IMPEDANCES FOR RADIALLY INHOMOGENEOUS VISCOELASTIC SOIL MEDIA

By Y. C. Han[1] and G. C. W. Sabin[2]

ABSTRACT: The dynamic impedances of a radially inhomogeneous viscoelastic soil layer with a central hole are formulated based on a new boundary zone model with a nonreflective boundary. A parabolic variation of the medium properties is assumed, so that the boundary zone has properties smoothly approaching those of the outer zone to alleviate wave reflections from the interface between the two media. Vertically, torsionally and radially excited soil layers have been examined in this paper. The results are evaluated over a wide range of the parameters involved and compared with those obtained for a homogeneous layer, as well as compared with Novak and Veletsos's solutions. Since this boundary-zone model includes mass and a nonreflective boundary, the impedances (soil stiffness and damping) presented in this paper are considered to be more suitable to practical applications.

INTRODUCTION

One of the more important problems in soil-structure interaction is how to model the soil. A number of approaches are available to account for dynamic soil-structure interaction but they are usually based on the assumptions that the soil behavior is governed by the laws of linear elasticity or viscoelasticity and the soil is perfectly bonded to an embedded foundation or a pile. In practice, however, the bonding between the soil and the footing is rarely perfect and slippage or even separation often occurs in the contact area. Furthermore, the soil region immediately adjacent to the footings can undergo a large degree of straining, which would cause the soil-structure system to behave in a nonlinear manner. Both theoretical and experimental studies have shown that the dynamic response of the footings is very sensitive to the properties of the soil in the vicinity of the footings (Han and Novak 1988; Han 1989; Han and Vaziri 1992; El-Marsafawi et al. 1992).

To account for the nonlinearities resulting from loss of contact between the soil and the footing, Novak and Sheta (1980) proposed including a cylindrical annulus of weakened soil (an inner weakened zone, or so-called boundary zone) around the footing in their plane strain analysis. Although their analysis allowed for different properties in the weakened boundary zone and the outer zone, each zone is assumed to be homogeneous. One of the simplifications involved in the original boundary zone concept was that the mass of the inner zone was neglected to avoid the wave reflection from the interface between the inner zone and the outer zone. To overcome this problem Lakshmanan and Minai (1981), Veletsos and Dotson (1986) and Dotson and Veletsos (1990) proposed schemes that can account for the mass of the boundary zone. Some of the effects of the boundary zone mass were investigated by Novak and Han (1990) who found that a homogeneous boundary zone with a nonzero mass yields undulating impedances due to wave reflections from the interface between the two media. Since in reality the interface between the two media is only fictitious, the undulating impedances may be not suitable for practical applications. The ideal boundary zone should have properties smoothly approaching those of the outer zone to alleviate wave reflections from the interface.

It should be mentioned that a continuously increasing modulus with radial distance was proposed to eliminate the undulations in the impedances by Veletsos and Dotson (1988) and Gazetas and Dobry (1984). However, in those schemes the concept of the boundary zone was not included and the modulus was increased unboundedly.

In this paper, the impedances for a composite soil layer are formulated based on a new model of the boundary zone with a nonreflective boundary. A parabolic variation of the medium properties is assumed, so that the boundary zone has properties smoothly approaching those of the outer zone to remove wave reflections from the interface. The impedances of the soil layer are presented for different modes of vibration: (1) Vertical vibration; (2) torsional vibration; and (3) axisymmetrical vibration (breathing vibration). The results are evaluated over a wide range of the parameters involved and compared with those obtained for a homogeneous layer, as well as compared with Novak and Veletsos's results. Since the boundary zone mass is accounted for in this model and the nonreflective boundary is included, the impedances (soil stiffness and damping) are considered to be more suitable for practical applications than previous ones.

COMPOSITE LAYER WITH NONREFLECTIVE BOUNDARY

The system examined is a linear viscoelastic layer of unit depth and infinite extent with a circular hole of radius r_o, as shown in Fig. 1(a). The impedances of the composite layer

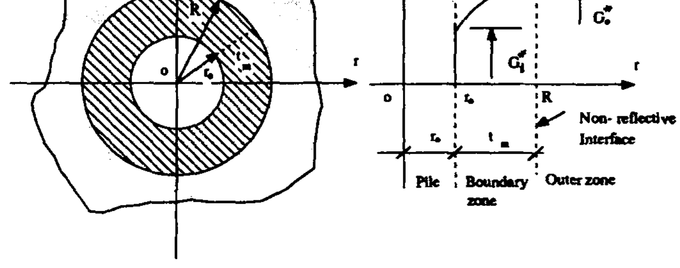

FIG. 1. Model of Boundary Zone with Nonreflective Boundary: (a) Composition of Zones; and (b) Variation of Shear Modulus with Radial Distance

[1]Grad. Res. Assoc., Fac. of Engrg., Memorial Univ. of Newfoundland, Canada A1B 3X5.

[2]Prof., Fac. of Engrg., Memorial Univ. of Newfoundland, Canada A1B 3X5.

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are derived based on the plane-strain assumption. The outer zone medium is homogeneous, isotropic, and viscoelastic, with frequency independent material damping; within the boundary zone, the complex shear modulus, $G^*(r)$, varies parabolically, as expressed by the function $f(r)$, shown in Fig. 1(b). The variation of $G^*(r)$ is continuous at the boundary, both the function itself and its derivatives, so that no reflective wave can be produced at the interface (this is referred to as nonreflection boundary).

The properties of the soil medium for each region are defined by the complex-valued modulus

$$G^*(r) = \begin{cases} G_i^*; & r = r_o \\ G_o^* f(r); & r_o < r < R \\ G_o^*; & r \geq R \end{cases} \quad (1a\text{–}c)$$

$$\text{and} \quad G_i^* = G_i(1 + i2\beta_i); \quad G_o^* = G_o(1 + i2\beta_o) \quad (2a,b)$$

in which G_i and G_o = shear moduli of the inner and outer zones; r_o = radius of the cylindrical cavity in the medium; R = radius of boundary zone; r = radial distance to an arbitrary point; β_i and β_o = damping ratio for the two zones; and $i = \sqrt{-1}$. The parabolic function, $f(r)$, can be expressed as

$$f(r) = 1 - m^2\left(\frac{r - R}{r_o}\right)^2 \quad (3)$$

$$\text{and} \quad m^2 = \frac{1 - G_i^*/G_o^*}{(t_m/r_o)^2} \quad (4)$$

where t_m = thickness of boundary zone; m = a constant whose value depends on G_i^*/G_o^* and t_m/r_o, as shown in (4). It should be explained that the soil in the boundary zone may be weakened—as well as strengthened in some cases, such as pile driving. When the soil is weakened, $G_i/G_o < 1$; when the soil is strengthened, $G_i/G_o > 1$. Denoting

$$\xi = r/r_o \quad (5)$$

$$\text{then} \quad f(\xi) = 1 - m^2(\xi - R/r_o)^2 \quad (6)$$

VERTICAL VIBRATION

Within Boundary Zone

As the composite layer is excited vertically, $u = v = 0$, where u, v = radial and tangential displacement, respectively. The governing equations can be derived from the standard form, written in cylindrical coordinates (Sokolnikoff 1956)

$$G^*(r)\frac{\partial^2 w}{\partial r^2} + \left[\frac{dG^*(r)}{dr} + \frac{G^*(r)}{r}\right]\frac{\partial w}{\partial r} = \rho\frac{\partial^2 w}{\partial t^2} \quad (7)$$

where w = vertical displacement; t = time; and ρ = mass density. The mass density for the inner zone is assumed to be equal to that of the outer zone.

Under harmonic excitation

$$w = w(r)e^{i\omega t} \quad (8)$$

Eq. (7) can be written as

$$f(\xi)\frac{d^2w}{d\xi^2} + \left[\frac{df(\xi)}{d\xi} + \frac{f(\xi)}{\xi}\right]\frac{dw}{d\xi} - \lambda_o^2 w = 0 \quad (9)$$

$$\text{in which} \quad \lambda_o = \frac{ia_o}{\sqrt{1 + i2\beta_o}} \quad (10)$$

where dimensionless frequency, $a_o = \omega r_o/V_s$, ω is circular frequency, and V_s is shear wave velocity of soil. Let

$$x = m(R/r_o - \xi) \quad (11)$$

$$\text{then} \quad f(\xi) = 1 - x^2 \quad (12)$$

Substituting (11) and (12) into (9), yields

$$(x^2 - 1)\frac{d^2w}{dx^2} + \left(2x + \frac{x^2 - 1}{x - mR/r_o}\right)\frac{dw}{dx} + \left(\frac{\lambda_o}{m}\right)^2 w = 0 \quad (13)$$

Denoting $a = mR/r_o$ and $b = (\lambda_o/m)^2$. Eq. (13) can be rewritten as

$$(x^2 - 1)\frac{d^2w}{dx^2} + \left(2x + \frac{x^2 - 1}{x - a}\right)\frac{dw}{dx} + bw = 0 \quad (14)$$

The displacement, w, can be expressed by a power series as

$$w = \sum_{n=0}^{\infty} A_n x^n \quad (15)$$

Substituting (15) into (14), the coefficients in the power series can be determined as

$$A_o = C_1; \quad A_1 = C_2; \quad A_2 = \frac{abA_o + A_1}{2a} \quad (16a\text{–}c)$$

$$A_n = \{(n - 1)^2A_{n-1} + a[b + (n - 2)(n - 1)]A_{n-2} - [b + (n - 3)(n - 1)]A_{n-3}\}/[n(n - 1)a] \quad (16d)$$

where C_1 and C_2 = complex-valued constants that can be determined by considering the boundary conditions.

Finally, the shear stress is

$$\tau_i = G^*(r)\frac{dw}{dr} = -\frac{m}{r_o}G^*(r)\frac{dw}{dx} \quad (17)$$

Outer Medium

The governing equation can also be derived from the standard form, written in cylindrical coordinates, but G^* is taken as constant in the outer zone. The equation can be written

$$\xi^2\frac{d^2w}{d\xi^2} + \xi\frac{dw}{d\xi} - \lambda_o^2\xi^2 w(\xi) = 0 \quad (18)$$

This is a Bessel equation whose solution is

$$w(\xi) = C_3K_o(\lambda_o\xi) + C_4I_o(\lambda_o\xi) \quad (19)$$

where I_o and K_o = modified Bessel functions of zero order of the first and second kind, respectively; C_3 and C_4 = complex-valued constants of integration that can be determined from the boundary conditions.

The displacement amplitude should be unit at the boundary of the hole and the displacements vanish as $\xi \to \infty$; the displacements and stresses are continuous at the interface of the two zones. Then, the boundary conditions can be written as

$$w_i = 1 \quad \text{at } \xi = 1 \quad (20a)$$

$$w_o = 0 \quad \text{at } \xi \to \infty \quad (20b)$$

$$w_i = w_o \quad \text{at } \xi = R/r_o \quad (20c)$$

$$\tau_i = \tau_o \quad \text{at } \xi = R/r_o \quad (20d)$$

To satisfy these boundary conditions, $C_4 = 0$ must hold. Eq. (19) can now be written

$$w(\xi) = C_3K_o(\lambda_o\xi) \quad (21)$$

At the boundary of the hole, $r = r_o$, so $\xi = 1$. Thus, (11) reduces to

$$x_1 = \sqrt{1 - G_i^*/G_o^*} \quad (22)$$

and likewise (15) becomes

$$C_1 + C_2x_1 + A_2x_1^2 + \cdots + A_nx_1^n = 1 \quad (23)$$

At the interface of the two zones, $\xi = R/r_o$, so $x = 0$. From (15) and (21), it follows that:

$$C_1 = C_3 K_o[\lambda_o(R/r_o)] \tag{24}$$

and the shear stresses are

$$\tau_i = C_2 \left(-\frac{m}{r_o}\right) G_o^* \tag{25}$$

$$\tau_o = -C_3 \frac{\lambda_o}{r_o} K_1 \left(\lambda_o \frac{R}{r_o}\right) G_o^* \tag{26}$$

where K_1 = a modified Bessel function of first order and the second kind. Using $\tau_i = \tau_o$ at the interface it follows that:

$$C_2 = C_3 \frac{\lambda_o}{m} K_1 \left(\lambda_o \frac{R}{r_o}\right) \tag{27}$$

From (23), (24), and (27), C_1, C_2, and C_3 can be calculated.

The impedances of the composite layer for vertical vibration are defined as

$$K_v = -2\pi r_o \tau_{i(r=r_o)} \tag{28}$$

then, K_v can now be determined from

$$K_v = \pi G_i 2m(1 + i2\beta_i) \left.\frac{dw}{dx}\right|_{x=x_1} \tag{29}$$

It is desirable to express K_v in the following form:

$$K_v = \pi G_i(S_{v1} + ia_i S_{v2}) \tag{30}$$

$$\text{in which} \quad a_i = \omega r_o/V_{si} \tag{31}$$

where $V_{si} = \sqrt{G_i/\rho}$ = shear wave velocity for the boundary zone; S_{v1} and S_{v2} are dimensionless factors that depend on a_o, t_m/r_o, G_i/G_o, β_i, and β_o. The factors S_{v1} and S_{v2} are referred to as the vertical stiffness and damping of soil, respectively.

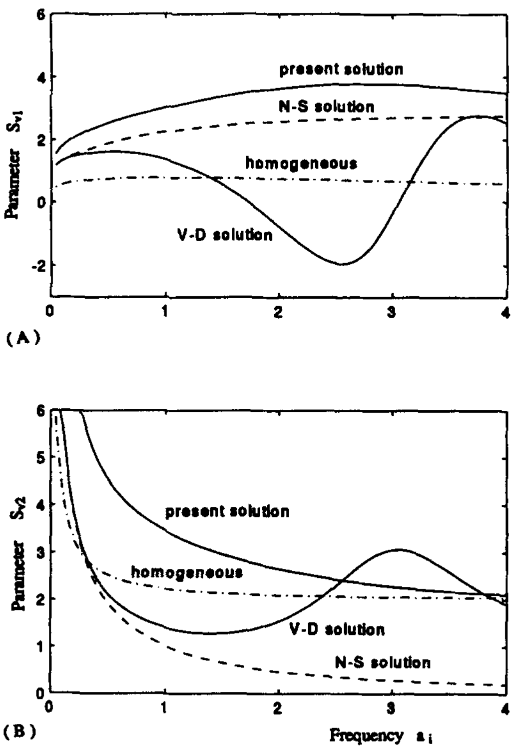

FIG. 2. Comparison of Impedance Functions for Vertical Vibration with Novak-Sheta (N-S) and Veletsos-Dotson (V-D) Solutions, t_m/r_o = 1.0; G_i/G_o = 0.25; β_i = 0.1; β_o = 0.05: (a) Stiffness Factor S_{v1}; and (b) Damping Factor S_{v2}

In this paper, the impedances of the soil layer were expressed in terms of the shear modulus of the inner region, G_i, following the format employed in Veletsos and Dotson (1988). The stiffness and damping factors, S_{v1} and S_{v2}, obtained from the present analysis are compared with those obtained for the Novak-Sheta (N-S) and Veletsos-Dotson (V-D) idealizations, as shown in Fig. 2. These solutions are for a soil layer with t_m/r_o = 1.0; G_i/G_o = 0.25; β_i = 0.1; and β_o = 0.05. The mass density for the inner zone is taken to be the equal to that for the outer region in the present solution and the V-D solution, while for the N-S solution the mass density for the inner zone is assumed to be zero, an unrealistic assumption. It can be observed that the three sets of results are significantly different; the V-D solution results in pronounced oscillations (undulations) caused by wave reflections from the interface between the two media. It should be noted that Veletsos and Dotson (1988) also included a model with a continuously increasing shear modulus with radial distance to remove the undulations in the impedances. The differences between the present solution and that solution are that the modulus was varied unboundedly, i.e., without a boundary zone in their model, and in the present model the modulus varies only in the boundary zone and is constant in the outer zone as one might expect. The nonreflective boundary between the boundary zone and the outer zone is formulated in the present solution. It should be explained that the convergence of the power series used to express the displacement in this study is rapid; however, the convergence for the derivative of the series is slow. To obtain a satisfactory solution, the first 10 to 12 terms of the series are needed. The results from the present analysis are smooth curves over a wide range, the value of a_i from 0.0 to 4.0, which indicates that the wave reflections from the interface are alleviated because it embodies a continuous variation in soil properties in the boundary zone with smooth (continuous derivatives) transition into the outer zone. For comparison, the results for a homogeneous layer are also included in Fig. 2.

To illustrate the influence of parameters involved, the stiffness and damping factors for a vertically excited layer are plotted in Fig. 3 and Fig. 4 as a function of a_i for several different combinations of t_m/r_o and G_o/G_i, with material damping, β_i = 0.1 and β_o = 0.05. It should be noticed that the undulations caused by wave reflection vanish as expected in all of the cases presented, owing to the nonreflective boundary. The influence of the material properties in the boundary zone is sensitive to the stiffness and damping of the soil layer. The stiffness factor, S_{v1}, increases with the level of G_o/G_i and is smallest for the homogeneous case (G_o/G_i = 1). The damping factor, S_{v2}, at lower frequency levels becomes larger as the magnitude of G_o/G_i increases; however, at higher frequencies this tendency is diminished.

The effects of material damping on the impedances of the soil layer are shown in Fig. 5. Several values of damping ratio are selected, in one case both β_i and β_o are zero and in other cases β_o = 0.05 and β_i = 0.05, 0.1, and 0.2, respectively. It can be seen that the stiffness factor, S_{v1}, reduces with material damping increasing, but the effect to the damping factor, S_{v2}, is small. This trend in damping response can be explained by reference to the fact that the radiation damping becomes more dominant (relative to the material damping) at higher frequency levels.

TORSIONAL VIBRATION

Within Boundary Zone

Because the composite layer is excited torsionally, $w = u = 0$, the governing equations can be derived from the standard form, written in cylindrical coordinates (Sokolnikoff 1956).

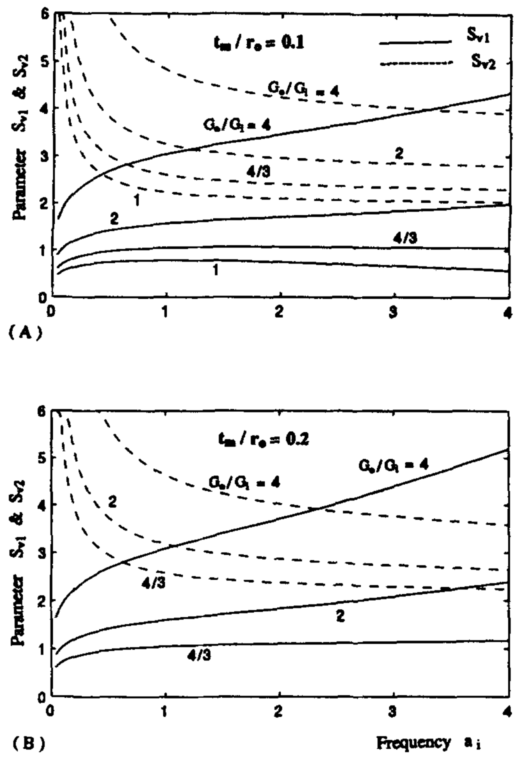

FIG. 3. Vertical Impedances for Composite Layer with Material Damping β_i = 0.1, β_o = 0.05 and Different Parameters: (a) for t_m/r_o = 0.1; (b) for t_m/r_o = 0.2

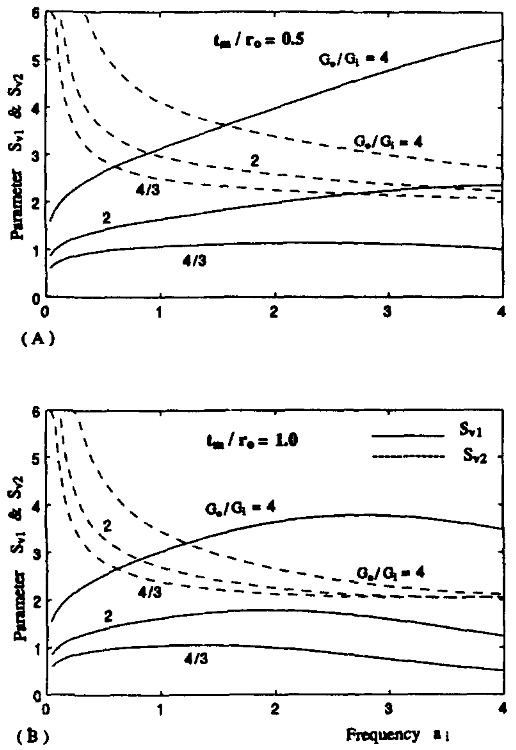

FIG. 4. Vertical Impedances for Composite Layer with Material Damping β_i = 0.1, β_o = 0.05 and Different Parameters: (a) for t_m/r_o = 0.5; (b) for t_m/r_o = 1.0

$$G^*(r)\left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r}\frac{\partial v}{\partial r} - \frac{v}{r^2}\right) + \frac{dG^*(r)}{dr}\left(\frac{\partial v}{\partial r} - \frac{v}{r}\right) = \rho\frac{\partial^2 v}{\partial t^2} \quad (32)$$

where v = tangential displacement; t = time; and ρ = mass density. The mass density for the inner zone is assumed to be equal to that of the outer zone.

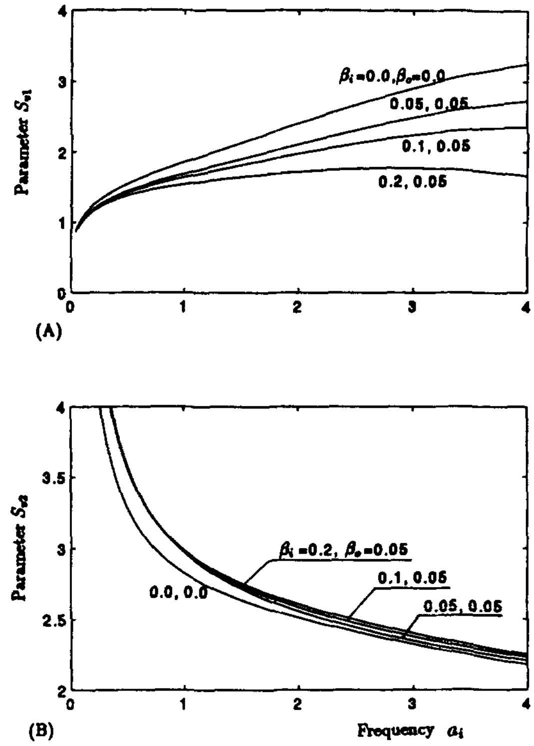

FIG. 5. Effects of Material Damping on Vertical Impedances of Soil Layer: (a) Stiffness Factor S_{v1}; and (b) Damping Factor S_{v2}

Under harmonic excitation

$$v = v(r)e^{i\omega t} \quad (33)$$

Eq. (32) can be written

$$f(\xi)\frac{d^2 v}{d\xi^2} + \left[\frac{df(\xi)}{d\xi} + \frac{f(\xi)}{\xi}\right]\frac{dv}{d\xi} - \left[\frac{1}{\xi}\frac{df(\xi)}{d\xi} + \frac{f(\xi)}{\xi^2} + \lambda_o^2\right]v = 0 \quad (34)$$

Substituting (11) and (12) into (34), yields

$$(x^2 - 1)\frac{d^2 v}{dx^2} + \left[2x + \frac{x^2 - 1}{x - a}\right]\frac{dv}{dx} + \left[\frac{2x}{a - x} + \frac{1 - x^2}{(x - a)^2} + b\right]v = 0 \quad (35)$$

The displacement, v, can be expressed by a power series as

$$v = \sum_{n=0}^{\infty} A_n x^n \quad (36)$$

Substituting (36) into (35), the coefficients in the power series can be determined as

$$A_o = C_1; \quad A_1 = C_2 \quad (37a,b)$$

$$A_2 = \frac{(a^2 b + 1)A_o + aA_1}{2a^2} \quad (37c)$$

$$A_3 = \frac{2a(1 - b)A_o + a^2(b + 2)A_1 + 6aA_2}{6a^2} \quad (37d)$$

and $\delta 1 = (n - 1)(2n - 3)a$ (38a)

$$\delta 2 = (n - 1)(n - 2)a^2 - (n - 2)^2 + ba^2 + 1 \quad (38b)$$

$$\delta 3 = -a[(n - 3)(2n - 3) + 2(b - 1)] \quad (38c)$$

$$\delta 4 = (n - 2)(n - 4) + b - 3 \quad (38d)$$

the general term can be expressed as

$$A_n = \frac{\delta 1 A_{n-1} + \delta 2 A_{n-2} + \delta 3 A_{n-3} + \delta 4 A_{n-4}}{n(n-1)a^2} \quad (39)$$

where C_1 and C_2 are complex-valued constants that can be determined by considering the boundary conditions.

Finally, the shear stress is

$$\tau_i = G^*(r)\left(\frac{dv}{dr} - \frac{v}{r}\right) = -G^*(r)\left(\frac{m}{r_o}\frac{dv}{dx} + \frac{v}{r}\right) \quad (40)$$

Outer Medium

The governing equation can also be derived from the standard form, written in cylindrical coordinates, but G^* is taken as constant in the outer zone. The equation can be written

$$\xi^2 \frac{d^2 v}{d\xi^2} + \xi \frac{dv}{d\xi} - (\lambda_o^2 \xi^2 + 1)v(\xi) = 0 \quad (41)$$

This is a Bessel equation, for which its solution is

$$v(\xi) = C_3 K_1(\lambda_o \xi) + C_4 I_1(\lambda_o \xi) \quad (42)$$

where I_1 and K_1 = modified Bessel functions of order one of the first and second kind, respectively; C_3 and C_4 = complex-valued constants of integration, which can be determined from the boundary conditions.

The boundary conditions are

$$v_i/r_o = 1 \quad \text{at } \xi = 1; \quad v_o = 0 \quad \text{at } \xi \to \infty \quad (43a,b)$$

$$v_i = v_o \quad \text{at } \xi = R/r_o; \quad \tau_i = \tau_o \quad \text{at } \xi = R/r_o \quad (43c,d)$$

From these boundary conditions, C_1, C_2, C_3, and C_4 can be calculated.

The impedance of the composite layer for torsional vibration is defined as the moment of the shear stresses around the cylinder axis with respect to a unit torsional angle ($v_i/r_o = 1$)

$$K_\theta = -2\pi r_o^2 \tau_{i(r=r_o)} \quad (44)$$

then, K_θ can now be determined from

$$K_\theta = 2\pi r_o^2 G_i(1 + i2\beta_i)\left(\frac{m}{r_o}\frac{dv}{dx} + 1\right)\Big|_{x=x_1} \quad (45)$$

It is desirable to express K_θ in the following form:

$$K_\theta = 2\pi r_o^2 G_i(S_{\theta 1} + i a_i S_{\theta 2}) \quad (46)$$

where $S_{\theta 1}$ and $S_{\theta 2}$ are dimensionless factors that depend on a_o, t_m/r_o, G_i/G_o, β_i, and β_o. The factors $S_{\theta 1}$ and $S_{\theta 2}$ are referred to as the torsional stiffness and damping of soil, respectively.

The stiffness and damping factors, $S_{\theta 1}$ and $S_{\theta 2}$, obtained from the present analysis are compared with those obtained for the Novak-Sheta (N-S) and Veletsos-Dotson (V-D) idealizations, as shown in Fig. 6. These solutions are for a soil layer with $t_m/r_o = 1.0$; $G_i/G_o = 0.25$; $\beta_i = \beta_o = 0.0$. The mass density for the inner zone is taken to be the equal to that for the outer region in the present solution and the V-D solution; for the N-S solution the mass density for the inner zone is assumed to be zero. It can be observed that similar to the case of vertical vibration, the V-D solution results in pronounced oscillations (undulations) caused by wave reflections from the interface between the two media. In the same way as in the vertical vibration case, Veletsos and Dotson (1988) proposed a continuously increasing shear modulus to eliminate the undulations in the impedances. The results from the present analysis are smooth curves over a wide range parameter values, indicating that the wave reflections from the interface have been removed. For comparison, the results for a homogeneous layer are also included in Fig. 6.

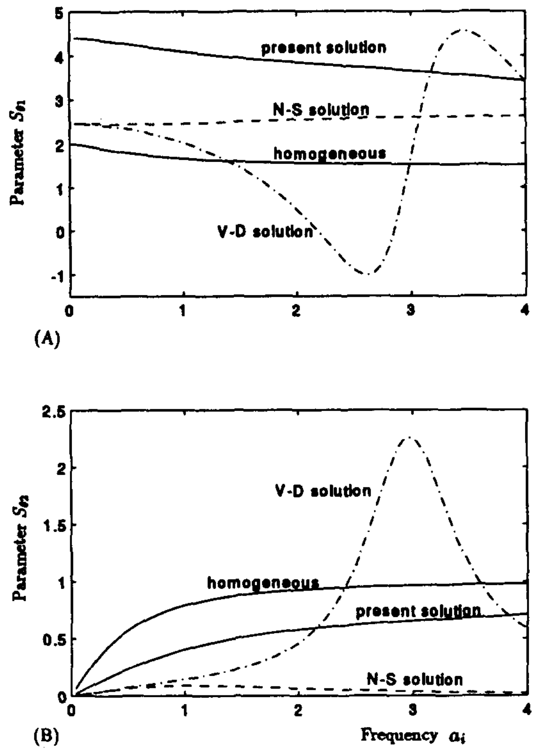

FIG. 6. Comparison of Torsional Impedance Functions with Novak-Sheta (N-S) and Veletsos-Dotson (V-D) Solutions, $t_m/r_o = 1.0$, $G_i/G_o = 0.25$, $\beta_i = \beta_o = 0.0$: (a) Stiffness Factor $S_{\theta 1}$, and (b) Damping Factor $S_{\theta 2}$

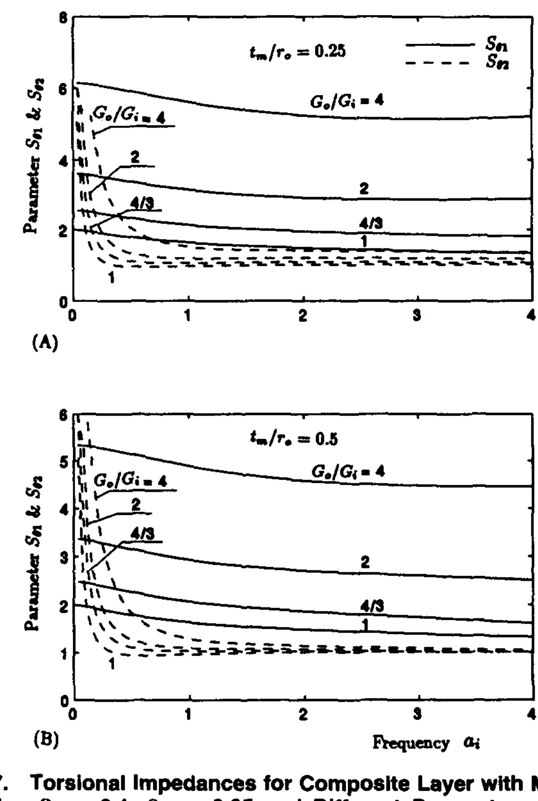

FIG. 7. Torsional Impedances for Composite Layer with Material Damping $\beta_i = 0.1$, $\beta_o = 0.05$ and Different Parameters: (a) for $t_m/r_o = 0.25$; (b) for $t_m/r_o = 0.5$

To illustrate the influence of the parameters involved, the stiffness and damping factors for a torsionally excited layer are plotted in Fig. 7 and Fig. 8 as a function of a_i for several

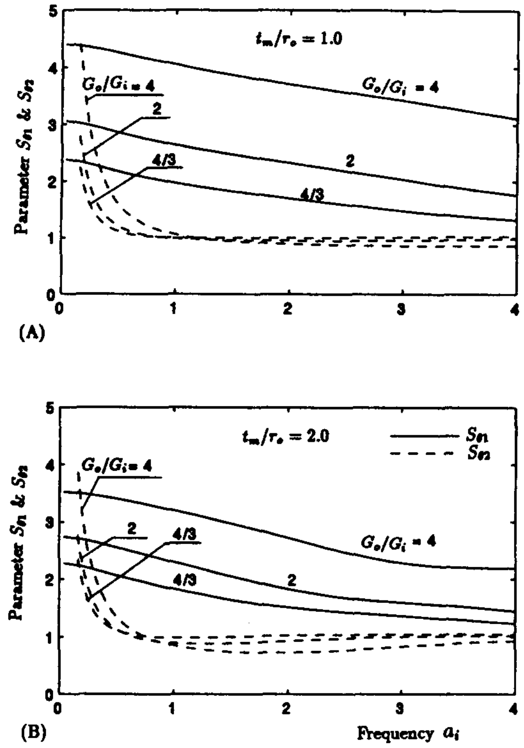

FIG. 8. Torsional Impedances for Composite Layer with Material Damping $\beta_i = 0.1$, $\beta_o = 0.05$ and Different Parameters: (a) for $t_m/r_o = 1.0$; (b) for $t_m/r_o = 2.0$

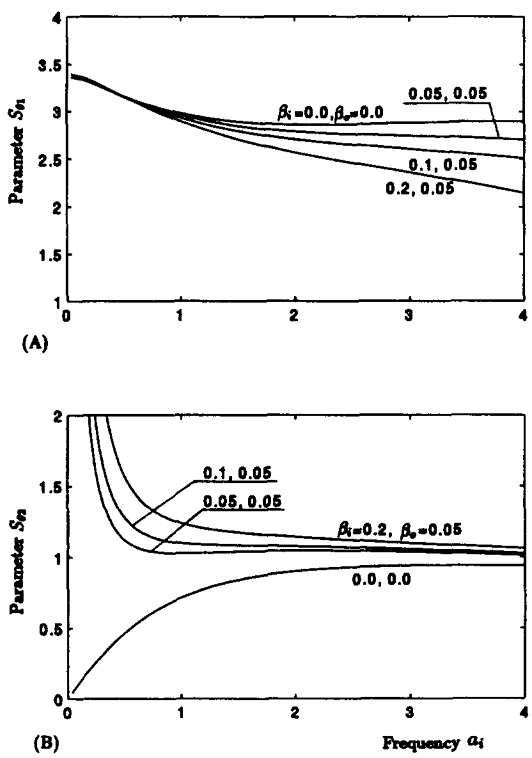

FIG. 9. Effects of Material Damping on Torsional Impedances of Soil Layer: (a) Stiffness Factor $S_{\theta 1}$; and (b) Damping Factor $S_{\theta 2}$

different combinations of t_m/r_o and G_o/G_i, with material damping, $\beta_i = 0.1$ and $\beta_o = 0.05$. It should be noticed that the undulations caused by wave reflection vanish as expected in all of the cases presented. The torsional stiffness factor, $S_{\theta 1}$, increases with the level of G_o/G_i and is smallest for the homogeneous case ($G_o/G_i = 1$).

The effects of material damping on the torsional impedances of the soil layer are shown in Fig. 9. Several values of the damping ratio are selected, in one case both β_i and β_o are zero and in the other cases $\beta_o = 0.05$ and $\beta_i = 0.05$, 0.1, and 0.2, respectively. It can be seen that the stiffness factor, $S_{\theta 1}$, reduces with material damping increasing, but the effect to the damping factor, $S_{\theta 2}$, is small. This trend in damping response can be explained by reference to the fact that the radiation damping becomes more dominant (relative to the material damping) at higher frequency levels.

RADIAL VIBRATION (BREATHING VIBRATION)

Within Boundary Zone

The composite layer is subjected to an axisymmetrical, volumetrical deformation associated with the propagation of a P-wave in the radial direction or, say, a breathing vibration, such as in the cases of cavity expansion, pile vibration and driving. In this case, $w = v = 0$, and the governing equations can be derived from the standard form (Sokolnikoff 1956)

$$\frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\theta}{r} = \rho \frac{\partial^2 u(r, t)}{\partial t^2} \tag{47}$$

where σ_r = normal stress; σ_θ = tangent stress; and $u(r, t)$ = radial displacement. The stresses, σ_r and σ_θ, can be expressed as

$$\sigma_r = (\lambda^* + 2G^*) \frac{\partial u(r, t)}{\partial r} + \lambda^* \frac{u(r, t)}{r} \tag{48}$$

$$\sigma_\theta = \lambda^* \frac{\partial u(r, t)}{\partial r} + (\lambda^* + 2G^*) \frac{u(r, t)}{r} \tag{49}$$

where λ^* = complex Lamé constant of the medium in the boundary zone, and expressed as

$$\lambda^* = \frac{2\nu}{1 - 2\nu} G^* \tag{50}$$

where ν = Poisson's ratio, to be assumed a constant in the boundary zone and the same as that in the outer medium.

Substituting (48) and (49) into (47), yields

$$\frac{\partial}{\partial r}\left[(\lambda^* + 2G^*) \frac{\partial u(r, t)}{\partial r} + \lambda^* \frac{u(r, t)}{r}\right] + \frac{2G^*}{r}\left[\frac{\partial u(r, t)}{\partial r} - \frac{u(r, t)}{r}\right] = \rho \frac{\partial^2 u(r, t)}{\partial t^2} \tag{51}$$

Within the boundary zone, λ^* and G^* are variable, (51) can be written

$$(\lambda^* + 2G^*)\left[\frac{\partial^2 u(r, t)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r, t)}{\partial r} - \frac{1}{r^2} u(r, t)\right] + \frac{d(\lambda^* + 2G^*)}{dr}\frac{\partial u(r, t)}{\partial r} + \frac{d\lambda^*}{dr}\frac{u(r, t)}{r} = \rho \frac{\partial^2 u(r, t)}{\partial t^2} \tag{52}$$

Under harmonic excitation

$$u(r, t) = u(r)e^{i\omega t} \tag{53}$$

Eq. (52) becomes

$$f(\xi)\frac{d^2 u}{d\xi^2} + \left[\frac{df(\xi)}{d\xi} + \frac{f(\xi)}{\xi}\right]\frac{du}{d\xi} - \left[\frac{f(\xi)}{\xi^2} - \frac{df(\xi)}{d\xi}\frac{\eta_1^2}{\xi} + (sr_o)^2\right]u = 0 \tag{54}$$

$$\text{in which} \quad sr_o = \frac{ia_o}{\eta\sqrt{1 + i2\beta_o}} \tag{55a}$$

$$\eta = \sqrt{2(1-\nu)/(1-2\nu)}; \quad \eta_1 = \sqrt{\nu/(1-\nu)} \qquad (55b,c)$$

$$\text{and denoting} \quad B = \left(\frac{sr_o}{m}\right)^2 \qquad (56)$$

With reference to (11) and (12), (54) can be written

$$(x^2-1)\frac{d^2u}{dx^2} + \left(2x + \frac{x^2-1}{x-a}\right)\frac{du}{dx} + \left[\frac{1-x^2}{(x-a)^2} + \eta_1^2\frac{2x}{x-a} + B\right]u = 0 \qquad (57)$$

The displacement, u, can be expressed by a power series as

$$u = \sum_{n=0}^{\infty} A_n x^n \qquad (58)$$

Substituting (58) into (57), the coefficients in the power series can be determined as

$$A_o = C_1; \quad A_1 = C_2 \qquad (59a,b)$$

$$A_2 = \frac{(a^2B+1)A_o + aA_1}{2a^2} \qquad (59c)$$

$$A_3 = [-2a(B+\eta_1^2)A_o + a^2(B+2)A_1 + 6aA_2]/6a^2 \qquad (59d)$$

$$\text{and} \quad \delta 1 = (n-1)(2n-3)a \qquad (60a)$$

$$\delta 2 = (n-2)(n-3)(a^2-1) + (n-2)(2a^2-1) + Ba^2 + 1 \qquad (60b)$$

$$\delta 3 = -a[(n-3)(2n-3) + 2(B+\eta_1^2)] \qquad (60c)$$

$$\delta 4 = (n-2)(n-4) + B + 2\eta_1^2 - 1 \qquad (60d)$$

the general term can be expressed as

$$A_n = (\delta 1 A_{n-1} + \delta 2 A_{n-2} + \delta 3 A_{n-3} + \delta 4 A_{n-4})/[n(n-1)a^2] \qquad (61)$$

where C_1 and C_2 = complex-valued constants which can be determined by considering the boundary conditions. Finally, the normal stress is

$$\sigma_r = -\frac{m}{r_o}(\lambda^* + 2G^*)\frac{du}{dx} + \lambda^*\frac{u}{r} \qquad (62)$$

Outer Medium

The governing equations of the homogeneous medium can be derived from (51), but λ^* and G^* are taken as constants. For the axisymmetric, volumetric deformation associated with the propagation of P-wave in the radial direction, breathing vibration, the equation is

$$(\lambda_o^* + 2G_o^*)\left[\frac{\partial^2 u(r,t)}{\partial r^2} + \frac{1}{r}\frac{\partial u(r,t)}{\partial r} - \frac{1}{r^2}u(r,t)\right] = \rho\frac{\partial^2 u(r,t)}{\partial t^2} \qquad (63)$$

where λ_o^* = complex Lamé constant of medium in outer zone. For harmonic excitation, (63) becomes

$$\frac{d^2u(r)}{dr^2} + \frac{1}{r}\frac{du(r)}{dr} - \left(s^2 + \frac{1}{r^2}\right)u(r) = 0 \qquad (64)$$

This is a modified Bessel equation, for which its solution is

$$u(r) = C_3K_1(sr) + C_4I_1(sr) \qquad (65)$$

where I_1 and K_1 = modified Bessel function of order one, the first and second kind, respectively; C_3 and C_4 = complex-valued constants of integration that can be determined from the boundary conditions.

The boundary conditions are the displacement amplitude is unity at the boundary of the hole and displacements vanish as $r \to \infty$; displacements and stresses are equal at the interface between the two zones. To satisfy these boundary conditions, $C_4 = 0$ must hold. Eq. (65) can now be written as

$$u(r) = C_3K_1(sr) \qquad (66)$$

At the boundary of the hole, $r = r_o$, and $\xi = 1$

$$x_1 = \sqrt{1 - G_i^*/G_o^*} \qquad (67)$$

and likewise (58) becomes

$$C_1 + C_2x_1 + A_2x_1^2 + \cdots + A_nx_1^n = 1 \qquad (68)$$

At the interface of the two zones, $r = R$ and $x = 0$, from (58) and (66), it follows that:

$$C_1 = C_3K_1(sR) \qquad (69)$$

From (1) and (50) one knows that $\lambda^* = \lambda_o^*$ and $G^* = G_o^*$ at the interface ($r = R$), and the normal stresses $\sigma_i = \sigma_o$, then

$$C_2 = C_3\left[\frac{r_o}{mR}K_1(sR) + \frac{sr_o}{m}K_o(sR)\right] \qquad (70)$$

From (68), (69) and (70), C_1, C_2 and C_3 can be obtained. Since the displacement amplitude is unity at $r = r_o$, the radial stiffness is defined as

$$K_r = -\sigma_{r(r=r_o)} \qquad (71)$$

then, K_r can now be determined from

$$K_r = \frac{2(1-\nu)}{1-2\nu}\frac{m}{r_o}G_i^*\left.\frac{du}{dx}\right|_{x=x_1} - \frac{2\nu}{1-2\nu}\frac{1}{r_o}G_i^* \qquad (72)$$

Separating the real and imaginary parts of (72), the complex-valued radial stiffness can be written as

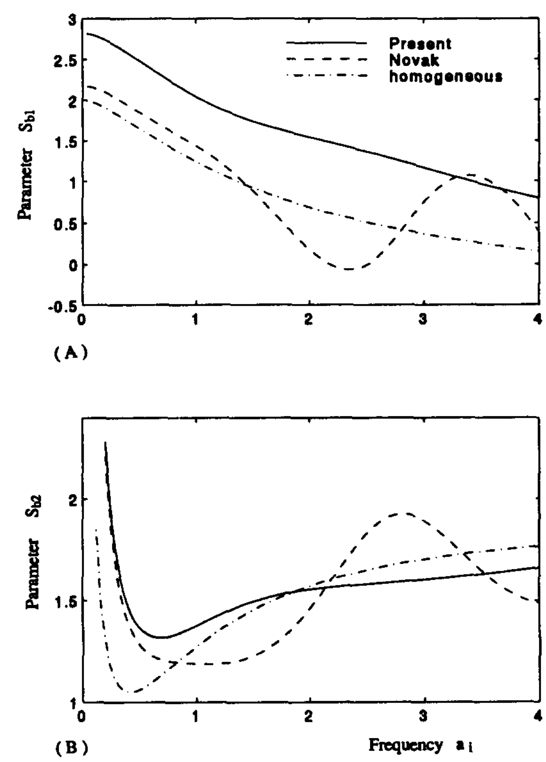

FIG. 10. Comparison of Radial Impedance Functions with Novak-Sheta (N-S) and Veletsos-Dotson (V-D) Solutions, t_m/r_o = 2.0, G_i/G_o = 0.5, ν = 0.25, β_i = 0.1 and β_o = 0.05: (a) Stiffness Factor S_1; and (b) Damping Factor S_{b2}

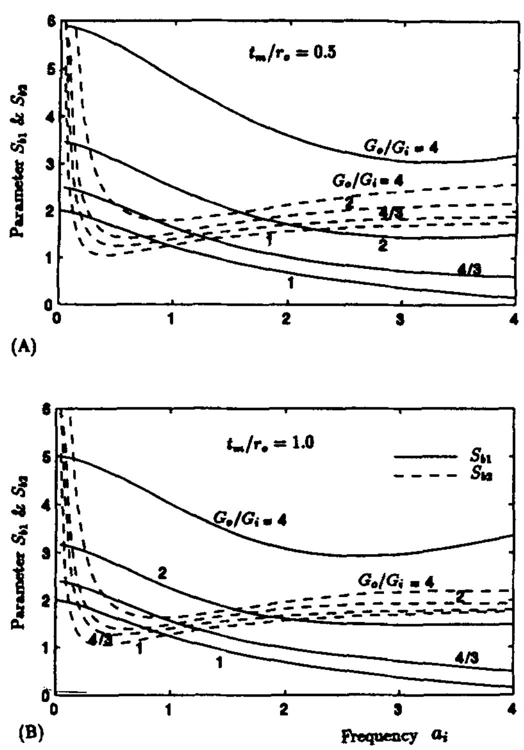

FIG. 11. Radial Impedances for Composite Layer with Material Damping β_i = 0.1, β_o = 0.05 and Different Parameters: (a) for t_m/r_o = 0.5; (b) for t_m/r_o = 1.0

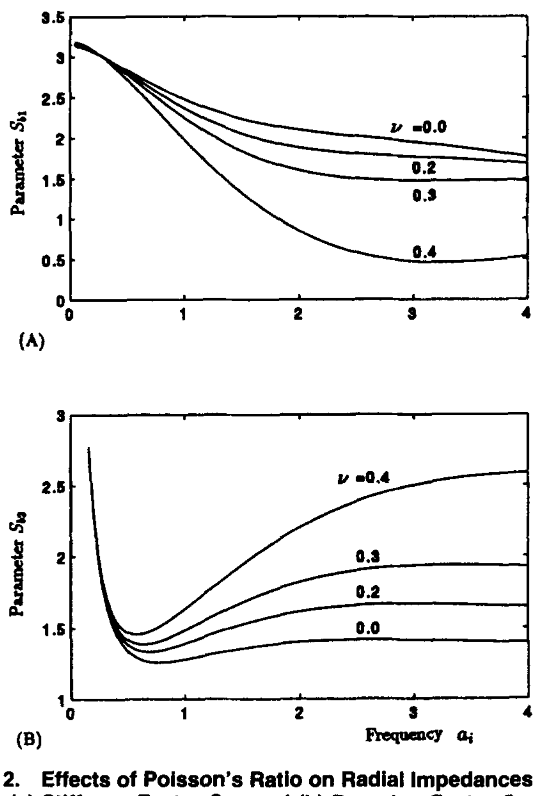

FIG. 12. Effects of Poisson's Ratio on Radial Impedances of Soil Layer: (a) Stiffness Factor S_{b1}; and (b) Damping Factor S_{b2}

$$K_r = (G_i/r_o)[S_{b1} + ia_i S_{b2}] \tag{73}$$

The real part S_{b1} is a dimensionless stiffness factor and S_{b2} is a damping factor. The values of S_{b1} and S_{b2} depend on frequency a_o, damping ratio β_o and Poisson's ratio ν; also on the boundary zone parameters, such as, the modulus ratio G_i/G_o, thickness ratio t_m/r_o and damping ratio β_i.

The stiffness and damping factors, S_{b1} and S_{b2}, obtained from the present analysis are compared with those obtained for the Novak and Mitwally idealizations (1988), as shown in Fig. 10. These solutions are for a soil layer with t_m/r_o = 2.0, G_i/G_o = 0.5, β_i = 0.1, β_o = 0.05, and Poisson's ratio ν = 0.25. The mass for the inner zone is accounted for in both Novak's solution and the present solution. However, the properties of the soil for the inner zone were assumed to be constant in the former, which results in pronounced oscillations (undulations) caused by wave reflections from the interface between the two media. The results from the present analysis are smooth curves over a wide range of parameters, indicating that the wave reflections from the interface have been removed. For comparison, the results for a homogeneous layer are also included in Fig. 10.

To illustrate the influence of the parameters involved, the stiffness and damping factors for a radially excited layer are plotted in Fig. 11 as a function of a_i for several different combinations of t_m/r_o and G_o/G_i, with material damping, β_i = 0.1 and β_o = 0.05. It should be noticed that the undulations caused by wave reflection vanish as expected in all of the cases presented. The radial stiffness factor, S_{b1}, increases with the level of G_o/G_i and is smallest for the homogeneous case (G_o/G_i = 1).

The effects of Poisson's ratio on the radial impedances of the soil layer are shown in Fig. 12. Several values of Poisson's ratio are selected, ν = 0.0, 0.2, 0.3, and 0.4, respectively. It can be seen that the stiffness factor, S_{b1}, reduces with Poisson's ratio increasing, but the damping factor, S_{b2}, increased with ν in the higher frequency range.

CONCLUSION

With the information and insight into the response that have been provided in this paper, the dynamic impedances of a radially inhomogeneous viscoelastic soil layer with a central hole may be evaluated readily. Vertically, torsionally, and radially excited soil layers have been examined. The proposed boundary zone model allows a continuous variation of soil properties with a smooth transition into the outer zone, so that the wave reflections from the interface between the two media are removed. The impedances of a composite layer do not show the oscillations that are generally observed with the other boundary zone models that take account of the soil mass. In this regard, the proposed formulations have satisfactorily corrected the problems associated with wave reflections in the boundary zone.

Using the impedances of the composite layer with a nonreflective boundary, the stiffness and damping of piles or embedded foundations can be formulated readily. Since the problems associated with wave reflections in the boundary zone have been solved in this paper, the impedances may be more suitable for practical applications.

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