transfer. This is reflected in Equation 9.6.4.2.3-1.

$$\Delta f_{pES} = \frac{E_p}{E_{ci}} f_{cgp} \qquad (9.6.4.2.3\text{-}1)$$

where:

f_{cgp} = sum of concrete stresses at the center of gravity of prestressing tendons due to the prestressing force at transfer and the self-weight of the member at the sections of maximum moment (MPa)

E_p = modulus of elasticity of prestressing steel (MPa)

E_{ci} = modulus of elasticity of concrete at transfer (MPa)

Losses due to elastic shortening may also be calculated in accordance with other published guidelines. Losses due to elastic shortening for external tendons may be calculated in the same manner as for internal tendons.

For typical pretensioned members, f_{cgp} may be calculating assuming the prestressing steel stress to be 65% of tendon's tensile strength for stress-relieved strand or high-strength bars, or 70% of the tensile strength for low-relaxation strand. These assumed steel stresses represent a slight increase over current practice as represented by the Standard Specifications where the prestressing steel force is assumed to be 63% for stress-relieved strand and 69% for low-relaxation strand.

For post-tensioned members other than slab systems, the loss in prestress due to elastic shortening is taken as that for pretensioned members multiplied by a term which is a function of the number of identical tendons. This is reflected in Equation 9.6.4.2.3-2.

$$\Delta f_{pES} = \frac{N-1}{2N} \frac{E_p}{E_{ci}} f_{cgp} \qquad (9.6.4.2.3\text{-}1)$$

where:

N = number of identical prestressing tendons

f_{cgp} = sum of concrete stresses at the center of gravity of prestressing tendons due to the prestressing force after jacking and the self-weight of the member at the sections of maximum moment (MPa)

If the identical tendons cannot be used, an equation is given in the Commentary to convert the different tendons to an equivalent tendon.

For post-tensioned members with bonded tendons, f_{cgp} may be taken at mid-span, or for continuous construction, at the maximum moment section. For members with unbonded tendons, f_{cgp} may be taken as the concrete stress at the centroid of the steel averaged along the length of the member.

For post-tensioned slab systems, the loss due to elastic shortening may be taken as 25% of that given by Equation 9.6.4.2.3-1.

9.6.4.3 TIME-DEPENDENT LOSSES

9.6.4.3.1 Simplified Lump Sum Estimate

The lump sum time-dependent prestress losses are given in Table 9.6.4.3.1-1. These simplified estimates may be used for: (1) pretensioned members stressed after attaining a concrete compressive strength, f_{cu}', of 24 MPa, or (2) post-tensioned non-segmental members with spans up to 50 000 mm and stressed at a concrete age of between 10 and 30 days. Additional requirements are that the concrete members: (a) be of normal-weight concrete, (b) steam or moist cured, (c) reinforced with bars or strands with normal relaxation properties, and (d) be sited in average exposures and temperatures.

Table 9.6.4.3.1-1 - Time-Dependent Losses in MPa

Type of Beam Section	Level	For Wires and Strands with f_{pu} = 1620, 1725 or 1860 MPa	For Bars with f_{pu} = 1000 or 1100 MPa
Rectangular Beams and Solid Slabs	Upper Bound Average	200 + 28 PPR 180 + 28 PPR	130 + 41 PPR
Box Girder	Upper Bound Average	145 + 28 PPR 130 + 28 PPR	100
I-Girder	Average	$230\left[1 - 0.15\frac{f'_c - 41}{41}\right] + 41\,PPR$	130 + 41 PPR
Single T, Double T, Hollow Core and Voided Slab	Upper Bound Average	$270\left[1.0 - 0.15\frac{f'_c - 41}{41}\right] + 41\,PPR$ $230\left[1.0 - 0.15\frac{f'_c - 41}{41}\right] + 41\,PPR$	$210\left[1.0 - 0.15\frac{f'_c - 41}{41}\right] + 41\,PPR$

For concrete members of lightweight concrete, the tabularized values should be increased by 35 MPa.

For low-relaxation strands, the tabularized values may be reduced by: (1) 28 MPa for box girders, (2) 41 MPa for rectangular beams and I-girders, and (3) 55 MPa for single T's, double T's and solid, hollow-core and voided slabs.

For unusual exposure conditions, the tabularized values are not appropriate.

9.6.4.3.2 Refined Itemized Estimate

Refined itemized time-dependent losses are specified in Article S5.9.5.4. These estimates of losses due to each individual time-dependent source can provide a better estimate of total losses than those of Table 9.6.4.3.1-1.

These itemized losses are appropriate for prestressed non-segmental concrete members of: (1) span not greater than 75 000 mm, (2) normal-weight concrete and (3) strength at the time of prestress in excess of 24 MPa.

9.6.4.3.2a Shrinkage

The expressions for prestress loss due to shrinkage are a function of average annual ambient relative humidity, H, and are given as Equations 9.6.4.3.2a-1 and 9.6.4.3.2a-2 for pretensioned and post-tensioned members, respectively.

- for pretensioned members:

Δf_{pSR} = (117 - 1.03 H) (MPa) (9.6.4.3.2a-1)

- for post-tensioned members:

Δf_{pSR} = (93 - 0.85 H) (MPa) (9.6.4.3.2a-2)

where:

H = the average annual ambient relative humidity (%)

The average annual ambient relative humidity may be obtained from local weather statistics or taken from the map of Figure 9.6.4.3.2a-1.

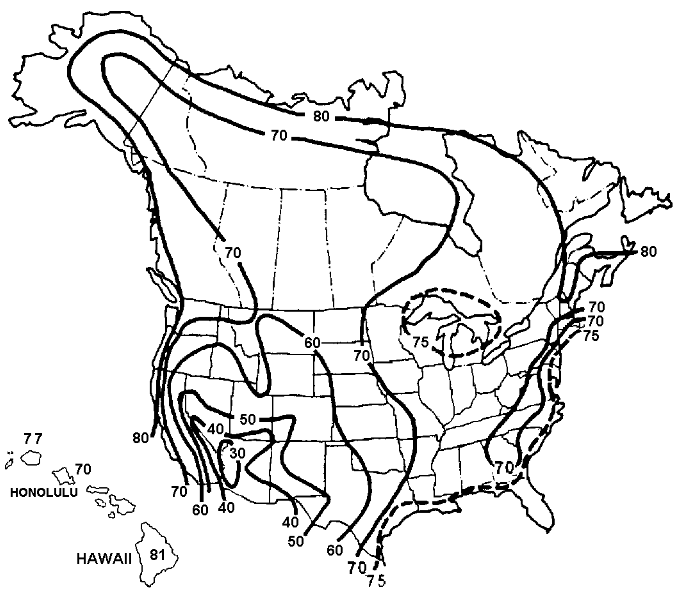


Figure 9.6.4.3.2a-1 - Annual Average Ambient Relative Humidity in %

9.6.4.3.2b Creep

The expression for prestress loss due to creep are a function of the concrete stress at the centroid of the prestressing steel at transfer, f_{cgp}, and the change in concrete stress at the centroid of the prestressing steel due to all permanent loads except those present at transfer, Δf_{cdp}, and is given as Equation 9.6.4.3.2b-1.

$\Delta f_{pCR} = 12.0\ f_{cgp} - 7.0\Delta f_{cdp}\ \$\ 0$ (9.6.4.3.2b-1)

where:

f_{cgp} = concrete stress at center of gravity of prestressing steel at transfer (MPa)

Δf_{cdp} = change in concrete stress at center of gravity of prestressing steel due to permanent loads, except the load acting at the time the prestressing force is applied. Values of Δf_{cdp} should be calculated at the same section or sections for which f_{cgp} is calculated (MPa)

The "greater than or equal to 0.0" in Equation 9.6.4.3.2b-1 is needed because a negative value could result in some cases of partial prestressing, but Δf_{pCR} should not be taken as less than 0.0.

The values of f_{cgp} and Δf_{cdp} should be determined at the same section for use in the equation.

The itemized prestress loss due to creep should never be taken as less than zero. In some cases of partial prestressing, Equation 9.6.4.3.2b-1 may yield a negative number. In these cases, the loss due to creep should be taken as zero.

For camber and deflection calculations, the values of f_{cgp} and Δf_{cdp} may be computed as the stress at the centroid of the prestressing steel averaged along the length of the member, within the limitations specified in Article S5.9.5.5.

9.6.4.3.2c Relaxation

The total relaxation at any time after transfer is composed of two components: (1) relaxation at transfer and (2) relaxation after transfer.

Generally, the initial relaxation loss is now determined by the Fabricator. Where the Engineer is required to make an independent estimate of the initial relaxation loss, or chooses to do so as provided in Article S5.9.5.1, the provisions of this article may be used as a guide. If project-specific information is not available, the value of f_{pj} may be taken as 0.80 f_{pu} for the purpose of this calculation.

Article S5.9.5.4.4b provides equations to estimate relaxation at transfer for pretensioned members, initially stressed in excess of 50% of tendons tensile strength. Equations 9.6.4.3.2c-1 and 9.6.4.3.2c-2 are for stress-relieved strand and low-relaxation strand, respectively.

- for stress-relieved strand:

$$\Delta f_{pR1} = \frac{\log(24.0t)}{10.0}\left[\frac{f_{pj}}{f_{py}} - 0.55\right] f_{pj} \qquad (9.6.4.3.2c\text{-}1)$$

- for low-relaxation strand:

$$\Delta f_{pR1} = \frac{\log(24.0t)}{40.0}\left[\frac{f_{pj}}{f_{py}} - 0.55\right] f_{pj} \qquad (9.6.4.3.2c\text{-}2)$$

where:

t = time estimated in days from stressing to transfer (DAYS)

f_{pj} = initial stress in the tendon at the end of stressing (MPa)

f_{py} = specified yield strength of prestressing steel (MPa)

The relaxation loss for low-relaxation strand merely represents one quarter of that for stress-relieved strand with equation identical otherwise.

Article S5.9.5.4.4c provides equations to estimate relaxation after transfer for pretensioned members with stress-relieved or low-relaxation strands, and post-tensioned members with stress-relieved strand or 1000 MPa to 1100 MPa bars.

Relaxation losses increase with increasing temperatures. The expressions given for relaxation are appropriate for normal temperature ranges only.

9.6.4.3.3 Rigorous Analysis

For segmental construction, lightweight concrete construction, staged prestressing with spans greater than 50 m and other bridges where more exact evaluation of prestress losses are desired, losses should be calculated in accordance with a method supported by proven research data.

For multi-stage construction and/or prestressing, the losses should be computed considering the elapsed time between each stage. Such computation can be handled with the time-steps method.

9.7 SHEAR AND TORSION

9.7.1 Introduction

Two design procedures are available in the LRFD Specification for shear and torsion design of components: (1) the sectional model as specified in Article S5.8.3 and (2) the strut-and-tie model as specified in Article S5.6.3. Both of these models are new to the LRFD Specification, replacing the sectional model of the Standard Specifications. Just as the expressions for flexural resistance, these models are applicable to concrete members reinforced with conventional rebars, prestressing tendons or any combination thereof.

The sectional model is more appropriate for the design of regions of members where plane sections remain plane after loading, and the response of the section depends only upon the sectional force effects, moment, shear, axial load and torsion, and not upon how the force effects are introduced into the member. Such regions are termed "flexural regions" in the LRFD Specification.

The strut-and-tie model is the only appropriate procedure for regions of members where the assumption of plane sections remaining plane is not valid, and where how the force effects are introduced into the members is significant. Such regions are termed "regions near discontinuities" and include regions adjacent to abrupt changes in cross-section, openings and dapped ends, deep beams and corbels.

9.7.2 Sectional Model

In the sectional design approach, the member is designed by comparing the factored shear force and the factored shear resistance at a number of sections along the members length. Traditionally for bridge design, this check is made at the tenth points along the span and at locations near the supports.

The sectional model of the LRFD Specification was developed by Professor M. P. Collins of the University of Toronto and Professor D. Mitchell of McGill University and is based upon modified compression field theory including a variable angle of concrete compression.

The sectional model of the Standard Specifications is based upon an assumption of 45E-angle of principal compressive stress inclination. Research since 1971 has demonstrated that the angle of inclination of the compression is not 45E.

9.7.2.1 MODIFIED COMPRESSION FIELD THEORY

The modified compression field theory is a behavioral model of cracked reinforced concrete developed through non-traditional experiments utilizing elements subjected to uniform stresses instead of traditional member tests.

The theory assumes that a member can be modeled as a variable-angle truss where the angle of inclined cracks defines the angle of the assumed diagonal truss members. This angle is also assumed to coincide with the angle of inclination of the principal tensile strains. While the theory assumes that after cracking the concrete can no longer resist tension, it assumes that shear can be resisted by a field of diagonal compression, hence the name, compression field theory. Examining equilibrium of a lower joint of the hypothesized truss, one sees that the vertical reinforcement must resist the vertical component of the diagonal compression, and the longitudinal reinforcement the horizontal component.

The equilibrium conditions for cracked reinforced concrete are expressed in terms of average stresses. Likewise, the compatibility conditions are expressed in terms of average strains. In both cases, the equilibrium stresses and compatibility strains are averaged over lengths greater than the spacing of the cracks.

An important simplifying assumption of the modified compression field theory is that the direction of largest average compressive stress in the cracked concrete is identical to that of the largest average compressive strain. In other words, the direction of the principal average stress is assumed to be identical to that of the principal average strain.

The non-traditional shear tests demonstrate several important principles which are applied in the sectional model procedures of the LRFD Specifications.

The principal compressive stress in the cracked concrete is not only a function of the principal compressive strain, but also the principal tensile strain. The compressive resistance of the cracked concrete decreases as its principal tensile strain increases.

Significant tensile stresses exist in cracked concrete and increase the shear resistance of the cracked concrete.

9.7.2.2 NOMINAL SHEAR RESISTANCE

9.7.2.2.1 General

The nominal shear resistance, V_n, of Equation 9.7.2.2.1-1 is the summation of the various components of the shear resistance of a concrete member.

$$V_n = V_c + V_s + V_p \qquad (9.7.2.2.1\text{-}1)$$

These components are the shear resistances due to tensile stress in the concrete, V_c; tensile stress in the transverse reinforcement, V_s; and the vertical component of the prestressing, V_p. While V_p is clearly only applicable for prestressed members, the expressions for V_c and V_s are applicable to both non-prestressed and prestressed concrete members.

Equation 9.7.2.2.1-2 is an upper bound on nominal shear resistance, V_n, which makes sure that the concrete in the web will not crush before the transverse reinforcement yields.

$$V_n = 0.25\, f'_c\, b_v\, d_v + V_p \qquad (9.7.2.2.1\text{-}2)$$

The nominal shear resistance due to tensile stress in the concrete, V_c, of Equation 9.7.2.2.1-3 is a function of β, the residual tensile stress factor, which indicates the ability of the cracked concrete to transmit tensile stress.

$$V_c = 0.083\,\beta\sqrt{f'_c}\,b_v d_v \qquad (9.7.2.2.1\text{-}3)$$

The nominal shear resistance due to tensile stress in the transverse reinforcement, V_s, of Equation 9.7.2.2.1-4 is a function of θ, the angle of inclination of the diagonal compressive stresses.

$$V_s = \frac{A_v f_y d_v (\cot\theta + \cot\alpha)\sin\alpha}{s} \qquad (9.7.2.2.1\text{-}4)$$

where:

b_v = effective web width taken as the minimum web width measured parallel to the neutral axis, between the resultants of the tensile and compressive forces due to flexure, or for circular sections, the diameter of the section, modified for the presence of ducts where applicable (mm)

d_v = effective shear depth taken as the distance, measured perpendicular to the neutral axis, between the resultants of the tensile and compressive forces due to flexure, but it need not be taken less than the greater of $0.9d_e$ or 0.72h (mm)

A_v = area of all legs of one stirrup (mm^2)

s = spacing of stirrups (mm)

β = factor indicating ability of diagonally cracked concrete to transmit tension

Previous editions of these Specifications permitted d for prestressed members to be taken as 0.8h. The 0.72 factor is 0.9 x 0.8.

In determining b_v, at a particular level, the diameters of ungrouted ducts or one-half the diameters of grouted ducts, at that level, shall be subtracted from the web width.

9.7.2.2.2 Simplified Procedure for Non-prestressed Sections

For non-prestressed sections not subjected to axial tension and reinforced with at least the minimum amount of transverse steel specified in Article S5.8.2.5, or having an overall depth less than 400 mm, β may be taken as 2.0 and θ as 45°. With these assumed values, the expressions for nominal shear resistance are essentially identical to those traditionally used. However, recent large-scale experiments have suggested that these traditional expressions for shear resistance can be unconservative.

9.7.2.2.3 General Procedure

In general, values of β and θ can be taken from Table 9.7.2.2.3-1 or Table 9.7.2.2.3-2 for sections with or without transverse reinforcement, respectively.

Table 9.7.2.2.3-1 - Values of θ and β for Sections with Transverse Reinforcement

$\frac{v}{f'_c}$	ε_x x 1,000										
	#-0.20	#-0.10	#-0.05	#0	#0.125	#0.25	#0.50	#0.75	#1.00	#1.50	#2.00
#0.075	22.3 6.32	20.4 4.75	21.0 4.10	21.8 3.75	24.3 3.24	26.6 2.94	30.5 2.59	33.7 2.38	36.4 2.23	40.8 1.95	43.9 1.67
#0.100	18.1 3.79	20.4 3.38	21.4 3.24	22.5 3.14	24.9 2.91	27.1 2.75	30.8 2.50	34.0 2.32	36.7 2.18	40.8 1.93	43.1 1.69
#0.125	19.9 3.18	21.9 2.99	22.8 2.94	23.7 2.87	25.9 2.74	27.9 2.62	31.4 2.42	34.4 2.26	37.0 2.13	41.0 1.90	43.2 1.67
#0.150	21.6 2.88	23.3 2.79	24.2 2.78	25.0 2.72	26.9 2.60	28.8 2.52	32.1 2.36	34.9 2.21	37.3 2.08	40.5 1.82	42.8 1.61
#0.175	23.2 2.73	24.7 2.66	25.5 2.65	26.2 2.60	28.0 2.52	29.7 2.44	32.7 2.28	35.2 2.14	36.8 1.96	39.7 1.71	42.2 1.54
#0.200	24.7 2.63	26.1 2.59	26.7 2.52	27.4 2.51	29.0 2.43	30.6 2.37	32.8 2.14	34.5 1.94	36.1 1.79	39.2 1.61	41.7 1.47
#0.225	26.1 2.53	27.3 2.45	27.9 2.42	28.5 2.40	30.0 2.34	30.8 2.14	32.3 1.86	34.0 1.73	35.7 1.64	38.8 1.51	41.4 1.39
#0.250	27.5 2.39	28.6 2.39	29.1 2.33	29.7 2.33	30.6 2.12	31.3 1.93	32.8 1.70	34.3 1.58	35.8 1.50	38.6 1.38	41.2 1.29

Table 9.7.2.2.3-2 - Values of θ and β for Sections without Transverse Reinforcement

s_{xe} (mm)	ε_x x 1000										
	#-0.20	#-0.10	#-0.05	#0	#0.125	#0.25	#0.50	#0.75	#1.00	#1.50	#2.00
#130	25.4 6.36	25.5 6.06	25.9 5.56	26.4 5.15	27.7 4.41	28.9 3.91	30.9 3.26	32.4 2.86	33.7 2.58	35.6 2.21	37.2 1.96
#250	27.6 5.78	27.6 5.78	28.3 5.38	29.3 4.89	31.6 4.05	33.5 3.52	36.3 2.88	38.4 2.50	40.1 2.23	42.7 1.88	44.7 1.65
#380	29.5 5.34	29.5 5.34	29.7 5.27	31.1 4.73	34.1 3.82	36.5 3.28	39.9 2.64	42.4 2.26	44.4 2.01	47.4 1.68	49.7 1.46
#500	31.2 4.99	31.2 4.99	31.2 4.99	32.3 4.61	36.0 3.65	38.8 3.09	42.7 2.46	45.5 2.09	47.6 1.85	50.9 1.52	53.4 1.31
#750	34.1 4.46	34.1 4.46	34.1 4.46	34.2 4.43	38.9 3.39	42.3 2.82	46.9 2.19	50.1 1.84	52.6 1.60	56.3 1.30	59.0 1.10
#1000	36.6 4.06	36.6 4.06	36.6 4.06	36.6 4.06	41.2 3.20	45.0 2.62	50.2 2.00	53.7 1.66	56.3 1.43	60.2 1.14	63.0 0.95
#1500	40.8 3.50	40.8 3.50	40.8 3.50	40.8 3.50	44.5 2.92	49.2 2.32	55.1 1.72	58.9 1.40	61.8 1.18	65.8 0.92	68.6 0.75
#2000	44.3 3.10	44.3 3.10	44.3 3.10	44.3 3.10	47.1 2.71	52.3 2.11	58.7 1.52	62.8 1.21	65.7 1.01	69.7 0.76	72.4 0.62

In these tables, β and θ are given as functions of the shear stress on the concrete, v (from Equation 9.7.2.2.3-1), the strain in the reinforcement on the flexural tension side of the member, ε_x (from Equations 9.7.2.2.3-2 through 9.7.2.2.3-4) and the crack spacing parameter, s_{xe}. (From Equation 9.7.2.2.3-5).

The shear stress on the concrete is determined as:

$$v = \frac{V_u - \varphi V_p}{\varphi b_v d_v} \qquad (9.7.2.2.3\text{-}1)$$

The strain in the reinforcement on the flexural tension side of the member is determined as follows:

- If the section contains at least the minimum transverse reinforcement as specified in Articles 5.8.2.5:

$$\varepsilon_x = \frac{\left(\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}\right)}{2(E_sA_s + E_pA_{ps})} \le 0.002$$

(9.7.2.2.3-2)

- If the section contains less than the minimum transverse reinforcement as specified in Articles 5.8.2.5:

$$\varepsilon_x = \frac{\left(\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}\right)}{E_sA_s + E_pA_{ps}} \le 0.002$$

(9.7.2.2.3-3)

- If the value of ε_x from Equations 1 or 2 is negative, the strain shall be taken as:

$$\varepsilon_x = \frac{\left(\frac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}\right)}{2\left(E_cA_c + E_sA_s + E_pA_{ps}\right)} \qquad (9.7.2.2.3\text{-}4)$$

$$s_{xe} = s_x \frac{1.38}{a_g + 0.63} \le 2000 \text{ mm} \qquad (9.7.2.2.3\text{-}5)$$

where:

φ = resistance factor for shear specified in Article S5.5.4.2

A_c = area of concrete on the flexural tension side of the member as shown in Figure S5.8.3.4.2-1 (mm^2)

A_{ps} = area of prestressing steel on the flexural tension side of the member, shown in Figure S5.8.3.4.2-1 (mm^2)

A_s = area of non-prestressed reinforcing steel on flexural tension side of member, as shown in Figure S5.8.3.4.2-1. In calculating A_s for use in this equation, bars which are terminated at a distance less than their development length from the section under consideration shall be ignored. (mm^2)

f_{po} = a parameter taken as modulus of elasticity of prestressing tendons multiplied by the locked in difference in strain between the prestressing tendons and the surrounding concrete (MPa). For the usual levels of prestressing, a value of 0.7 f_{pu} will be appropriate for both pretensioned and posttensioned members. Within the transfer length, f_{po} shall be increased linearly from zero at the location where the bond between the strands and concrete commences to its full value at the end of the transfer length.

a_g = maximum aggregate size (mm)

s_x = the lesser of either d_v or the maximum distance between layers of longitudinal crack control reinforcement, where the area of the reinforcement in each layer is not less than $0.003b_v s_x$ (mm)

The values of β and θ are based upon the ability to transmit stresses across diagonally cracked concrete. As cracks become wider, the ability to transmit stress decreases.

The strain, ε_x, is an indication of the longitudinal stiffness of the section and the magnitude of the moment, axial force and prestressing force at the section. For sections reinforced with large percentages of mild steel or with prestressing tendons, or subject to small moments, the values of ε_x are relatively low. For many prestressed sections, it has been found that ε_x is essentially zero. Lower values of ε_x correspond to small web deformations and higher values of V_c.

While it is most appropriate that coincident values of V_u and M_u be used in the shear resistance determination, it is not necessarily required. In determining ε_x at a section, it is conservative to take M_u as the highest factored moment that will occur at the section rather than the coincident moment.

Since ε_x is a function of θ in Equations 2 through 4, and θ is related to ε_x in Tables 1 and 2, an iterative solution is required. A flow chart for shear design is shown in Figure 1 which indicates the iterative solution for β using θ and ε_x. Alternatively, as a conservative approach, the values of β & θ corresponding to the shear stress on the section, or S_{xe} in case of sections without transverse reinforcement, and a value of ε_x = 0.002 may be used for design. However, this approach may grossly underestimate V_c.

The specifications also presents a more accurate method for calculating ε_x. The more accurate method is recommended for evaluating existing members and may result in higher shear resistance than calculated using ε_x from Equations 2 through 4.

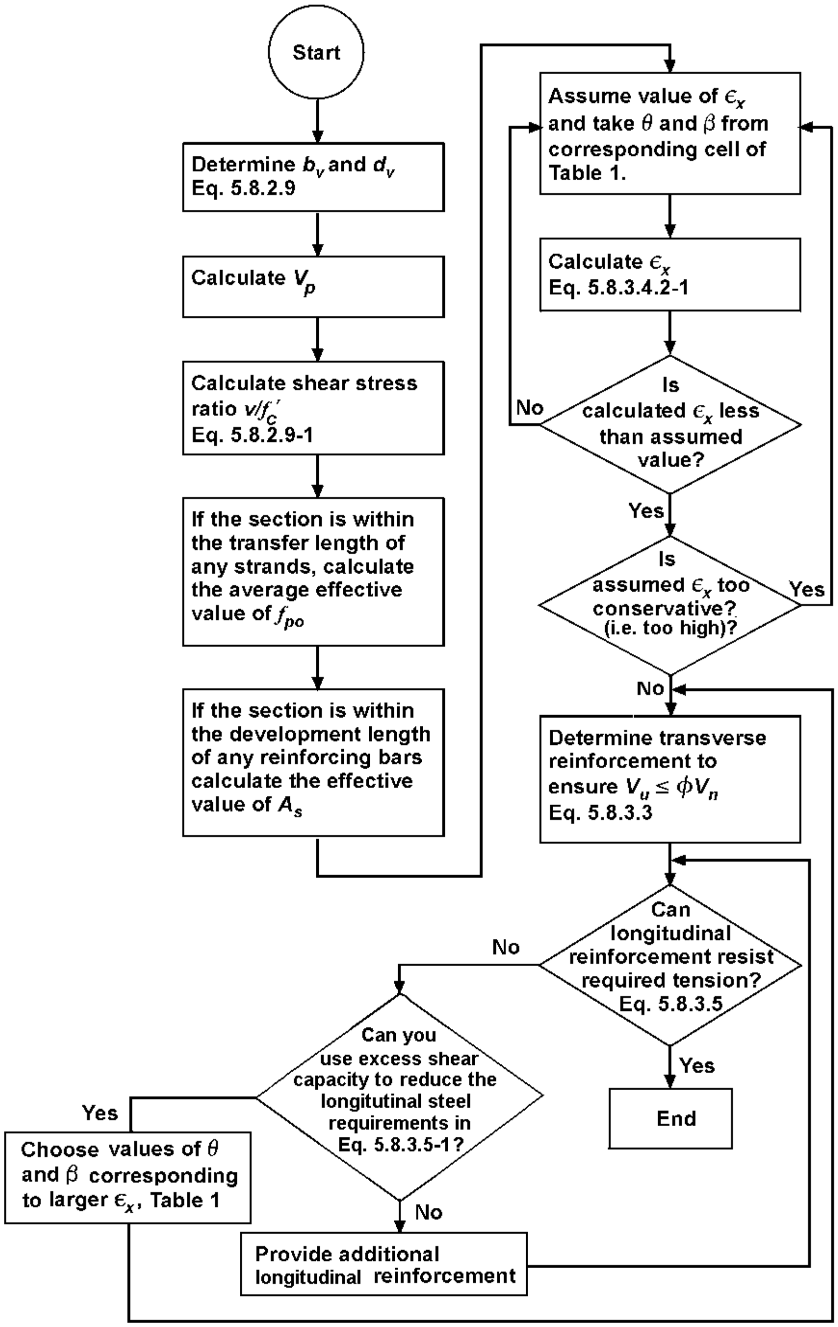


Figure 9.7.2.2.3-1 - Flow Chart for Shear Design

9.7.2.3 LONGITUDINAL REINFORCEMENT

Carrying part of the shear by tensile stresses in the concrete reduces the required amount of web reinforcement, but increases the stresses in the longitudinal reinforcement at a crack. Consequently, the longitudinal reinforcement may have to be increased above that traditionally required to resist bending moment alone by either adding additional tendons and/or mild steel.

Therefore, longitudinal reinforcement in sections not subject to torsion must satisfy the requirements of Article S5.8.3.5.

The tensile resistance of the reinforcement on the flexural tension side of the member, taking into account any lack of full development of that reinforcement, shall be greater than or equal to the force T calculated as:

$$T = \frac{M_u}{d_v \phi} + 0.5 \frac{N_u}{\phi} + \left(\frac{V_u}{\phi} - 0.5 V_s - V_p \right) \cot \theta \qquad (5.8.3.5\text{-}1)$$

The tensile resistance of the reinforcement when fully developed = $A_s f_y + A_{ps} f_{ps}$.

where:

V_s = shear resistance provided by the transverse reinforcement at the section under investigation as given by Equation S5.8.3.3-4, except V_s shall not be taken as greater than V_u/φ (N)

θ = angle of inclination of diagonal compressive stresses used in determining the nominal shear resistance of the section under investigation as determined by Article S5.8.3.4 (DEG)

φ = resistance factors taken from Article S5.5.4.2 as appropriate for moment, shear and axial resistance

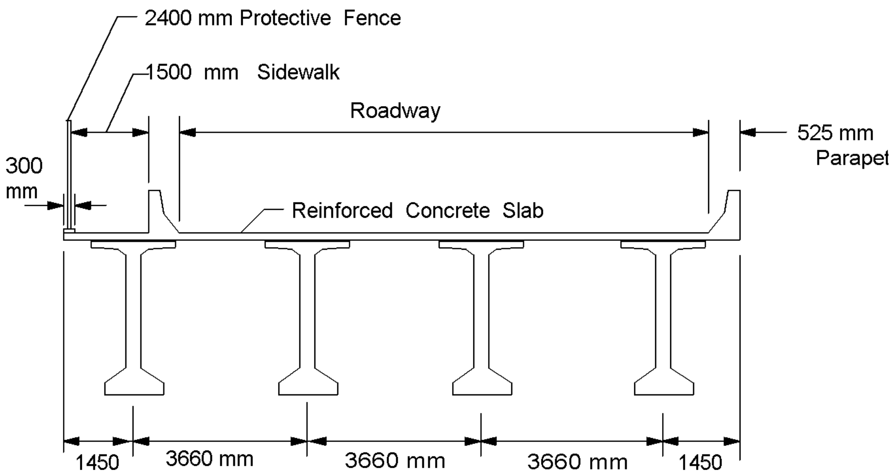
This requirement avoids yielding of the longitudinal reinforcement for combined loading of moment, M_u, axial load, N_u, and shear, V_u.

9.8 DURABILITY

The LRFD Specification has a complete set of provisions dedicated to increased durability of concrete bridges contained within Article S5.12. Some of the provisions are new provisions, while others previously appeared elsewhere in the Standard Specifications. Design considerations for durability include concrete quality, protective coatings, minimum cover, distribution and size of reinforcement, details and crack widths. The principal aim of the provisions of Article S5.12, with regard to durability, is the prevention of corrosion of reinforcing steel and prestressing tendons.

The provisions under this article include general requirements related to concrete quality, alkali-silica reactive aggregates, concrete cover, protective coatings and protection for prestressing tendons.

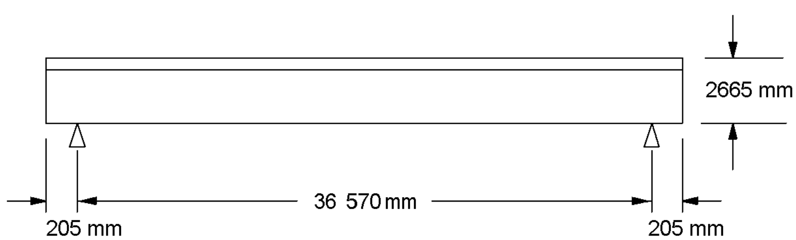
9.9 DESIGN EXAMPLE - PRESTRESS CONCRETE I-BEAM



Use $\eta = \eta_D \eta_R \eta_I = 1.0$

SUPERIMPOSED LOADS

Parapet and Fence Per Beam = 3.77 N/mm



FWS = 3.94 N/mm

BASIC BEAM SECTION PROPERTIES

Span	= 36 570 mm
Depth	= 2435 mm
Top Flange	= 1065 mm
Thickness of Web	= 205 mm
Area	= 8.26E+05 mm^2
Moment of Inertia	= 6.313E+11 mm^4
N.A. to Top, Y_t	= 1218.3 mm
N.A. to Bottom, Y_b	= 1216.7 mm

S_{TOP}	= 5.182E+08 mm^3
S_{BOT}	= 5.189E+08 mm^3
Drape Point	= 12190 mm from CL BRG
Prestressing Force Eccentricity at End	= 974 mm
Prestressing Force Eccentricity at Center	= 1072 mm
Distance from Prestressing Force to Bottom of Beam:	
at end of beam	= 242.7 mm
at center	= 144.7 mm

COMPOSITE BEAM SECTION PROPERTIES

Since the haunch is used to adjust the elevation of the slabs, its actual thickness may be smaller than that assumed in the design. In many cases, the haunch is considered in determining the loads on the beam, but are ignored in determining section properties and stresses. This assumption will be followed in this example. Also notice that the concrete of the slab is transformed into its equivalent beam concrete.

Effective Slab Width	= 2993 mm
Slab Thickness, t_s	= 205 mm
Haunch Thickness, t_h	= 25 mm
Total Depth (including the haunch)	= 2665 mm
S_{TOP} Slab	= 1.12E+09 mm^3
S_{TOP} Beam	= 1.568E+09 mm^3
S_{BOT} Beam	= 6.858E+08 mm^3
Shift in N.A., d'	= 477.4 mm
Beam Tension Area	= 507 762 mm^2
Moment of Inertia	= 1.162E+12 mm^4
N.A. to Slab Top	= 971 mm
N.A. to Beam Top	= 741 mm
N.A. to Beam Bot	= 1694 mm
Prestressing Strand Eccentricity:	
at end	= 1451.4 mm
at center	= 1549.4 mm

MATERIAL PROPERTIES

Beam Conc, F'_{cb}	= 55 MPa
Slab Conc, F'_{cs}	= 28 MPa
Beam Conc Init, F'_{ci}	= 50 MPa
Beam Conc Modulus, E_c	= 35 598 MPa
Initial Conc Modulus, E_{ci}	= 33 941 MPa
Slab Concrete E_c	= 25 426 MPa
Strand Area	= 99 mm^2
Strand Diameter	= 12.7 mm
Strand Yield, F_y=0.9 f_u	= 1674 MPa
Strand Ult, F_u	= 1860 MPa
Strand Modulus, E_p	= 197 000 MPa

Concrete density may vary depending on the materials used in the mix and on the amount of voids in the concrete. To produce the critical cases, some designers base the value of the modulus of elasticity on a density lower than that used to calculate dead loads. In this example, the concrete density is assumed to be 2320 kg/m^3 for modulus of elasticity calculations and 2400 kg/m^2 for load calculations.

TRANSVERSE REINFORCEMENT

Steel Area = 258 mm^2
Steel Yield, F_y = 420 MPa

Time of Transfer = 3 Days
Average Humidity = 70%

Design calculations are presented in summary form for several points along the beam. Complete calculations are provided for the point along the beam identified by the symbol "**". These complete calculations follow the group of summary calculations to which they apply.

NOTATIONS

Diaph: concrete diaphragm at mid-span, weight per girder = 36 240 N
SIP: deck slab stay-in-place metal forms taken as 100 kg/m^2
FWS: future wearing surface on the bridge
DL_1: noncomposite dead loads
DL_2: composite dead loads
LL: live loads
I: dynamic load allowance (impact)

DEAD LOAD FORCES

Section@ mm		Unfactored DL Moments (N.mm) and Shears (N)				
		Dead Load Components				
		Girder	Diaphragm	Slab + SIP	FWS	Parapet & Fence
0	M	0.000E+00	0.000E+00	0.000E+00	0.000E+00	0.000E+00
	V	3.552E+05	1.812E+04	4.002E+05	7.209E+04	6.898E+04
2195	M	7.327E+08	3.376E+07	8.255E+08	1.487E+08	1.414E+08
	V	3.126E+05	1.812E+04	3.522E+05	6.344E+04	6.070E+04
3657	M	1.169E+09	6.626E+07	1.317E+09	2.373E+08	2.271E+08
	V	2.841E+05	1.812E+04	3.202E+05	5.767E+04	5.518E+04
7314	M	2.079E+09	1.325E+08	2.342E+09	4.219E+08	4.037E+08
	V	2.131E+05	1.812E+04	2.401E+05	4.325E+04	4.138E+04
10971	M	2.728E+09	1.988E+08	3.074E+09	5.537E+08	5.298E+08
	V	1.421E+05	1.812E+04	1.601E+05	2.884E+04	2.760E+04
DRAPE PT	M	2.887E+09	2.209E+08	3.253E+09	5.860E+08	5.607E+08
	V	1.184E+05	1.812E+04	1.334E+05	2.403E+04	2.299E+04
14628	M	3.118E+09	2.651E+08	3.513E+09	6.328E+08	6.055E+08
	V	7.103E+04	1.812E+04	8.004E+04	1.442E+04	1.380E+04
CENTERLINE	M	3.248E+09	3.313E+08	3.659E+09	6.592E+08	6.308E+08
	V	0.000E+00	1.812E+04	0.000E+00	0.000E+00	0.000E+00

CALCULATE DISTRIBUTION FACTORS FOR INTERIOR BEAM -
AASHTO TYPE I-BEAM "28/96"

S = 3660 mm
L = 36 570 mm
A = 8.260E+05 mm^2
I = 6.313E+11 mm^4
n = $(f_{cbeam}/f_{cslab})^{½}$ = 1.402

$$e_g = NA_{yT} + t_h + \frac{t_s}{2} \quad \text{(S4.6.2.2.1)}$$

$$e_g = 1218.3 + 25 + \frac{205}{2} = 1345.8 \ mm$$

$K_g = n\,(I + Ae_g^2)$ (S4.6.2.2.1-1)

$K_g = 1.402(6.313E{+}11 + 8.260E{+}05(1345.8^2)) = 2.983E{+}12$

For Moment - Multiple Lanes Loaded

$$DF_M = 0.075 + \left(\frac{S}{2900}\right)^{0.6}\left(\frac{S}{L}\right)^{0.2}\left(\frac{K_g}{Lt_s^3}\right)^{0.1} \qquad \text{(Table S4.6.2.2.2b-1)}$$

$$DF_m = 0.075 + \left(\frac{3660}{2900}\right)^{0.6}\left(\frac{3660}{36570}\right)^{0.2}\left(\frac{2.983E{+}12}{36570(205)^3}\right)^{0.1} = 0.984$$

For Shear - Multiple Lanes Loaded

$$DF_v = 0.2 + \frac{S}{3600} - \left(\frac{S}{10700}\right)^{2.0}$$

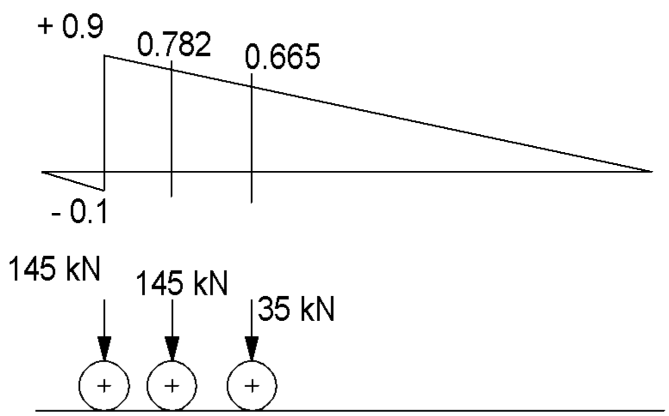
$$DF_v = 0.2 + \frac{3660}{3600} - \left(\frac{3660}{10700}\right)^{2.0} = 1.100$$

Section @ mm	Design Moments (N·mm) & Shears (N)				
		Total DL1	Total DL2	LL+I	Σ Factored*
0	M. V.	0.000E+00 7.735E+05	0.000E+00 1.410E+05	0.000E+00 6.250E+05	0.000E+00 2.255E+06
2195	M. V.	1.592E+09 6.828E+05	2.908E+08 1.241E+05	1.150E+09 5.748E+05	4.403E+09 2.030E+06
**3657	M. V.	2.553E+09 6.224E+05	4.641E+08 1.128E+05	1.830E+09 5.420E+05	7.033E+09 1.882E+06
7314	M. V.	4.553E+09 4.713E+05	8.250E+08 8.459E+04	3.227E+09 4.627E+05	1.247E+10 1.515E+06
10971	M. V.	6.001E+09 3.203E+05	1.083E+09 5.639E+04	4.189E+09 3.872E+05	1.632E+10 1.156E+06
DRAPE PT	M. V.	6.360E+09 2.699E+05	1.146E+09 4.699E+04	4.420E+09 3.629E+05	1.726E+10 1.037E+06
14628	M. V.	6.896E+09 1.692E+05	1.238E+09 2.820E+04	4.758E+09 3.154E+05	1.865E+10 8.022E+05
Centerline	M. V.	7.238E+09 1.812E+04	1.289E+09 0.000E+00	4.913E+09 2.473E+05	1.942E+10 4.554E+05

*Load factors for Strength I limit state are 1.25 for DL_1 and weight of parapet and fence, 1.5 for weight of future wearing surface (FWS) and 1.75 for live load.

Sample Calculation - Max. Shear @ 3657 mm

Influence line for shear

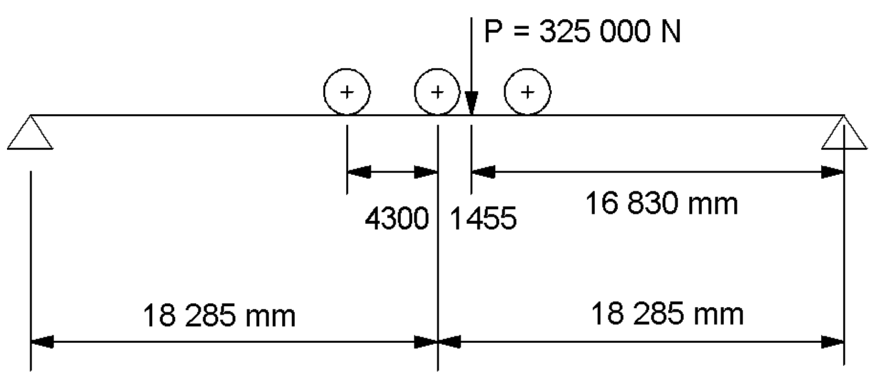


Max.V_{truck}=.9(145 000)+.782(145 000)
+0.665(35 000)=2.67E+5 N

Max. V_{Lane}=W*(positive influence area)
=9.3(0.5 x 0.9(36 570-3657))=1.377E+05 N

V_{max}=DF(V_{Lane}+IM*V_{Truck})
=1.100((1.377E+05)+1.33(2.67E+05))=5.423E+05 N

Sample Calculation - Moments at Centerline



Design Lane Load (S3.6.1.2.4)

W = 9.3 N/mm

$$M_{max} = \frac{WL^2}{8} = \frac{9.3(36\ 570)^2}{8} = 1.555E{+}09\ N \cdot mm$$

Design Truck (S3.6.1.2.4)

$$M_{max\,centerline} = \frac{325\ 000(16\ 830)(18\ 285)}{36\ 570} - (35\ 000)4300 = 2.584E+09\ N\cdot mm$$

M_{LL+I} @ Centerline

$M_{LL+I} = DF(M_{Lane}+IM*M_{Truck})$

M_{LL+I} = 0.984(1.555E+09 + 1.33(2.584E+09))
=4.913E+09 N·mm

Sample Calculation of Factored Moment at 3657 mm from Support

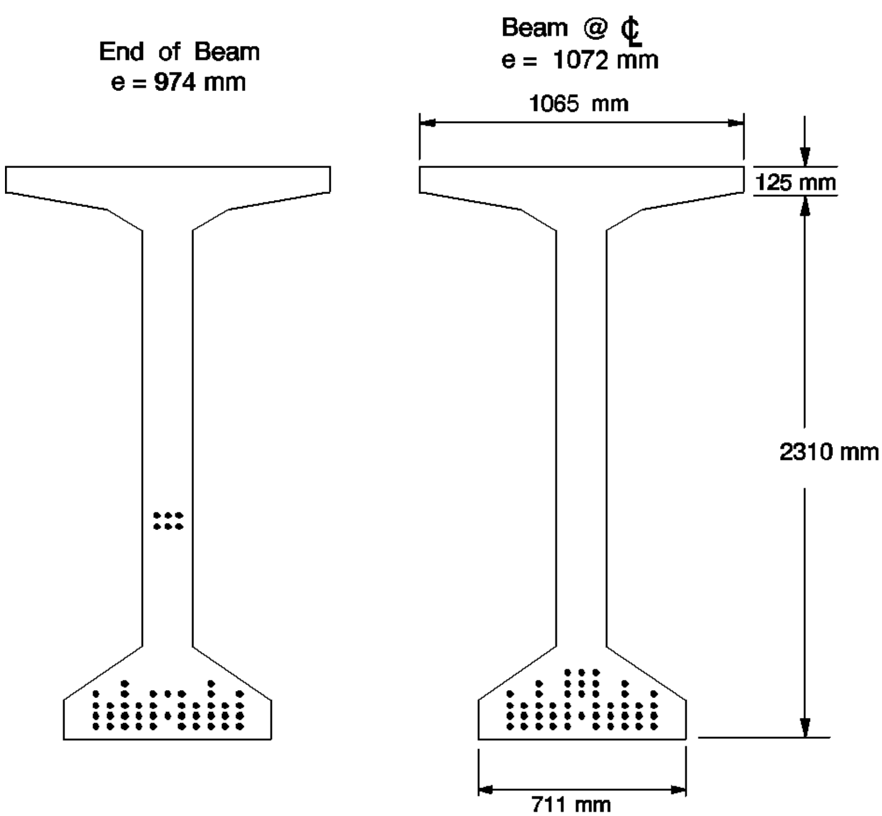
M_u = 1.25(1.169E+09 + 6.626E+07 + 1.317E+09 + 2.271E+08)
+1.50(2.373E+08)+1.75(1.830E+09)
= 7.033E+09 N·mm

Sample Calculation - Prestress Loss Calculations

Calculate Total Loss Relative to Immediately After Transfer

$\Delta f_{pT} = \Delta f_{pES}+\Delta f_{pSR}+\Delta f_{pCR}+\Delta f_{pR2}$ (S5.9.5.1-1)

Calculate Stress Immediately Prior To Transfer



$0.75\ f_{pu} = 0.75(1860) = 1395$ MPa

<u>Calculate Δf_{pES}</u>

$$\Delta f_{pES} = \frac{E_p}{E_{ci}}\left(f_{cgp}\right) \quad \text{(S5.9.5.2.3a-1)}$$

In calculating the section properties, the presence of the steel strands may be ignored or the transformed area of the steel may be considered.

Most of the calculations in this example are based on the concrete-only properties of the beam. However, transformed section properties may also be used. To illustrate this, f_{cgp} will be calculated based on transformed sections. This is accomplished by including the denominator in the equations used to calculate f_{cgp}. In this case, the difference is about 8%.

$$f_{cgp} = \frac{\dfrac{A_{ps}\left(0.7 f_{pu}\right)}{A}\left(1 + \dfrac{e^2 A}{I}\right) - \dfrac{M_G e}{I}}{1 + \dfrac{A_{ps}}{A}\left(\dfrac{E_p}{E_{ci}}\right)\left(1 + \dfrac{e^2 A}{I}\right)}$$

where:

f_{cgp}: stress in concrete at center of gravity of the prestressing strands at transfer (MPa)

A_{ps}: area of prestressing strands (mm^2)

f_{pu}: ultimate strength of prestressing strands (MPa)

e: the eccentricity of the strands in the non-composite beam (mm)

A: cross-sectional area of the noncomposite beam (mm^2)

I: moment of inertia of the noncomposite beam (mm^4)

M_G: moment due to self weight of the beam at the center of the span (N·mm)

E_p: modulus of elasticity of prestressing strand (MPa)

E_{ci}: modulus of elasticity of the beam at the time of transfer (MPa)

$$f_{cgp} = \frac{\left(\frac{46(99)(0.7(1860))}{826\ 000}\right)\left(1+\frac{1072^2(826\ 000)}{6.313E+11}\right) - \frac{3.248E+09(1072)}{6.313E+11}}{1+\frac{46(99)}{826\ 000}\left(\frac{197\ 000}{33\ 941}\right)\left(1+\frac{(1072)^2 826\ 000}{6.313E+11}\right)}$$

$$f_{cgp} = 11.53\ MPa$$

$$\frac{E_p}{E_{ci}} = \frac{197\ 000}{33\ 941} = 5.8$$

$$\Delta f_{pES} = (5.8)(11.53) = 66.9\ MPa$$

PRESTRESSING FORCE IMMEDIATELY AFTER TRANSFER

Strand stress immediately before transfer = 0.75 F_{pu}
= 0.75 x 1860 = 1395 MPa

Stress immediately after transfer f_{pt} =1395-66.9 = 1328.1 MPa

Prestressing force for sections beyond the transfer length

P_t = N A_{strand} f_{pt} = 46x94x1328.1 = 6.048E+06 N

Prestressing Force at Release

Transfer length = 60 strand diameters (S5.11.4.1)
= 60 x 12.7 = 762 mm

Assume force changes linearly along the transfer length.

Distance from CL of bearing to end of beam = 205 mm.

Prestressing force at centerline of bearing immediately after transfer = 6 048 000 x 205/762 = 1.627E+06 N.

Eccentricity of prestressing strands at CL bearings µ the eccentricity at the end = 974 mm.

For sections at a distance $ 762 mm from the end of beams, prestressing force immediately after transfer = 6.048E+06N and the eccentricity may be calculated for each section.

Limit stresses at transfer (S5.9.4.1) (assuming no bonded auxiliary reinforcement):

Tension is concrete = 0.25 / f'_{ci} = 0.25 / 50 = 1.77 MPa

Compression in concrete = 0.6 f'_{ci} = 0.6 50 =30 MPa

STRESS AT RELEASE

STRESSES AT RELEASE
MPa (+TENSION -COMPRESSION)

		TOP FIBER SLAB	TOP FIBER BEAM	BOTTOM FIBER BEAM
AT C.L. BRG.	COMPUTED	0.000	1.089	-5.025
	ALLOW.	0.000	1.77	-30.000
AT DRAPE POINT	COMPUTED	0.000	-0.38	-14.255
	ALLOW.	0.000	-30.0	-30.000
AT POINT OF MAX.	COMPUTED	0.000	-1.077	-13.560
**MOMENT	ALLOW.	0.000	-30.0	-30.000

<u>Sample Stress Calculations</u>

<u>Bottom Stresses at Release</u>

At Point of Max. Moment at CL of beam

$$Actual\ Stress = \frac{p_t}{A} - \frac{p_t e}{S_b} + \frac{M_G}{S_b}$$

$$Actual\ Stress = \frac{-1328.1(46)99}{826\ 000} - \frac{1328.1(46)(99)1072}{5.189E+08} + \frac{3.248E+09}{5.189E+08} = -13.56\,MPa$$

Allow. Stress = -0.60 f'_{ci} (S5.9.4.1.1)

Allow. Stress = -.60(50) = -30.00 MPa

PRESTRESSING LOSSES

<u>Shrinkage Losses, Δf_{pSR}</u>

Δf_{pSR} = 117-1.03H (S5.9.5.4.2-1)

Δf_{pSR} = 117-1.03(70) = 44.90 MPa

<u>Creep Losses, Δf_{pCR}</u>

$\Delta f_{pCR} = 12.0 f_{cgp} - 7.0 \Delta f_{cdp} \geq 0$ (S5.9.5.4.3-1)

$$\Delta f_{cdp} = \frac{(M_{dia}+M_{slab})e_{noncomposite}}{I} + \frac{(M_{Parapet+Fence})(e_{composite})}{I_c}$$

$$\Delta f_{cdp} = \frac{(3.313E+08 + 3.659E+09)1072}{6.313E+11} + \frac{(6.308E08)(1549.4)}{1.162E+12}$$

Δf_{cdp} = 7.62 MPa

Δf_{pCR} = 12.0(11.53)-7.0(7.62) = 85.02 MPa

<u>Relaxation Loss after Release, Δf_{pR2}</u>

Δf_{pR2} = 138 - 0.4Δf_{pES} - 0.2(Δf_{pSR}+Δf_{pCR}) (S5.9.5.4.4c-1)

Δf_{pR2} = 138 - 0.4(66.9)-0.2(44.90+85.02) = 85.26 MPa

For a pretensioned member

Δf_{pR2} = 0.3Δf_{pR2} = 0.3(85.26) = 25.57 MPa

<u>Total Loss, Δf_{pT}</u>

Δf_{pT} = Δf_{pES}+Δf_{pSR}+Δf_{pCR}+Δf_{pR2} (S5.9.5.1-1)

Δf_{pT} = 66.9+44.90+85.02+25.57 = 222 MPa

<u>Calculate Effective Prestressing Force</u>

f_{py} for low relaxation strands = 0.9 f_{pu} = 0.9x1860 = 1674 MPa (S5.9.3)

max. allowable f_{pe} = 0.80 f_{py} (Table S5.9.3-1)

max. allowable f_{pe} = 0.80 (1674) = 1339.2 MPa

f_{pe} = 0.75 f_{pu}-Δf_{pT}
= 1395-222 = 1173 MPa < max. allowable f_{pe} OK

P_e = 5.342E+06 N

SERVICE LIMIT STATE

Prestressing Force at Service Limit State

Assuming force changes linearly along the transfer length of 762 mm. Final prestressing force at center of bearing = 5.342E+06 x 205/762 = 1.437E+06N.

For sections at a distance $ 762 mm from the end of the beam, effective prestressing force at service limit state = 5.342E+06 N.

FINAL STRESSES UNDER DESIGN PERMANENT LOADS

MPa (+TENSION -COMPRESSION)

		TOP FIBER SLAB	TOP FIBER BEAM	BOTTOM FIBER BEAM
AT C.L. BRG.	COMPUTED	0.000	0.962	-4.436
	ALLOW.	-12.600	3.710	-24.750
AT DRAPE POINT	COMPUTED	-0.730	-8.433	-3.567
	ALLOW.	-12.600	-24.750	-24.75
**AT POINT OF MAX. MOMENT	COMPUTED	-0.822	-10.209	-1.666
	ALLOW.	-12.600	-24.750	-24.75

Sample Calculations

Final Bottom Stresses Under Permanent Loads

At Point of Max. Moment:

$$Prestress\ Stress = \frac{-P_e}{A} - \frac{P_e e}{S_b} = \frac{-5.342E+06}{826\ 000} - \frac{5.342E+06(1072)}{5.189E+08} = -17.5\ MPa$$

$$External\ Permanent\ Moment\ Stress = \frac{M_{TDL1}}{Z_b} + \frac{M_{TDL2}}{Z_{bc}} = \frac{7.238E+09}{5.189E+08} + \frac{1.289E+09}{6.858E+08} = 15.83\ MPa$$

Stress Under Prestress and Permanent Loads = -17.50+15.83
= -1.67MPa

$$Allowable\ Tensile\ Stress = 0.50\sqrt{f'_c}\quad \text{(S5.9.4.2.2b)}$$

$$Allowable\ Tensile\ Stress = 0.5\sqrt{55} = +3.71\ MPa$$

$$Allowable\ Compressive\ Stress = 0.45 f'_c = 24.75\ MPa\ \ (S5.9.4.2.1)$$

FINAL STRESSES UNDER DESIGN PERMANENT AND LIVE LOADS

MPa (+TENSION -COMPRESSION)

		TOP FIBER SLAB	TOP FIBER BEAM	BOTTOM FIBER BEAM
AT C.L. BRG.	COMPUTED	0.000	0.962	-4.437
	ALLOW.	-16.800	3.708	-33.000
AT DRAPE POINT	COMPUTED	-3.839	-11.242	1.588
	ALLOW.	-16.800	-33.000	3.708
**AT POINT OF	COMPUTED	-4.282	-13.342	4.061
MAX. MOMENT	ALLOW.	-16.800	-33.000	3.708

Sample Final Stresses Calculations Under Permanent and Live Loads

At Point of Max. Moment

$$Total\ Stress = \left[\frac{-P_e}{A} - \frac{P_e e}{S_b} + \frac{M_{TDL1}}{S_b} + \frac{M_{TDL2}}{S_{bc}}\right] + 0.80\left(\frac{M_{LL+I}}{S_{bc}}\right)$$

$$= -1.67 + 0.80\left(\frac{4.913E+09}{6.858E+08}\right) = +4.061\ MPa$$

$$Allowable\ Stress = 0.50\sqrt{f'_c}\ \ (S5.9.4.2.2)$$

$$Allowable\ Stress = 0.5\sqrt{55} = +3.708\ MPa$$

$$Allowable\ Compressive\ Strength = 0.6 f'_c = 33\ MPa\ \ (S5.9.4.2.1)$$

The stress at the point of maximum moment exceeds the allowable. More strands are required. For example purposes, we will continue with the current strand pattern.

CHECK RESISTANCE AT STRENGTH LIMIT STATE

Sample Calculations of Stress in Prestressing Steel at Nominal Flexural Resistance at Center of Beam

$$f_{ps} = f_{pu}\left(1 - K\frac{c}{d_p}\right)\ \ (S5.7.3.1.1\text{-}1)$$

d_p = distance from top of beam to prestressing force
= 2665 -144.7 = 2520.3 mm

A_{ps} = 46 x 99 = 4554 mm^2

β_1 = 0.85 (N.A. assumed in deck, i.e., rectangular section behavior)

<u>Calculate K</u>

$$K = 2\left(1.04 - \frac{f_{py}}{f_{pu}}\right) \quad \text{(S5.7.3.1.1-2)}$$

$$K = 2\left(1.04 - \left(\frac{1674}{1860}\right)\right) = 0.28$$

<u>Calculate Depth from Top of Section to Neutral Axis, c</u>

$$c = \frac{\left(A_{ps}f_{pu} + A_s f_y - A'_s f'_y\right)}{.85 f'_c \beta_1 b + K A_{ps}\dfrac{f_{pu}}{d_p}} \quad \text{(S5.7.3.1.1-4)}$$

N.A. assumed in the deck (rectangular behavior with the compression block within the slab thickness). Use f'_c = 28 MPa (f'_c for the slab) and width of compression block, b = effective flange width = 2993 mm

The area of the mild steel reinforcement that may exist in the section is ignored.

$$c = \frac{[46(99)(1860) + 0 - 0]}{(0.85)(28)(0.85)(2993) + 0.28(46)(99)(1860/2520.3)} = 137.75\ mm$$

$c<t_s$, i.e, neutral axis is in the deck. Rectangular section behavior is obtained as assumed.

<u>Calculate Prestressing Steel Stress at Moment Resistance, f_{ps}</u>

$$f_{ps} = 1860\left(1 - 0.28\left(\frac{137.75}{2520.3}\right)\right) = 1832\ MPa$$

<u>Calculate Depth of Stress Block, a</u>

$a = c\beta_1$ (S5.7.3.2.2)

a = 137.75(0.85) = 117.1 mm

Calculate Nominal Resistance, M_n

$$M_n = A_{ps} f_{ps}\left(d_p - \frac{a}{2}\right) + A_s f_y\left(d_s - \frac{a}{2}\right) - A_s' f_y'\left(d_s' - \frac{a}{2}\right) + .85 f_c'(b - b_w)\beta_1 h_f\left(\frac{a}{2} - \frac{h_f}{2}\right) \qquad \text{(S5.7.3.2.2-1)}$$

$$M_n = 46(99)(1832)\left(2520.3 - \frac{117.1}{2}\right) + 0 - 0 + 0 = 2.054E+10\ N \cdot mm$$

Calculate Factored Resistance, M_r

$M_r = \varphi M_n$

$M_r = 1.0(2.054E+10) = 2.054E+10$ N·mm

$M_r > M_u = 1.942E+10$ N·mm   OK

SHEAR DESIGN

SECTION @	ECC	V_p	.9*De	d_v
mm	mm	N	mm	mm
2195	992	42943	2196	2381
3657	1003	42943	2207	2393
7314	1033	42943	2233	2423
**10971	1062	42943	2260	2452
DRAPE PT	1072	0	2267	2461
14628	1072	0	2268	2462
CENTERLINE	1072	0	2268	2462

Sample Calculation, V_p and d_v

Calculate Eccentricity of Prestressing at the Section

Section at 10 971 mm from centerline of bearing.

$$e = e_{end} + \frac{e_{cL} - e_{end}}{Drape\ Pt}(Distance\ to\ Design\ Section)$$

$$e = 974 + \left(\frac{1072 - 974}{12\ 190}\right)(10\ 971) = 1062\ mm$$

Calculate V_p

V_p = vertical component of prestressing force

V_p acts as an upward force applied at drape points. Drape point distance from end of beam = 1/3 span length + distance from CL of bearings to end of beams. This is approximately equal to 1/3 span length.

$$V_p = \frac{f_{pe} A_{ps} (e_{CL} - e_{end})}{\sqrt{(e_{CL} - e_{end})^2 + Drape\ point\ distance^2}} \qquad \text{(S5.8.2.4)}$$

$$= \frac{1173 \times 46 \times 99 \times (1549.4 - 1451.4)}{\sqrt{(1549.4 - 1451.4)^2 + \left(\frac{36\ 570}{3}\right)^2}} = 42\ 943\ \text{N}$$

Calculate d_v

d_v is taken as the largest of 0.72h, 0.9 d_e and the distance between the center of the internal tension and compression force on the section at nominal moment resistance (S5.8.2.9).

.72h = .72(2665) = 1919 mm

$.9d_e$ = .90(d-(Y_b-e))
= .9(2665-(1216.7-1062)) = 2259 mm

distance between tension and compression forces = h-(Y_b-e) - a/2

$$= 2665 - (1216.7 - 1062) - \frac{117.1}{2} = 2451\ mm$$

Maximum Allowed Stirrup Spacing

SECTION @ mm	0.125 $f'_c b_v d_v$ (N)	SPACING mm (max)	f_{po} MPa
2195	3.356E+06	600	1302
3657	3.372E+06	600	1302
7314	3.415E+06	600	1302
** 10971	3.456E+06	600	1302
DRAPE PT	3.468E+06	600	1302
14628	3.468E+06	600	1302
CENTERLINE	3.468E+06	600	1302

Sample Calculation - Maximum Allowed Stirrup Spacing

If V_u < 0.125 f'_c, s_{max} = 0.8 d_v # 600 mm (S5.8.2.7-1)

For v = 0.125 f'_c

V = 0.125 f'_c b_v d_v
= 0.125 x 55 x 2058 x 2452
= 3.456E+06 N

V_u = 1.156E+06 N < 0.125 $f'_c b_v d_v$ § S_{max} = 600 mm

(Notice: For all sections, V_u < 0.125 f'_c b_v d_v)

Calculate f_{po}

For typical levels of prestressing, f_{po} may be taken as 0.7 f_{pu}.

f_{po} = 0.7x1860 = 1302.0 MPa

Determine θ

SECTION @ mm	ASSUMED THETA	STRAIN	v/f'c	*** ACTUAL THETA
2195	21.44	-0.000041	0.083	21.44
3657	21.76	-0.000018	0.077	21.76
7314	28.55	0.000315	0.061	28.55
**10971	35.05	0.000845	0.045	35.05
DRAPE PT	36.40	0.000995	0.042	36.40
14628	38.60	0.001195	0.032	38.60
CENTERLINE	38.80	0.001245	0 .018	38.80

Note: This particular beam has 6 #13 longitudinal bars in the web used for temperature and shrinkage reinforcement. They are below the composite neutral axis, i.e., within the tension side of the beam and may be included in the check of longitudinal steel and in the calculation of ε_x. It is customary to ignore these bars when determining ε_x during the design of new beams. However, these bars are usually considered in determining ε_x during the evaluation of existing beams.

Sample Calculation - β' and θ Values at 10 971 mm from Centerline of Bearings

$$v = \frac{V_u - \varphi V_p}{\varphi b_v d_v} \quad \text{(S5.8.2.9-1)}$$

$$v = \frac{(1.156E{+}06 - 0.9(42\ 943))}{0.9(205)(2452))} = 2.47\ MPa$$

$$\frac{v}{f'_c} = \frac{2.47}{55} = .045$$

Assume θ = 38°

$$\varepsilon_x = \frac{\dfrac{M_u}{d_v} + 0.5N_u + 0.5(V_u - V_p)\cot\theta - A_{ps}f_{po}}{2\left(E_sA_s + E_pA_{ps}\right)}$$

$$\varepsilon_x = \frac{\dfrac{1.632E+10}{2452} + 0 + 0.5(1.156E06 - 42\ 943)\cot 38° - 46(99)(1302)}{2(200\ 000(0) + 197\ 000(46)99)}$$

$$\varepsilon_x = 0.000809$$

Check Table S5.8.3.4.2-1 with:

ε_x = 0.000809, v/f'$_c$ = .045 → θ = 36.4°

A portion of Table S5.8.3.4.2-1 is repeated below for convenience. Also for convenience, the sample calculations will be based on the value of θ and β from the conservative box in the table. Interpolation for θ and β may produce slightly different values, however, it implies a high degree of accuracy that may not be achieved using an approximate method.

Table S5.8.3.4.2-1 - Values of θ and β for Sections with Transverse Reinforcement

$\frac{v}{f'_c}$	ε_x x 1000					
	≤0.25	≤0.5	≤0.75	≤1	≤1.5	≤2
≤ 0.075	26.6 3.94	30.5 2.59	33.7 2.38	36.4 2.23	40.8 1.95	43.9 1.67

Assume θ = 36.4°, β = 2.23

$$\varepsilon_x = \frac{\dfrac{1.632E10}{2452} + 0 + 0.5(1.156E+06 - 42\ 943)\cot 36.4° - 46(99)(1302)}{2(200\ 000(0) + 197\ 000(46)99)}$$

ε_x = 0.000825

Table S5.8.3.4.2-1 with:

ε_x = 0.000825, v/f'$_c$ = .045 → θ = 36.4°, β = 2.23

i.e., no change in β and θ from last iteration

Notice that if interpolation between the values in Table S5.8.3.4.2-1 was used, the values of θ and β would be 35.05° and 2.31, respectively. This represents a small difference from the values obtained without interpolation.

STIRRUP SPACING

SECTION @ mm	BETA	V_c N	V_s N	REQ'D S mm
2195	3.62	1 088 000	1 125 000	584
3657	3.67	1 108 000	940 000	691
7314	2.77	847 000	793 000	608
**10971	2.31	715 000	526 500	719
DRAPE PT	2.23	693 000	459 000	787
14628	2.09	649 000	242 000	1379
CENTERLINE	2.09	649 000	Negative Value	N/A

<u>Sample Calculation - Stirrup Spacing at 10 971 mm from centerline of bearings</u>

<u>Shear Resistance Provided by Concrete, V_c</u>

$$V_c = 0.083\beta\sqrt{f'_c}\,b_v d_v$$

$$V_c = 0.083(2.31)\sqrt{55}(205)(2452) = 715\ 000\,N$$

<u>Req'd V_s</u>

$$Req'd\ V_s = \frac{V_u}{0.9} - V_c - V_p$$

$$Req'd\ V_s = \frac{1.156E+6}{0.9} - 715\ 000 - 42\ 943 = 526\ 500\,N$$

<u>Req'd S</u>

$$S = \frac{A_v f_y d_v \cot\theta}{V_s} \quad \text{(SC5.8.3.3-1)}$$

$$S = \frac{258(420)(2452)\cot 35.05°}{526\ 500} = 719\,mm > 600\,mm \text{ max. } allowed$$

Use stirrup spacing 600 mm

SECTION @ mm	PROV'D SPACING mm c.c.	PROV'D V_s N
2195	580	1.13E+06
3657	600	1.08E+06
7314	600	8.04E+05
**10971	600	6.31E+05
DRAPE PT	600	6.03E+05
14628	600	5.57E+05
CENTERLINE	600	5.53E+05

Sample Calculation - V_s at 10971 mm from Centerline of Bearing

Prov'd V_s

Use S = 600 mm

$$V_s = \frac{A_v f_y d_v \cot\theta}{S} \quad \text{(SC5.8.3.3-1)}$$

$$V_s = \frac{258(420)(2452)\cot 35.05°}{600} = 6.31E+05N$$

Longitudinal Steel Requirements

Moments, shear and axial forces acting on a section affect the force in the longitudinal reinforcement. The force in the longitudinal reinforcement at moment resistance "T" should be smaller than the resistance of the reinforcement. In determining the resistance of mild steel bars and prestressing strands near the end of the beam, the effect of the lack of full development of the reinforcement should be taken into account. The force in mild steel reinforcement may be considered to increase linearly along the development length of the bars as determined using Article S5.11.2.1. The increase in the force in prestressing strands along the development and transfer lengths will be as specified in Article S5.11.4.

Assume that 6 #13 mild steel longitudinal reinforcement bars exist in the web within the tension side (½ depth of the beam).

LONGITUDINAL STEEL

MILD STEEL BAR AREA: 129 mm^2 BAR DIA: 12.7 mm
No. of bars = 6

SECTION@ mm	REQ'D TENSION FORCE (T) N	MILD STEEL BELOW NA mm2	MILD STEEL FORCE N	P/S STRAND FORCE N
153	4.828E+06	774	325 100	2.510E+06
2195	6.046E+06	774	325 100	8.343E+06
3657	6.714E+06	774	325 100	8.343E+06
7314	7.426E+06	774	325 100	8.343E+06

**10971	8.050E+06	774	325 100	8.343E+06
DRAPE PT	8.167E+06	774	325 100	8.343E+06
14628	8.345E+06	774	325 100	8.343E+06
CENTERLINE	8.175E+06	774	325 100	8.343E+06

SECTION @ mm	TOTAL FORCE N	ADD. TENSION REQ'D N
153	2.835E+06	1.993E+06
2195	8.668E+06	0.000E+00
3657	8.668E+06	0.000E+00
7314	8.668E+06	0.000E+00
**10971	8.668E+06	0.000E+00
DRAPE PT	8.668E+06	0.000E+00
14628	8.668E+06	0.000E+00
CENTERLINE	8.668E+06	0.000E+00

<u>Sample Calculation - Reinforcement Tensile Force (T) and Sufficiency of Reinforcement</u>

Applied force, T = at 10 971 from center of bearing

$$\frac{M_u}{d_v \varphi} + 0.5\frac{N_u}{\varphi} + \left(\frac{V_u}{\varphi} - .5V_s - V_p\right)\cot\theta \quad \text{(S5.8.3.5-1)}$$

$$\frac{1.632E+10}{2452(1.0)} + 0 + \left(\frac{1.156E+06}{0.9} - 0.5(526\ 500) - 42\ 943\right)\cot 35.05^{\circ} = 8.050E+06\ N$$

<u>RESISTANCE OF REINFORCEMENT</u>

<u>Sample Calculations at 10 971 mm from Center of Bearing</u>

Resistance of mild steel = (6)(129)(420) = 325 100 N

Resistance of prestressing strands = (46)(99)(1832) = 8.343E+06 N

Total reinforcement resistance = 8.343E+06+325 100
= 8.668 N > T = 8.050E+06N OK

<u>Sample Calculations at Inside Edge of Bearings</u>

Full resistance of prestressing strands = (46)(99)(1832)
= 8.343E+06 N

At inside edge of bearing, i.e., 153 mm from the centerline of bearing:

Transfer length of strands = 60 bar diameter = 762 mm

Development length of strands =
$(0.15 \times f_{ps} - 0.097\ f_{pe})\ d_b$ (S5.11.4.2-1)
(0.15x1832-0.097x1173) 12.7 = 2045 mm

Distance from centerline of bearing to end of beam =
153+205 = 358 mm < 762 mm.

Force in strands is assumed to increase from zero to f_{pe} along transfer length.

Resistance of strands 358 mm from end of beam =
1173 x 46x99x358/762 = 2.510E+06N.

Development length of mild steel bars = 320 mm (S5.11.2.1)

Mild steel bars will be fully developed.

Resistance of mild steel bars = 6x129x420 = 3.250E+05N

Total resistance of reinforcement = 2.510E+06+3.250E+05
= 2.835E+06N

Resistance (2.835E+06N) < applied force "T" (4.828E+06 N)

Difference = (4.828E+06 -2.835E+06) = 1.993E+06 N

Provide additional longitudinal reinforcement to resist this force or decrease the stirrup spacing to reduce "T" and eliminate the need for additional reinforcement.

Note: The size of the beam used in this example is larger than the optimum beam size for the applied loads and span considered. A smaller beam would have required more strands and, thus, would provide higher longitudinal reinforcement resistance than provided in this example. In general, most well designed beams will require no or minimal additional longitudinal reinforcement compared to this example.

INTERFACE SHEAR

Steel Area = 258 mm^2
Steel Yield, F_y = 420 MPa
Flange Width, B_v = 1065 mm
Cohesion Factor, C = 0.70
Friction Factor, μ = 1
Net Compression, P_c = 0

Interface shear reinforcement consists of the two legs of the stirrups, i.e., area = 258/stirrups spacing, s

Calculate Horizontal Shear

MAX

SECTION @ mm	V_{uh} N/mm	V_{nh} N/mm
153	528.0	932
2195	482.0	932
3657	450.0	926
7314	373.0	926
**10971	301.0	926
DRAPE PT	278.0	926
14628	234.0	926
CENTERLINE	172.0	926

<u>Sample Calculation - Horizontal Shear at 10971 mm from Centerline of Bearings</u>

<u>Calculate V_{uh}</u>

$$V_h = \frac{V_{u\,interface}}{d_e} \quad \text{(S5.8.4.1-8)}$$

$V_{u\ interface} = V_u - 1.25\ V_{DL1}$ = 1.156E+06 -1.25(3.203E+05)
= 7.556E+05 N

$$V_h = \frac{7.556E+05}{2511} = 301\,N/mm$$

<u>Calculate Maximum Shear Friction Permitted</u>

For stirrup spacing of 600 mm:

$V_n = CA_{cv} + \mu\,(A_{vf}\,f_y + P_c)$
= 0.7(1065)+1.0(258)(420)/600+0 = 926 N/mm

V_n should not exceed 0.2 $f'_c\ A_{cv}$ or 5.5 A_{cv} (S5.8.4.1)

$V_{nh,max}$ # 0.2 $f'_c\ A_{cv}$ (S5.8.4.1-2)

$V_{nh,\ max}$ # 0.2(28)(1065) = 5964 N/mm

and, $V_{nh,\ max}$ # 5.5 A_{cv} (S5.8.4.1-3)

$V_{nh,\ max}$ # 5.5(1065) = 5857 N/mm

$V_{nh,\ max}$ = smaller of 926, 5964 and 5857 N/mm

Use V_n = 926 N/mm

φV_{nh} = 0.9x926 = 833 N/mm > V_h = 301 N/mm OK

<u>Check Maximum Allowed Interface Shear Reinforcement Spacing</u>

Minimum required interface shear reinforcement area per unit length of beam = 0.35 b_v/f_y (S5.8.4.1).

For the provided stirrups at spacing 600 mm, minimum required stirrups area = 0.35 b_v/f_y x stirrup spacing.

This requirement is waived if $V_h/A_{cv} < 0.7$ MPa

V_h/A_{cv} = 301/1065 = 0.28 MPa < 0.7 MPa

Therefore, minimum reinforcement requirement waived.

The provided interface shear reinforcement (2 legs #13 bar each at 600 mm stirrup spacing) is adequate.