A POINT OF FIXITY MODEL FOR PILE AND SHAFT BENTS

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ABSTRACT: Pile bents are often used in bridge foundation systems. These sub-structural elements are constructed by driving a row of piles and connecting them with a concrete cap. One current design practice uses a point of fixity, which assumes the pile-soil system can be modeled as a cantilever of a particular length, forming a single column in a multi-column elastic frame. In this paper, current design practices are reviewed, and a new method for calculating point of fixity that better matches the maximum moments and displacements experienced by a pile under lateral loading of the bent is presented. An example that compares a pile bent designed as an equivalent elastic frame with a nonlinear analysis is also presented. It shows the elastic frame model with two equivalent lengths satisfactorily matches a full nonlinear model, while a frame with a single, deepest equivalent length provides a conservative approximation of the nonlinear model.

INTRODUCTION

Pile bents are often used as bridge piers or as a part of abutments. These sub-structures are designed and constructed such that a series of one or more rows of driven piles are connected to a single concrete bent cap. Elastomeric bearing pads are often used as connections between the pile bent and the superstructure, which consists of a girder-deck system. Depending on the lengths to be spanned and the abutment types used, multiple pile bents may be required in a particular bridge design.

These foundations are often designed by structural and geotechnical engineers in tandem. Geotechnical engineers are responsible for suggesting pile sizes and lengths for a given sets of axial and lateral loads. The structural engineers are then responsible for determining the required loads (dead, live, wind, impact, etc.) and designing the bent cap, the remainder of the bridge structure and perhaps checking the structural suitability of the piles. Many structural designs are based on elastic frame analyses, where the individual soil-pile systems are modeled as cantilevers with fixed bases and without soil. To determine the length of these cantilevers, various methods are used to determine a “point of fixity,” or a point somewhere above the pile toe that can adequately model the additional stiffness contributed by the soil. Ideally, the point of fixity would be chosen such that the design of the pile bent resulting from the elastic frame would be the same as a design resulting from more rigorous, but also more computationally expensive, analysis methods.

As part of a larger study (Robinson et al., 2006), four recently designed and constructed bridges in North Carolina were modeled using nonlinear finite element analysis programs that could capture the interaction between the soil and the bridge structure directly.

MultiPier (BSI, 2000) was selected due to its built-in P-y, t-z and q-z models for pile deformation and its ability to model the piles and bent cap with nonlinear material properties. SAP 2000 (Computers and Structures, Inc., 2004) was selected as a secondary analysis program to check and verify the MultiPier results.

EXISTING POINT OF FIXITY METHODS

In general, simplified formulas or selected results from a nonlinear lateral single pile analysis are used to estimate a point of fixity. Commonly used point of fixity equations were proposed by Davisson and Robinson (1965) and have been incorporated into the AASHTO LRFD Bridge Design Specifications (2004), where their use is recommended for the assessment of buckling effective length only. Equations 1 and 2 show slightly modified equations from Davisson and Robinson (1965) for lengths from the pile top to fixity below the ground surface for clay and sand profiles, respectively.

$$L_f = L_{free} + 1.4\left[\frac{E_p I_{py}}{E_c}\right]^{0.25} \qquad \text{(Clay)} \qquad (1)$$

$$L_f = L_{free} + 1.8\left[\frac{E_p I_{py}}{n_h}\right]^{0.20} \qquad \text{(Sand)} \qquad (2)$$

In equations 1 and 2, L_{free} is the distance from the pile top to the ground, E_p and I_{py} are the elastic modulus and inertia of the pile, E_c is an undrained elastic modulus for clays and n_h is the rate by which the soil modulus increases with depth in sands (see Chen, 1997 for representative values). Equations 1 and 2 are based on beam on elastic-foundation theory and assume a long, partially embedded pile in a single uniform layer of either clay or sand. The coefficients in Equations 1 and 2 are set so the model can approximately match bending and buckling response simultaneously.

A drawback of this approach is that the results from these models will not match lateral stiffness for deformation purposes. Also, for a multiple soil layer profile, the engineer has to determine an equivalent soil layer of either sand or clay in order to use the equations. It also assumes the soils can be modeled as perfectly elastic for the deformations induced by the pile bent. These models also do not distinguish between free-headed and fix-headed piles and they cannot be used to accurately assess lateral displacements.

Other methods to estimate point of fixity utilize the results of a single pile lateral analysis algorithm. These programs consider linear or nonlinear pile elements embedded in soil modeled by a series of nonlinear soil springs. The springs are defined by P-y curves that are empirically derived from a number of lateral load tests or back calculated from specific lateral load tests on a particular site. This method is summarized by Reese et al. (1991). Some engineers have used the point of maximum negative moment or the point of maximum negative displacement to define a point of fixity for elastic frame analysis purposes.

Thus, the current practice for pile bent design could proceed as follows in an engineering design office where full nonlinear analyses are unavailable, or considered unnecessary or unwieldy for a relatively small project:

- Assume typical lateral and axial loads for a particular bridge type.
- Once a pile size and type is determined, a single pile lateral analysis is run in a computer program which has built in P-y curves for modeling the lateral soil resistance and elastic or non-linear models for the pile materials, such as LPILE (Ensoft, 2004) or MultiPier,.
- Single pile lateral analysis results include moment, displacement and shear distributions along the pile length. If a maximum displacement at the pile top is exceeded, or if the pile is overloaded, the pile type is changed. A point of fixity is then determined as the distance from the top of the pile to the deepest of one of a few different points: the point of maximum negative displacement, the point of maximum negative moment, the points determined by Equation 1 or 2, or another point based on the engineer's judgment.
- The pile type and point of fixity is transmitted to the structural, where the columns are included as part of an elastic frame analysis. The live loads and other load cases are calculated by hand or by a computer program and applied to bearing locations where the bridge girders will be placed. The pile bent is assumed to act freely, that is, without additional connections to the bridge superstructure. The piles are modeled as columns connected to the bent cap with a length equal to the depth determined by the point of fixity. Each pile base is fixed against further rotation.
- The elastic frame is analyzed as a variety of AASHTO load cases are applied, and the resulting stresses in the bent cap are used in the frame analysis program to calculate the minimum required bent cap dimensions and reinforcement, and to check the structural suitability of the piles.

The main question, then, is how to best approximate the results of a full nonlinear model, which while relatively easy to use, lacks design modules to calculate live loads and required reinforcement in bent caps and piles, with an equivalent elastic frame. The point of fixity or equivalent length method was considered to fulfill this requirement.

PROPOSED POINT OF FIXITY/EQUIVALENT LENGTH METHOD

When the formulations for the point of fixity were reviewed in Robinson et. al (2006), it was noted that the results of a cantilevered column with an "equivalent" length determined by the point of fixity did not match the magnitudes of maximum moments, lateral pile top displacements or buckling behavior from the single pile lateral analysis when the pile was subjected to the same axial and lateral loads. Table 1 shows results from single pile lateral analyses in MultiPier from one bridge case study. Based on a free head lateral analyses and defining the point of fixity as using a method that included the maximum negative displacement from the single pile lateral analysis, the bridge was designed with a cantilever length of 31.55 ft.

The piles for this bent were 24 inch diameter steel pipe piles with 0.5 inch thick walls, which have a moment of inertia of 2549 in^4. An elastic modulus of 29000 ksi was used. These pipes were 60 ft long, with 13 ft of free length. The piles were subjected to an assumed 11 kip maximum lateral pile top load in the transverse (x) direction. A separate analysis was performed where a 6 kip maximum lateral load was applied in the longitudinal (y) direction. In both cases, the axial load was 150 kips.

To a depth of 6.8 ft, the sand was modeled using Reese's (1974) P-y model with a friction angle of 33^o, a unit weight of 120 pcf and a subgrade modulus of 100 lb/in^3. The water table was located at the bottom of this sand layer. From 6.8 to 36.8 ft, soft clay was modeled with Matlock's (1970) model for soft clay below the water table, with an undrained shear strength of 400 psf, a unit weight of 70 pcf and a major principal strain at 50% of the failure load of 0.02. The pile's toe was located in a sand layer located from 36.8 to 52 ft, which was also modeled using Reese's (1978) model with a friction angle of 36^o, a unit weight of 120 pcf and a subgrade modulus of 80 lb/in^3.

Table 1. Single Pile Lateral analysis results compared to cantilever length used in bridge's design.

24 inch Pipe	Lateral Load (kips)	Maximum Moment (kip-ft)	Pile Top Displacement (inches)
P-y analysis (fixed head)	11	123	0.25
Design 31.55 ft Cantilever (fixed head)	11	174	0.37
P-y analysis (free head)	6	97	0.54
Design 31.55 ft Cantilever (free head)	6	189	1.47

Due to the observed overprediction by the currently used point of fixity approach when compared to the single pile model, another method was developed. For design of free standing single pile bents (which, admittedly, will likely not have the same behavior as a bents with superstructures attached), lateral loads applied to the bent can be broken down into those acting in the bent's longitudinal direction (perpendicular to the row of piles) and those acting in the bent's transverse direction (in line with the piles). This naming convention is illustrated in Figure 1. Because the concrete cap connects the piles in the transverse direction, pile group effects will be an issue, and the piles will act more like piles with fixed top due to the interaction with others in the group. In the longitudinal direction, pile group effects will not be considered, and the pile tops are nominally free to rotate (this assumption could be most affected by including the superstructure in the analysis).

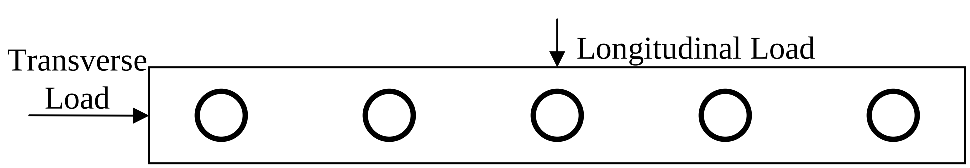


Figure 1. Plan View: 24 inch diameter pipe pile bent

If a bent is loaded laterally in any direction, there will be transverse and longitudinal components. To model a combined head condition, an equivalent column model was created that modeled the fixed head in the transverse direction and the free head in the longitudinal direction. In effect, there are two “points of fixity” to account for this difference: one in the longitudinal direction, one in the transverse direction. The proposed model derives its equivalent length based on the applied shear force and maximum moment from two single pile lateral analyses: one with a fixed head condition using transverse maximum loads and one with a free head condition using maximum longitudinal loads. If desired, the lateral displacements at the pile top from the lateral analyses can also be used to better match the equivalent model’s lateral displacements through inertia reduction factors. Figure 2 shows the equivalent model, while equations 3 through 6 show the calculations required to derive the equivalent model parameters. Not shown here, but developed in Robinson et al. (2006) are also inertia reduction factors for axial displacements that would also require more rigorous axial pile-soil deformation models.

Free head condition (longitudinal direction)

$$L_{e,free} = \frac{M_{\max,free}}{V_{free}} \tag{3}$$

$$\alpha_{free} = \frac{L^3_{e,free} V_{free}}{3 E_p I_p \Delta_{t,free}} \tag{4}$$

Fixed head condition

$$L_{e,fix} = \frac{2 M_{\max,fix}}{V_{fix}} \tag{5}$$

$$\alpha_{fix} = \frac{L^3_{e,fix} V_{fix}}{12 E_p I_p \Delta_{t,fix}} \tag{6}$$

Where:

L_e The length of a pile fixed at the base that will develop the same maximum moment, M_{max}, as the nonlinear soil-pile model under the lateral load, V.

M_{max} Maximum moment from the single pile lateral analysis for a head condition.

V Maximum lateral force applied at the top of the pile for a particular head condition.

α Inertia reduction factor that, when multiplied by the pile inertia, I_p, in the equivalent model, will result in the same lateral displacement as the nonlinear model.

E_p Elastic modulus of the pile material.

I_p Moment of inertia of the pile about the axis perpendicular to the applied load.

Δ_t Displacement at the top of the pile from the single pile lateral analysis caused by the application of the lateral load.

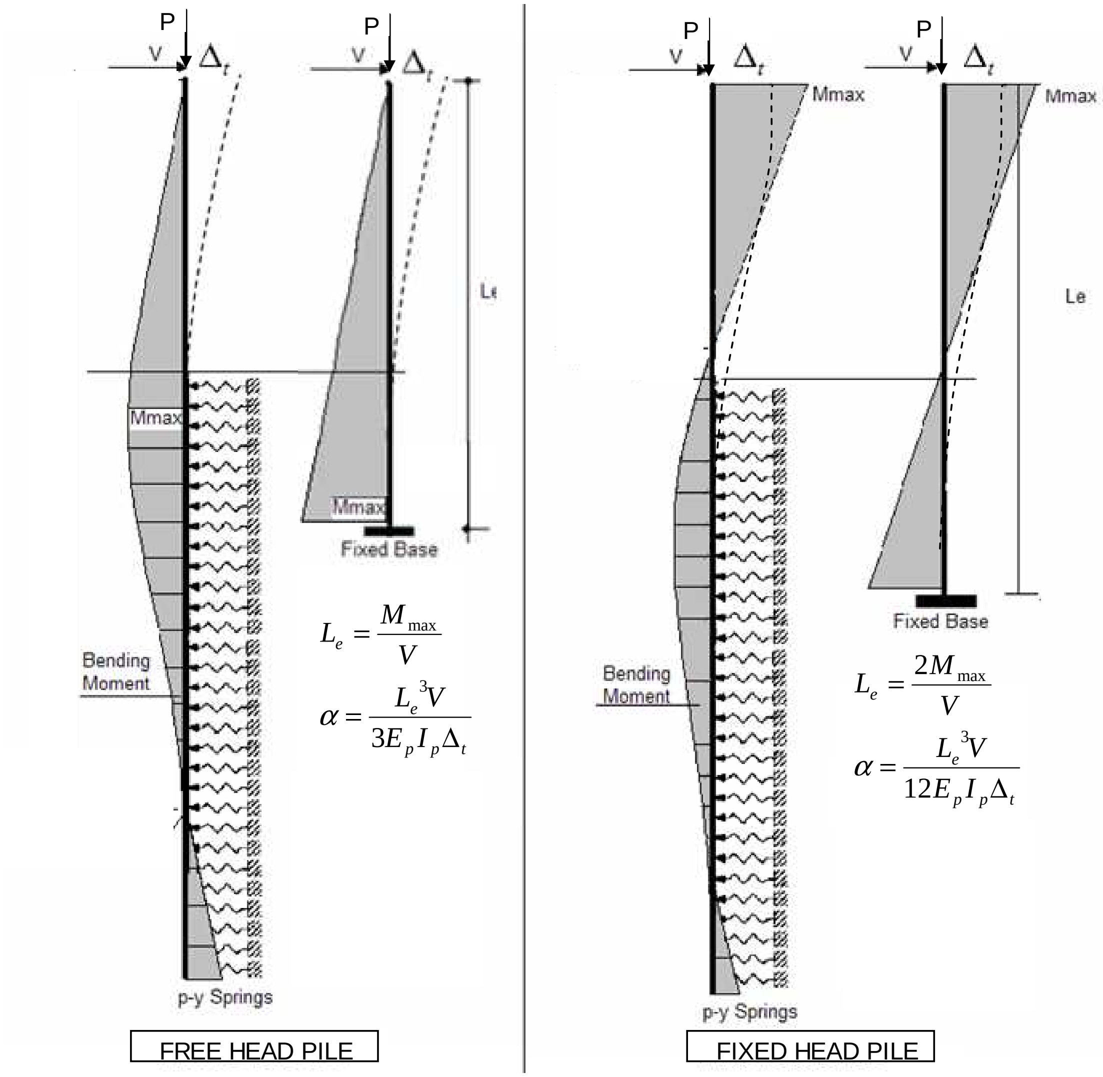


Figure 2. Equivalent Models for laterally loaded pile as a column with a fixed base. For each head condition, single pile lateral analysis is on the right, equivalent model is on the left.

Using the results of the free and fixed head P-y analyses shown in Table 1, the proposed equivalent model was used to determine new equivalent lengths and inertia reduction factors. These are summarized in Table 2.

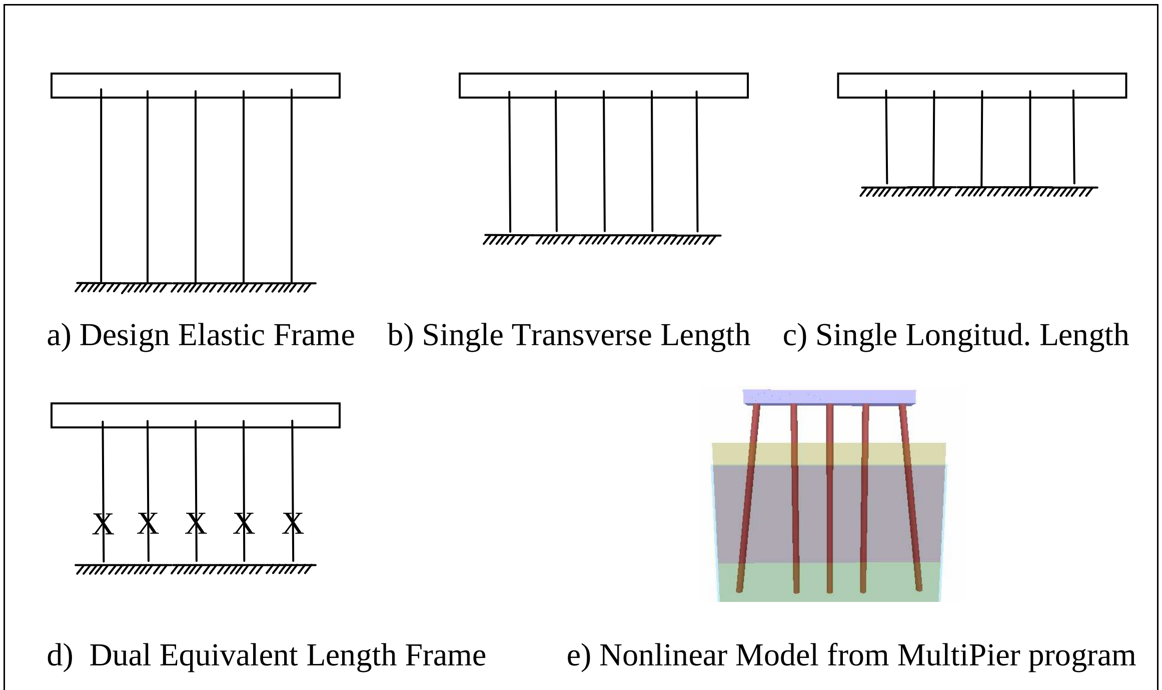
Table 2. Results of proposed equivalent length model. Axial Load 150 kips for both analyses.

Direction	Lateral Load (kips)	Maximum Moment (kip-ft)	Pile Top Disp. (inches)	Equivalent Length (ft)	Inertia Reduction Factor, α
Transverse (fixed head)	11	123	0.25	22.2	0.95
Longitudinal (free head)	6	97	0.54	16.2	0.37

One possible drawback of this method is the requirement for two equivalent cantilever lengths on a single pile. In programs like SAP 2000, this can be modeled by creating piles that are fixed at the base with the longest equivalent length. The second, shorter equivalent length can then be modeled by restraining the longitudinal displacement and rotation around the transverse axis of a special node at the depth of the shallower point of fixity. Many available frame analysis programs do not allow for input of such springs or for input of multiple inertia reduction factors on different axes, and thus only allow input of a single point of fixity. In this case, it would be conservative to use the deeper point of fixity only, as will be shown in the example below.

EXAMPLE

To show the results of application of the different models, a free standing pile bent is modeled using a variety of analysis methods. The results of an elastic frame analysis using the point of fixity from the original design are presented, followed by the results of a nonlinear analysis that directly includes the effects of the soil. Results from a model using the two proposed equivalent pile lengths and inertial reduction factors are also shown. Results from separate elastic frames that include only one set of equivalent lengths and inertia reduction factors, as could be taken by typical elastic frame software. Figure 3 shows diagrams of the five models.



a) Design Elastic Frame b) Single Transverse Length c) Single Longitud. Length

d) Dual Equivalent Length Frame e) Nonlinear Model from MultiPier program

Figure 3. Five Models of the pipe pile bent. "X" denotes a spring that restricts movement in the longitudinal direction.

Initially, the end piles were modeled in the equivalent models with the same batter as the nonlinear analysis. Battered piles take a portion of the applied lateral load and transmit it into the soil as axial loads. The amount of shear forces converted into axial forces depends on the batter angle and the axial pile-soil stiffness. If the axial pile-soil stiffness is not incorporated in the equivalent frame model by additional t-z and q-z analyses for the vertical pile, the axially stiffer battered piles in the equivalent model will take a greater portion of the shear, decreasing the shear demand and thus the moment demand in the vertical piles. This could lead to an unconservative design. Therefore, it is initially recommended to disregard the batter in the frame equivalent models, as shown in Figure 3, unless additional area reduction factors are calculated as described in Robinson et al. (2006).

Because the point of fixity models are nominally meant to capture the response of the pile to lateral loads applied to the bent, the two most critical lateral load cases from the AASHTO LFD Group II combination, which includes dead loads from the structure and wind loads on the structure were considered. These load cases consider a factored dead load in the vertical direction, and a factored wind load on the bridge's structure, which in this bridge case study are applied to four bearing locations. Table 3 shows the maximum pile moment, pile top displacement, shear and axial loads for the piles in the bent, as well as the maximum bending moment in the bent cap. It should be noted that, based on the calculated axial loads and moments, MultiPier can calculate a ratio of the demand from the loading to the structural capacity of the member. In these cases, the demand was no more than 25% of the structural capacity of the 24 inch diameter pipe piles.

Table 3. Numeric results from four elastic and one nonlinear models.

	As designed Elastic POF Model	MultiPier Nonlinear Analysis	Dual Equivalent Length Model	Transverse Equivalent Length, α_{fix} Model	Longitudinal Equivalent Length, α_{free} Model
Length of Piles/Columns	31.55	60	22.2/16.2	22.2	16.2
Group 2, LL1					
Maximum Moment in Piles (kip-ft)	169	136	121	121	90.2
Maximum Shear at Pile Top (kips)	10.7	11.9	10.9	10.9	10.9
Maximum Axial Force in piles (kips)	202	207	202	202	202
Transverse Pile Top Disp. (in)	0.66	0.24	0.24	0.24	0.24
Maximum Moment in Cap (kip-ft)	324	269	299	299	290
Group 2, LL5					
Maximum Moment in Piles (kip-ft)	218	125	123	159	123

Maximum Long. Shear at Pile Top (kips)	6.3	6.2	6.3	6.3	6.3
Maximum Axial Force in Piles (kips)	191	190.5	191	191	191
Long. Pile Top Disp. (in)	1.76	0.72	0.73	0.68	0.73

When comparing the full nonlinear analysis to the model used to build the bridge, it is clear the moments and lateral displacements predicted in the pile are higher than those predicted by the nonlinear analysis. Similarly, the moments in the bent cap from the elastic frame analysis are higher than those predicted by the nonlinear analysis, as well. Thus, the method used to design this particular bridge was conservative.

Comparing the equivalent models to the nonlinear model, the dual equivalent length model matches the moments and loads for both groups analyzed quite well. Since these are the load cases that will control design from a lateral standpoint, these should be the most critical. While moments or displacements due to lower loads applied to the equivalent model may not match the nonlinear analysis as well, these should also result in lower moments, shears and displacements, which would not tend to control design.

Using a single equivalent length shows some mixed results. If only the transverse equivalent length is used, the moments and displacements in the pile in the transverse direction, as well as the bent cap moment are matched well. In the longitudinal direction, however, maximum longitudinal pile moment is overpredicted, and the longitudinal displacement is slightly underpredicted. If, on the other hand, the longitudinal equivalent length is used, the pile moment in the transverse direction is underpredicted. This could lead to an under-designed section without some sort of additional calibration.

This equivalent frame method, of course, has limitations that a full nonlinear model does not. As long as the maximum expected load is applied to single pile analysis, the resulting equivalent lengths and elastic frame should size the beam correctly. However, if lower lateral or axial loads are used in the single pile lateral analysis, the equivalent length could be too short, the moments in the pile underpredicted, and the sections underdesigned. In this case, a new lateral analysis would be required. Similarly, if the assumed applied loads used to develop the equivalent model are higher than the actual loads, the equivalent lengths will be too long, and the piles could be overdesigned. In this case, a separate analysis with the more correct lateral and axial loads may be advisable to result in a more efficient section. Finally, the displacements predicted by the equivalent analysis will likely be overpredicted at loads lower than assumed and underpredicted at loads higher than assumed due to the nonlinear nature of the soil springs, even if the pile remains elastic.

SUMMARY AND CONCLUSIONS

The proposed design method uses the results of a single pile lateral analysis to estimate an equivalent column length for a fixed base cantilever column in an elastic frame analysis. While it would be more ideal to simply use a nonlinear model that automatically includes the nonlinear interactions between the soil and the piles, such a change may not allow existing, established design processes in a design group to proceed as normal or as efficiently as they have in the past. This proposed design model uses the results from a soil-pile model to capture the same moment, load and displacement behavior in the columns. This should lead to a better elastic frame model, where the moments predicted by the equivalent model are similar to those predicted by a more comprehensive nonlinear analysis.

A design example was presented that compared a nonlinear analysis to the current practice, as well as three possible implementations of the equivalent length method proposed here. The analysis showed the present design method in use to be conservative, with pile and bent cap moments higher than predicted by the nonlinear model. The dual equivalent length model requires, essentially, two different lengths and inertia reduction factors for the transverse and longitudinal directions. For the load cases analyzed, these match the results of the nonlinear analysis well. If a single equivalent length and inertia reduction factor is chosen, using the longer of the two equivalent lengths gives conservative results for both pile and bent cap moments, as well as pile top displacement.

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