Three-Dimensional Analysis of Lateral Pile Response using Two-Dimensional Explicit Numerical Scheme

Assaf Klar[1] and Sam Frydman[2]

Abstract: A procedure for exploiting a two-dimensional (2D) explicit, numerical computer code for the 3D formulation of dynamic lateral soil-pile interactions is considered. The procedure is applied to two models using simultaneous computation of a series of plane strain boundary value problems, each of which represents a horizontal layer of soil. The first model disregards the shear forces developed between the horizontal layers, and may be considered as a generalized Winkler model. The second model takes account of these forces by coupling the behavior of the horizontal layers. Several verification problems for a single pile and pile groups in a homogeneous soil layer modeled as a viscoelastic material were solved and compared to known solutions in order to assess the reliability of the models. Excellent agreement was observed between results of the present analyses and existing solutions.

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Introduction

Many methods have been proposed for analysis of dynamic laterally loaded piles and pile groups. Each method has its own advantages and disadvantages; rigorous analytical methods and boundary integral methods (e.g., Tajimi 1969; Kaynia and Kausel 1982; Mamoon et al. 1988) are restricted to viscoelastic materials and frequency domain analysis, true three-dimensional (3D) finite element method or finite difference method analyses require significantly large calculation time, and simple Winkler models (Mylonakis and Gazetas 1999) have difficulty in modeling the pile-soil-pile interaction for nonlinear materials. Furthermore, most of the soil-pile interaction analysis methods require computer codes designed specifically for that purpose, some of which are commercial, and others research codes [e.g., Blaney et al. 1976 (the 3D finite element research code *NONSPS*); Kagawa 1983 (the beam-on nonlinear Winkler foundation research code *DYNA*3); Novak et al. 1990 (the commercial code for pile and pile groups analysis *PILE*-3*D*); Wu and Finn 1997a, b (a quasi-3D finite element research code)]. This paper describes a procedure for utilizing explicit 2D finite element (FE) and finite difference (FD) codes for representation of a 3D soil-pile interaction under static and dynamic loading. This technique was developed using the commercially available 2D geotechnical finite difference code *FLAC* Ver. 3.4 (Itasca 1999) (hereafter referred to as the FD code). Although the procedure is not limited to this code, its user-available, internal programming feature makes the implementation of the technique quite simple. Furthermore, no extensive knowledge of numerical methods (creation of stiffness matrices, etc.) is required for implementation of the technique, and thus its use is feasible for any practicing engineer. Two models were developed using this technique; a generalized Winkler (uncoupled) model, and a coupled model. Several verification problems of single piles and pile groups under different loading conditions were analyzed and the results were compared with known solutions in order to assess the reliability of the models. In these verification problems, the soil was modeled as a homogeneous, viscoelastic material. However, the models are also applicable for nonlinear constitutive models. The work presented is part of broader research; the aim of the present paper is only to introduce the approach. Consequently, nonlinear behavior is not considered here, but will be the subject of future publications.

[1]Graduate Student, Faculty of Civil Engineering, Technion, Haifa, Israel.

[2]Professor, Faculty of Civil Engineering, Technion, Haifa, Israel.

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Outline of Proposed Technique

The technique is based on discretization of the 3D soil continuum into a series of horizontal layers, each layer represented by a 2D boundary value problem (BVP). The initial FD grid may be divided into several disconnected subgrids, thus allowing the simultaneous calculation of several BVPs. For representation of the physical problem, a cavity (or cavities in the case of a pile group) is inserted in each subgrid in order to model the pile cross section in that layer. Different soil properties and/or initial stresses may be introduced for each layer, as required. In addition, a separate grid consisting of a series of connected, unsupported beam elements, representing the pile, is defined. In the case of a pile group with a cap, another separate subsystem should be defined to represent the cap. The procedure advances in time, in correspondence with the FD explicit time marching calculation scheme, and develops the interaction between the two (or three) systems through transfer of velocities and forces from pile (beam) nodes to the BVP's grid point and vice versa. In each calculation cycle (time step) the pile's motion equations are solved and the velocities of each pile segment's nodes are applied to the corresponding

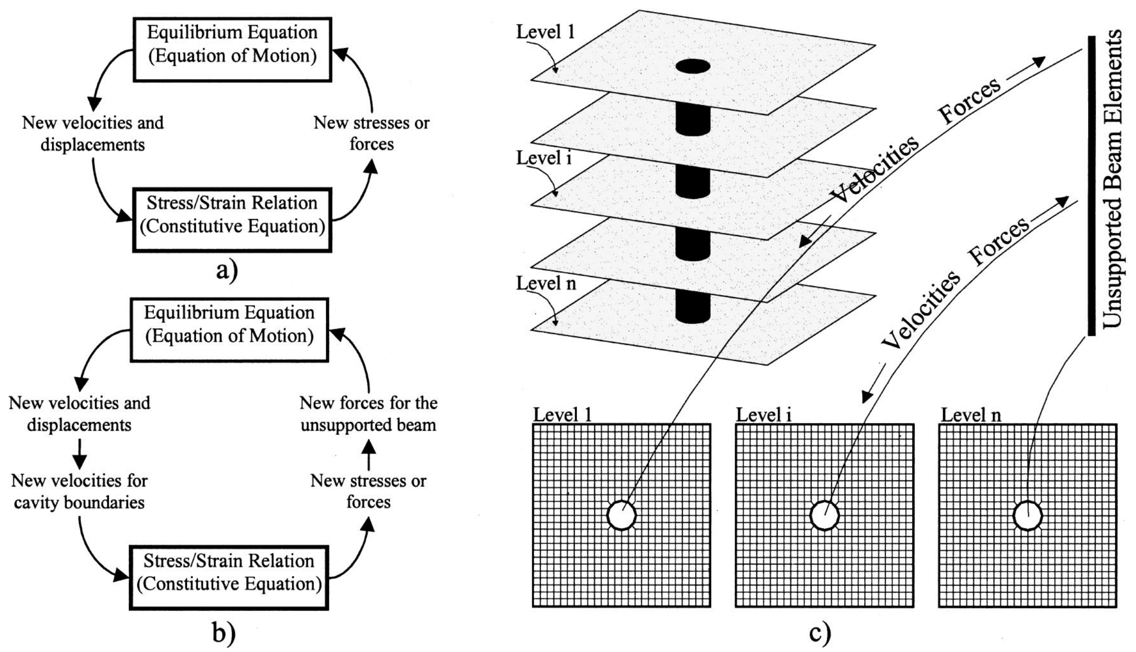

Fig. 1. Proposed procedure: (a) original *FLAC* calculation cycle; (b) modified calculation cycle; (c) subgrids and beam interaction

cavity boundary nodes. Next, in the same calculation cycle, the resulting forces acting on the cavity perimeter are extracted and applied to the appropriate pile segment's nodes. These forces are then used in solving the pile's equation of motion in the next time step, producing new velocities which are again transferred to the relevant cavity nodes. Fig. 1 demonstrates the concept of the procedure. It should be realized that, by applying the pile velocities to the cavity perimeter in the same calculation cycle, a full compatibility of displacement is achieved. This formulation describes only the interaction between the pile and the horizontal layers. The formulation of the BVP's coupling and the consideration of seismic loading will be described later.

Uncoupled Model

A 3D soil-pile interaction problem may be solved by discretization of the soil continuum into horizontal layers, where each layer is represented by a different uncoupled plane strain problem. This technique may be classified as a generalized Winkler approach. Although it is considered approximate, its predictions are in good agreement with rigorous solutions, within some limitations. Novak (1974) was the first to use this technique for evaluating the behavior of a single pile under lateral dynamic loading in a homogenous soil layer. He used an analytical solution of the plane strain problem in conjunction with the equation of equilibrium of the pile. Novak and El-Sharnouby (1983) extended these solutions to the case of soil properties varying with depth. Nogami and Novak (1980) showed that, for a frequency of loading higher than the fundamental natural frequency of the system, the soil medium can be treated as a Winkler model. The uncoupled model presented herein is based on this same assumption, but it is not restricted to viscoelastic material or frequency domain analysis. Since the BVPs are uncoupled, the formulation presented in the previous paragraph needs no modification for piles laterally loaded at their heads, and only minor modification for seismic loading. Several verification problems for different pile configurations and loading conditions are presented in the following sections.

Uncoupled Model: Lateral Dynamic Loading of Single Pile

Neglecting the internal damping of a pile embedded in a Winkler medium, the differential equation may be written as follows (Novak and Aboul-Ella 1978):

$$E_pI_p\frac{\partial^4 u(z,t)}{\partial z^4}+\mu\frac{\partial^2 u(z,t)}{\partial t^2}+k_{us}\cdot u(z,t)=0 \qquad (1)$$

$$k_{us}=G[S_{u1}(a_0,\nu,\beta_s)+iS_{u2}(a_0,\nu,\beta_s)] \qquad (2)$$

where E_pI_p, u, μ, and k_{us} are the pile flexural rigidity, lateral displacement along the pile, mass of the pile per unit length, and horizontal soil stiffness respectively. S_{u1} and S_{u2} are functions of the dimensionless frequency a_0 ($=\omega r_0/V_s$, where r_0=radius of the pile; ω=circular frequency of loading; and V_s=shear wave velocity of the soil), Poisson's ratio ν, and material damping β_s of the soil, as given by Novak et al. (1978). Assuming a harmonic, steady state motion $u(z,t)=u(z)e^{i\omega t}$ through the pile leads to the differential equation

$$E_pI_p\frac{\partial^4 u(z)}{\partial z^4}+(k_{us}-\mu\omega^2)u(z)=0 \qquad (3)$$

Solution of Eq. (3) results in a complex $u(z)$ and a typical solution is shown in Fig. 2. As can be seen, only a certain length is effective; this phenomenon is common in long piles, and is responsible for the length independency of the pile behavior. In the case of homogeneous soil properties with depth, a closed form solution is available. For an arbitrary variation of soil properties, there is no exact solution, but Eq. (3) may be solved numerically.

If the soil can be assumed to behave viscoelastically, the soil-pile-structure interaction problem may be solved by representing

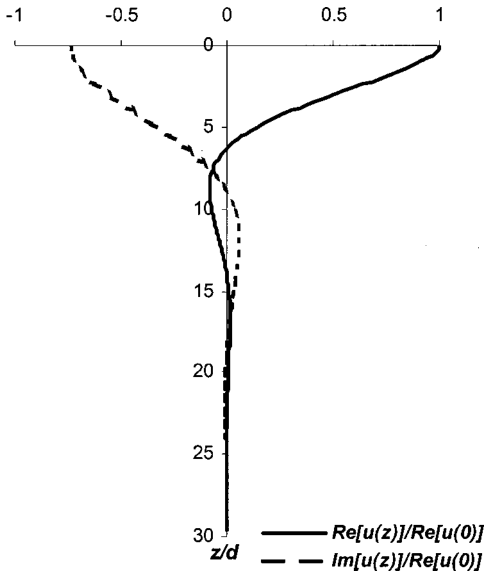

Fig. 2. Solution shape of Eq. (3) for $E_P/E_S = 1250$, $\nu = 0.25$, $a_0 = 0.3$, $\rho_s/\rho_p = 0.7$ (no material damping, $\beta_s = 0$)

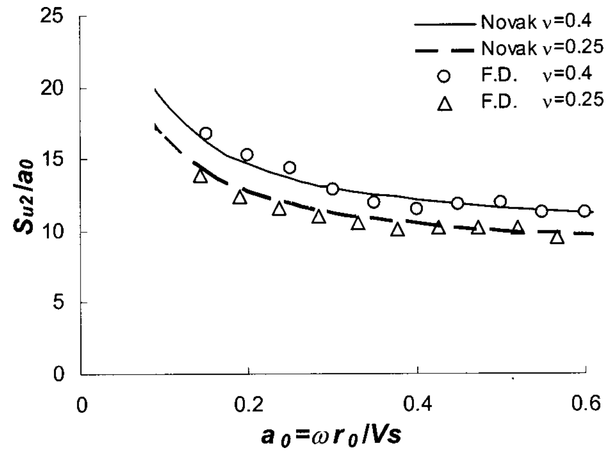

Fig. 3. Radiation damping of cavity motion in plane strain problem

the soil-pile system as an equivalent viscoelastic element (i.e., spring and dashpot). Considering two degrees of freedom at the top of the pile (U = horizontal translation and Φ = rotation) one can write the following relations for harmonic motion (Novak 1974):

$$\begin{Bmatrix} Pe^{i\omega t} \\ Me^{i\omega t} \end{Bmatrix} = \frac{E_p I_p}{r_0^3} \times \begin{bmatrix} (f_{11,1} + ia_0 f_{11,2}) & r_0(f_{9,1} + ia_0 f_{9,2}) \\ r_0(f_{9,1} + ia_0 f_{9,2}) & r_0^2(f_{7,1} + ia_0 f_{7,2}) \end{bmatrix} \times \begin{Bmatrix} Ue^{i\omega t} \\ \Phi e^{i\omega t} \end{Bmatrix} \quad (4)$$

where P and M are the lateral force and moment at the pile head, respectively; and $f_{i,j}$ are parameters extracted from the solution of Eq. (3) and given by Novak (1974) for different ratios of V_s/V_L and ρ_s/ρ_p (ratios of the soil shear wave velocity to the longitudinal wave velocity of the pile and the density of the soil to that of the pile, respectively) for a dimensionless frequency a_0 of 0.3. A more convenient way of presenting Novak's solution is through reference to the Young's modulus ratio of the pile and soil E_p/E_s (Kuhlemeyer 1979). Table 1 shows Novak's parameters $f_{i,j}$ for a ratio of pile length to diameter $L/d = 30$, $a_0 = 0.3$, $\nu = 0.25$, and $\rho_s/\rho_p = 0.7$. In order to verify the accuracy of the proposed model for dynamic problems, several runs were conducted and the results were compared with Novak's closed form

Table 1. Novak's Stiffness and Damping Parameters

		Stiffness Parameters			Damping Parameters		
V_S/V_L	E_P/E_S	$f_{11,1}$	$f_{9,1}$	$f_{7,1}$	$f_{11,2}$	$f_{9,2}$	$f_{7,2}$
0.01	5,714.286	0.0032	−0.0181	0.195	0.0076	−0.0262	0.135
0.02	1,428.571	0.009	−0.0362	0.275	0.0215	−0.0529	0.192
0.03	634.9206	0.0166	−0.0543	0.337	0.0395	−0.0793	0.235
0.04	357.1429	0.0256	−0.0724	0.389	0.0608	−0.1057	0.272
0.05	228.5714	0.0358	−0.0905	0.435	0.085	−0.1321	0.304

solutions. No material damping was introduced into the model, consistent with Novak's 1974 solutions. A quiet (nonreflecting) boundary was defined at the model edges, in order to prevent reflection of outward propagating waves back into the model from the boundaries, and to allow the necessary energy radiation. First, a plane strain model of a circular cavity motion was tested, for verification of the FD code's radiation damping, and the results were compared with Novak's parameter S_{u2} [see Eq. (2)], which represents the geometric (radiation) damping. From these calculations, it was found that a distance of approximately 0.7 wavelengths is required from the cavity to the boundary in order to provide satisfactory representation of the radiation damping. Fig. 3 shows a comparison between Novak's results, and those obtained from the FD computations; the agreement is excellent. Kuhlemeyer (1979) showed that Novak's $f_{i,j}$ parameters plot against E_p/E_s as a straight line on a log-log plot, and thus only a few values are needed for evaluating the validity of the proposed method. The FD analyses were performed at three different ratios of E_p/E_s (500, 1,250, 3,000); the results are presented in Fig. 4. The results were obtained by using two different techniques. In both techniques, the vibration was developed to the steady state condition smoothly, beginning from the static state. In the first technique, a force was applied to the pile head and displacement at the same node was monitored; in the second technique a velocity was applied to the pile head and the unbalanced force at the same node was monitored. In the latter case no forces were transferred from the subgrid corresponding to the pile head node, and thus it was necessary to subtract the reaction created in that subgrid (a velocity boundary and force boundary cannot coexist). The results obtained from both techniques were identical. The second technique is suitable for pile group analysis, where all pile heads move rigidly together. Deviation of the FD solutions from Novak's solutions are presented in Table 2. Novak's values f_{ij} for the tested E_p/E_s were calculated from the power function corresponding to Novak's straight line on the log-log plot. From the comparison presented in Table 2, it can be seen that the greatest deviation relates to the parameter $f_{11.1}$. Note that the distance between the cavity, which represents the pile, and the quiet boundary, was only 0.4 wavelengths in these calculation runs. The use of this small distance decreased the calculation time significantly, and thus justified the small deviation it possibly caused.

Uncoupled Model: Pile Group Interaction

The influence of pile-soil-pile interaction on the behavior of a pile group is well known for static loading. This interaction is quite

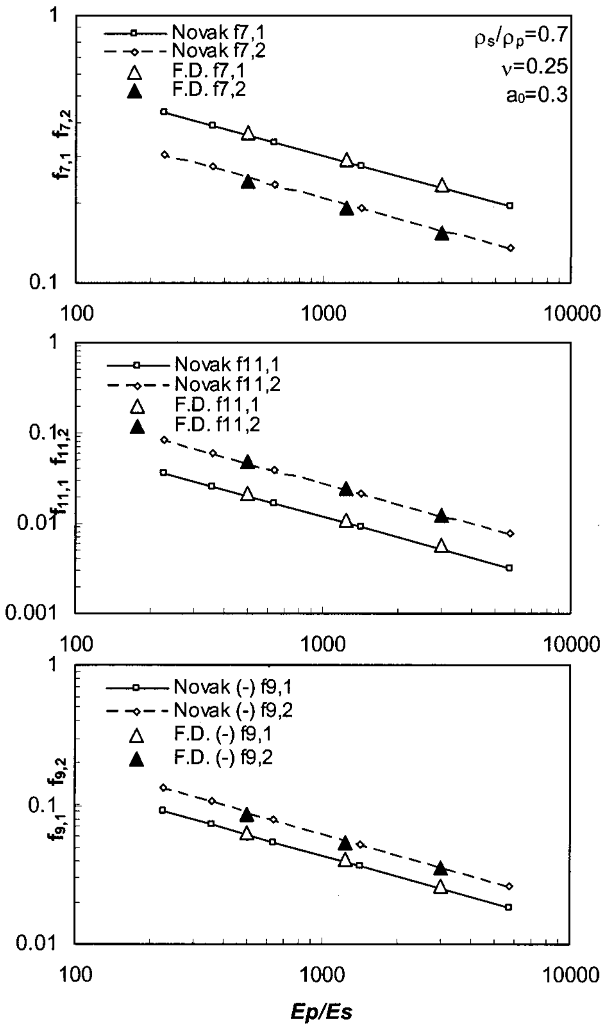

Fig. 4. Comparison between finite difference solutions and Novak's solutions

simple for linear elastic material, and many expressions for evaluating it have been suggested (e.g., Poulos 1971b). However, this is far from true for a pile group under dynamic loading; dynamic characteristics of pile groups are very complex, strongly frequency dependent, and often significantly different from those of a single pile (El-Marsafawi et al. 1992). A superposition approach is commonly employed, using complex interaction factors for two piles, in conjunction with known single pile dynamic characteristics. Clearly, in using superposition, the interaction between all the piles is approximated. The superposition method is implemented through a flexibility approach in conjunction with the dynamic interaction factors for two piles. For a pile group one can write the following relation:

$$\{U_i\}=\lambda^s[\alpha_{i,j}]\{P_j\} \tag{5}$$

Table 2. Deviation from Novak's Solution

	Stiffness Parameters			Damping Parameters		
E_P/E_S	$f_{11,1}$	$f_{9,1}$	$f_{7,1}$	$f_{11,2}$	$f_{9,2}$	$f_{7,2}$
500	9.66%	5.08%	2.44%	1.87%	−3.80%	−3.52%
1,250	9.52%	5.28%	2.41%	1.74%	−3.41%	−3.31%
3,000	10.20%	5.45%	2.42%	0.58%	−3.05%	−3.04%

where U_i = head displacement of pile i; λ^s = single pile dynamic flexibility; P_j = force at the head of pile j; and α_{ij} = complex interaction factor between pile i and pile j for a certain vibration mode, defined as the ratio of the dynamic displacement of pile i due to dynamic loading on pile j to the dynamic displacement of pile j. By considering the rigidity of the pile group cap ($U=U_i=\cdots=U_n$), one can derive the pile group complex stiffness from Eq. (5), as follows:

$$K_{\text{group}}=\frac{\Sigma_{j=1}^{n}P_j}{U}=\frac{1}{\lambda^s}\sum_{i=1}^{n}\sum_{j=1}^{n}\epsilon_{i,j}=k^s\sum_{i=1}^{n}\sum_{i=1}^{n}\epsilon_{i,j} \tag{6}$$

where k^s = single pile dynamic stiffness; and $\epsilon_{i,j}$ = elements of the inverse of matrix α. Design values of interaction factors for dynamic loading were suggested, for example, by Gazetas et al. (1991) and El-Marsafawi et al. (1992). It should be pointed out that, although the superposition method is very convenient for practical usage, a small error in evaluating the interaction factor will result in a much greater error in the group dynamic stiffness. As an example, the group stiffness of a 3×3 pile group is composed of the summation of 81 values of which 72 are evaluated from design graphs, some by interpolation functions. Winkler models for evaluating the horizontal dynamic pile-soil-pile interaction in pile groups have also been developed; they all show good agreement with more rigorous solutions based on 3D wave propagation. The simplest Winkler based model is quite new, suggested by Mylonakis and Gazetas (1999), yet it is limited by assumptions of linearity for soil and pile materials, and perfect bonding at the soil-pile interface. A more complex model, also based on the Winkler assumption, was proposed by El Nagger and Novak (1996). The model accounts for nonlinear behavior of the soil adjacent to the pile and gapping at the soil-pile interface. Although the computational effort is quite small, a computer code designed for that particular model is required. None of the Winkler model approaches used up to the present have taken account of the disturbance caused by a pile to the waves propagating through the soil medium. When the pile spacing is small, this effect may be of considerable significance. Even the more rigorous existing solutions relate the forces to the average displacement of the soil-pile interaction segment. Rajapakse and Shah (1989) reviewed continuum models for elastic soil pile systems and referred to the averaging procedure as problematic for dynamic problems. They claimed that with increasing frequency models based on the cross sectional average of the displacement fail to provide an accurate solution. To verify the reliability of the proposed method for the pile-soil-pile interaction, two simple interaction problems were considered—two fixed-head piles, and a 3×3 pile group. The solutions obtained from the FD analyses were compared with El-Marsafawi et al.'s (1992) solutions, which were based on a 3D boundary integral formulation, and with Mylonakis and Gazetas' (1999) solutions based on a Winkler model (propagation of waves in a horizontal manner only) in conjunction with superposition. For the case of interaction of two piles, the solutions were obtained for the case in which the horizontal translation of the pile head was in the direction of the line connecting the two pile centers. These solutions are presented as an interaction factor between the two piles in Fig. 5. The solution for the case of the 3×3 pile group is presented (Fig. 6) as a normalized dynamic stiffness defined as the dynamic stiffness of the pile group divided by the dynamic stiffness of a single pile multiplied by the number of piles in the group. In order to evaluate both the two-pile interaction factor and the normalized pile group stiffness, the dynamic stiffness of a single pile must be

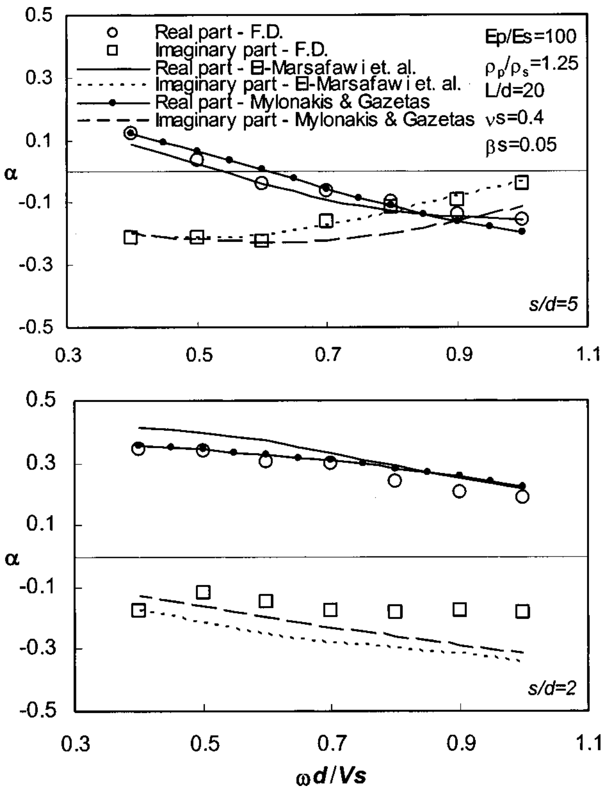

Fig. 5. Lateral interaction factors between two piles in homogeneous soil, for normalized distances $s/d = 2{,}5$

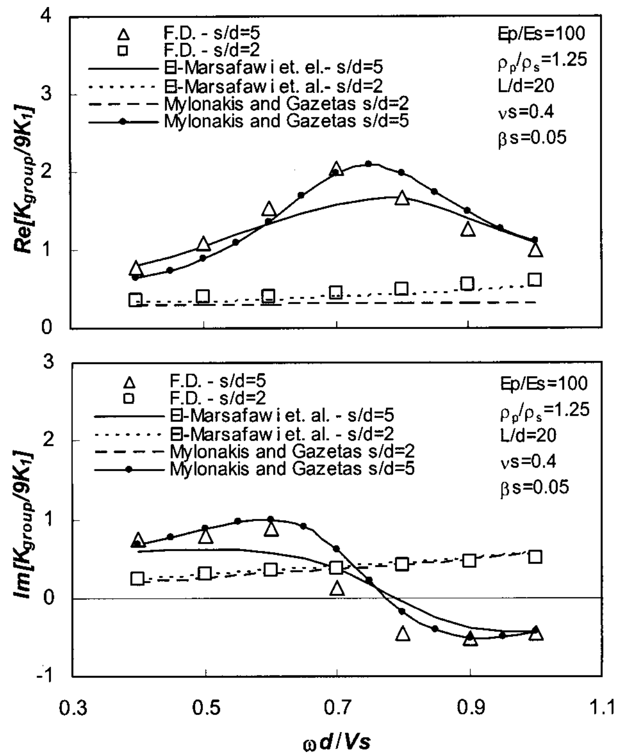

Fig. 6. Normalized lateral dynamic stiffness of 3×3 pile group

known. This was found by calculating, with the FD code, the dynamic stiffness of each frequency, in the same manner that was employed in the single-pile dynamic analysis. For the two-pile interaction, excellent agreement between the El-Marsafawi et al. (1992) solution and the FD solution exists for a normalized spacing (s/d) of 5. However, less good agreement exists for normalized spacing of 2, although the Mylonakis and Gazetas Winkler based model shows good agreement with the El-Marasafwi et al. solution. It may be seen that this deviation increases with frequency, implying that it is connected to the wave propagation factor. As mentioned previously, both the Winkler based model and the 3D solutions do not comprehensively account for the pile interference to wave propagation, and it is possible that the results from the present calculations take account of this interference. For the 3×3 pile group, general agreement exists for all the parameters; interaction of nine piles is quite complex and thus it is difficult to pinpoint the source of the observed deviations.

Uncoupled Model: Single-Pile Seismic Response

Using a Winkler based model for earthquake analysis requires an evaluation of shear wave propagation through the soil layers. Since the plane strain problems are uncoupled (disconnected) the vertical propagation of shear waves must be modeled separately and applied indirectly to the plane strain problems. This may be done by applying to each grid point, at each time step, an additional force which corresponds to the horizontal acceleration in the free field at the same depth and time. For viscoelastic material this procedure is completely legitimate; it is equivalent to superposition of forces. For nonlinear materials, it is considered to be a reasonable approximation, due to the difference in the polarity of the S waves in the free field (τ_{xz} and τ_{yz}) and the τ_{xy} stress waves and P waves that would emanate from the pile (Wang et al. 1998); where x is the horizontal direction of pile displacement and z is vertical. Since this formulation decouples the vertical shear propagation from the soil-pile interaction, the free field acceleration history may be calculated from an additional one-dimensional analysis within the FD code or an external program such as *SHAKE* (Idriss and Sun 1991). In this verification problem, the one-dimensional vertical shear propagation was modeled by an additional subgrid which represents the site. A subroutine, which monitors the velocities and acceleration of each grid point in the free field subgrid and inserts them in tables, was written. In every time step an acceleration value for each plane strain problem was interpolated from these tables, according to depth. The acceleration value was then multiplied for each internal (not on a boundary) grid point by its mass and was applied to it. For the plane strain boundary grid points, a force ΔF defined by Eq. (7) was added to the initial K_0 static forces in order to maintain nonreflecting boundaries

$$\Delta F = m a_{ff} + C(v_{ff} - v)s \tag{7}$$

where m = grid point mass; a_{ff} and v_{ff} = acceleration and velocity, respectively, of the free field at the same depth; v = grid point velocity; C = coefficient that is related to the S wave and P wave velocities (depending on the direction of the applied forces); and s = boundary length, which is related to the grid point. This formulation is very similar to that used in *FLAC* for free field boundaries, based on the viscous boundary developed by Lysmer and Kuhlemeyer (1969), yet the one implemented in *FLAC* cannot be used for this formulation. It should be noted that in case a material damping is defined externally (not within the constitutive relation) it should be activated only on strain rates, i.e., when using Rayleigh damping as in *FLAC*, the mass proportional damping coefficient should be set to zero. If a mass proportional damping

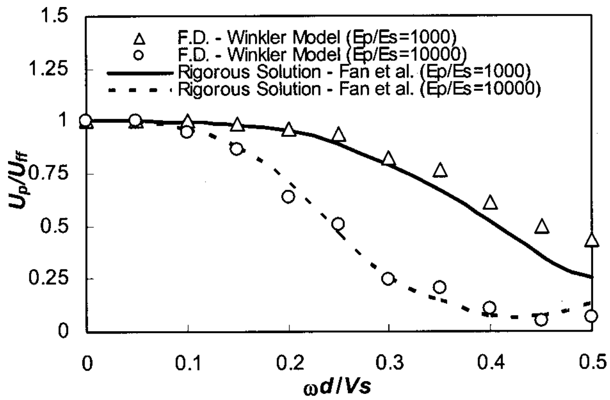

Fig. 7. Kinematic seismic response of single fixed-head pile in homogeneous soil layer ($\rho_s/\rho_p = 0.7$, $\beta_s = 0.05$, $\nu = 0.4$)

is defined, the free field vibration modes will be overdamped, since damping is applied both to the free field subgrid, and also to the plane strain system masses.

For the verification problem, a fixed-head pile embedded in a homogeneous, viscoelastic soil layer was considered. The base of the soil layer was excited by a harmonic shear wave. In order to represent a homogeneous half space, the shear wave was applied to a quiet boundary. After reaching a steady state motion, the amplitudes of the horizontal oscillation of the pile cap U_p and free field surface U_{ff} were extracted. The FD analyses were performed at two different ratios of E_p/E_s (1,000, 10,000) for a complete range of frequencies. The results were compared with the rigorous solution of Fan et al. (1991) based on the boundary integral based formulation developed by Kaynia and Kausel (1982). The results are presented in Fig. 7 as a ratio between the pile cap oscillation amplitude and the free field surface oscillation amplitude.

Good agreement is observed between the results. However, it can be seen that better agreement exists for the higher ratio of E_p/E_s. This is consistent with Nogami and Novak's (1980) conclusion that a soil medium can be treated more favorably as a Winkler based model for stiffer piles.

Limitations of Uncoupled Model

The uncoupled model is based on Winkler's approach. It may be classified as a first order, dynamic, subgrade model, according to Nogami et al.'s (1992) definition. Being a first order model, it is bounded by the first order model limitations. It encounters difficulties in modeling frequencies lower than the fundamental frequency of the soil medium. For machine foundations, this limitation is almost irrelevant since the frequencies involved in such cases are usually considerably higher than the fundamental frequency of the soil. Also, for earthquakes, most of the soil-pile interaction is noticeable for frequencies around the structure-foundation resonance, which are usually higher than the fundamental natural frequency of the ground. For floating piles, the first order model is quite accurate. Usually, in order to overcome this limitation of the first order dynamic model, it is modified by using a static stiffness for frequencies lower than the natural one, and ignoring the radiation damping (Nogami et al. 1992). A similar procedure may be invoked in the present procedure for lateral loading of piles, by using a fixed boundary instead of a quiet boundary, so that no energy is absorbed by the boundaries. One must, of course, be careful in using this procedure, since the location of the boundaries is crucial in determining the static stiffness in a plane strain problem. Baguelin et al. (1977) have conducted a theoretical study of the static lateral reaction mechanism of piles using plane strain analysis. Their recommendation for the boundary locations may be adopted, but it should be appreciated that unnecessary amplification is possible in dynamic analyses. However, it is possible to overcome all limitation of first order models by coupling the plane strain problems. This modification is presented in the following section.

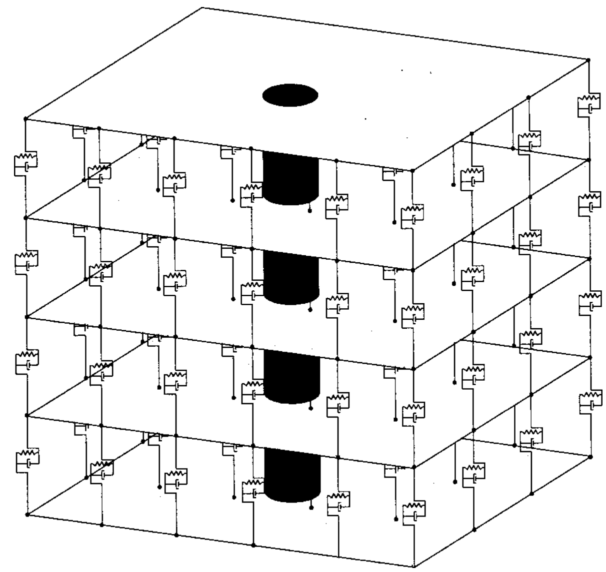

Fig. 8. Coupling mechanism of plane strain problems

Coupled Model

A coupling of the plane strain problems is feasible by connecting shear springs and dashpots between every plane strain problem grid point and the upper/lower plane strain problem grid point. A schematic representation of coupling the plane strain problems with springs and dashpots is presented in Fig. 8. This coupling may or may not actually be a part of the constitutive model for the soil. The use of springs and dashpots for homogenous elastic material is equivalent to the use of terms obtained from the equation of motion for a continuous body

$$\rho \frac{\partial^2 u_j}{\partial t^2} = \rho g_j + \frac{\partial \sigma_{ij}}{\partial x_i} \tag{8}$$

where u_j = displacement vector; ρ = density; t = time; g_j = gravitation vector; and σ_{ij} = stress tensor. By restricting the motion to the horizontal planes, only the variation of displacement with depth needs to be considered for the coupling. An additional force vector should be applied to the concentrated masses located at the corners of the FD zones (grid points)

$$F_j = \frac{M}{\rho}\frac{\partial \sigma_{zj}}{\partial z} = \frac{M}{\rho}\frac{\partial(2G\epsilon_{zj} + 2G\eta\dot{\epsilon}_{zj})}{\partial z}$$

$$= \frac{M}{\rho} G\left(\frac{\partial^2 u_j}{\partial z^2} + \sigma \frac{\partial^2 \dot{u}_j}{\partial z^2}\right) \tag{9}$$

where F_j = additional force vector applied to the mass; M = mass value of the grid point; G = shear modulus; η

=parameter related to the viscous damping and frequency; z =vertical axis; and j takes the index of the horizontal axes. Rewriting Eq. (9) using finite difference approximations of the derivatives results in the same expression that is obtained by using springs and dashpots with values of $K=G/l$ and $C=\eta G/l$, respectively, and correcting from stress to force by applying an "area factor" of $M/\rho d$ to the applied grid point force (l is the distance between the layers connected by the springs and dashpots and d is the thickness of the horizontal layer that the plane strain problem represents). One can regard this model as a true 3D model where all the grid points are constrained (fixed) in the vertical direction. Wu and Finn (1997a, b) presented a quasi-3D finite element model, where the grid point motion was fixed both in the vertical direction and in the direction perpendicular to the pile motion. However, instead of using a nonreflecting (quiet) boundary they applied dashpots to the pile shaft to model the radiation damping. Although the form of these dashpot coefficients was based on a simple, one-dimensional "cone" model (Gazetas et al. 1993), they were calibrated by curve-fitting results from rigorous, finite element analyses, where no limitation was made on grid point movements. Reduction of degrees of freedom in an implicit integral method is more computationally effective than it is in explicit integral methods where no factorization of matrices is necessary. Consequently, Wu and Finn's approach may be more justified for implicit schemes than for explicit schemes.

Several problems with different pile configurations and loading conditions are presented in the following sections for verification of the proposed coupled model.

Coupled Model: Static Lateral Loading

One of the disadvantages of the uncoupled model is that it cannot model a case of static loading, since there is no solution for plane strain loading in an infinite homogeneous material [i.e., in a numerical analysis the solution is boundary dependent; Baguelin et al. (1977) studied the necessary distance of a fixed boundary for simulation of static stiffness]. However, with the coupled model, such a static case is solvable, and was therefore chosen as one of the verification problems. The analysis was conducted for a homogeneous soil modeled as a linear elastic material. The single pile was pinned at its base to a solid rock. However, this end condition had no influence on the results and the solution may be regarded as that for a floating pile, since the pile length was significantly longer than its effective length. Since the calculation involved a linear elastic material, it was more convenient to define behavior of the coupling springs (Voigt elements) separately from the constitutive relation of the soil. In order to assess accuracy, a comparison to Poulos's (1971a) results (based on coupling Mindlin's solution for a concentrated horizontal load with the pile flexure equation), and to Kuhlemeyer's (1979) 3D finite element results, was conducted. The solutions are presented in Fig. 9 in terms of Kuhlemeyer's formulations

$$U=a_{11}\frac{P}{E_s r_0}+a_{12}\frac{M}{E_s r_0^2}$$
$$\Phi=a_{21}\frac{P}{E_s r_0^2}+a_{22}\frac{M}{E_s r_0^3} \quad (10)$$

where a_{ij} are parameters that are functions of the Poisson ratio and (E_p/E_s) ratio. Poulos presented his solutions in the form

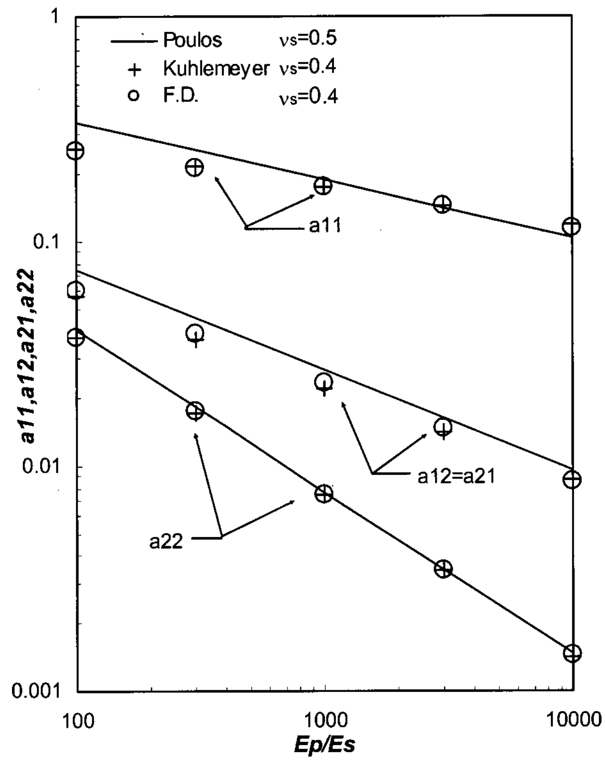

Fig. 9. Comparison of coupled model with Kuhlemeyer finite element solution and Poulos's solutions

$$U=I_{US}\frac{P}{E_s L}+I_{UR}\frac{M}{E_s L^2}$$
$$\Phi=I_{\theta S}\frac{P}{E_s L^2}+I_{\theta R}\frac{M}{E_s L^3} \quad (11)$$

where I_{US}, I_{UR}, $I_{\theta S}$, and $I_{\theta R}$ are functions of L/r_0 and the pile flexibility factor $K_R=E_p I_p/E_s L^4$. Presenting Poulos's results according to Kuhlemeyer's formulation leads to the identity

$$a_{11}=\frac{I_{US}}{L/r_0};\quad a_{12}=a_{21}=\frac{I_{UR}}{(L/r_0)^2};\quad a_{22}=\frac{I_{\theta R}}{(L/r_0)^3} \quad (12)$$

Excellent agreement exists between the coupled model results and Kuhlemeyer's results, the largest deviation being smaller than 5%. It should be noted that Kuhlemeyer's results are rigorous, obtained by a real, 3D analysis where every grid point is free to move in all directions, and not only horizontally. This small deviation might suggest that the restraint of movement in the vertical direction causes the soil-pile system to stiffen only slightly. The cause for deviation of Poulos's results was explained by Kuhlemeyer (1979), and will not be discussed here. However it should be noted that some of Poulos's results are considered to be in error due to numerical discretization. Comparisons with other static stiffness values obtained by finite element methods and boundary integral formulation (Randolph 1981; Dobry et al. 1982; Kaynia and Kausel 1991) were also conducted. The agreement with Randolph's solution is more or less as with Kuhlemeyer's solution. The deviations from Dobry et al.'s results (based on a finite element code developed by Blaney et al. 1976) are up to 20%. It should be noted that a similar difference exists between the Dobry et al. and Kuhlemeyer solutions, probably due to different discretization and boundary conditions. The present static value of stiffness is about 10% higher than that reported by Kaynia and Kausel (1991).

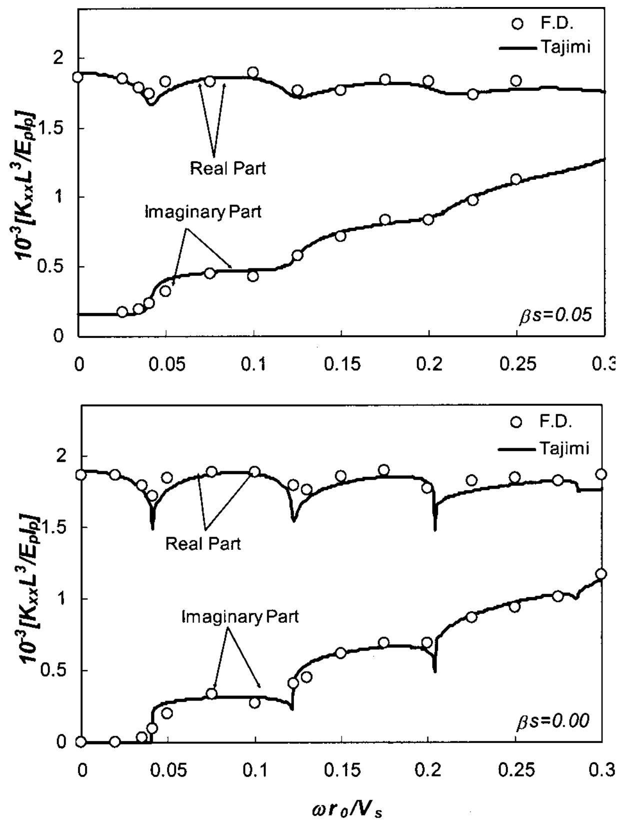

Fig. 10. Comparison of dynamic lateral stiffness computed by finite difference to values based on Tajimi

Coupled Model: Dynamic Lateral Loading

In order to verify the behavior of the coupled model under dynamic excitation, a simple problem of dynamic lateral loading was considered, and the results were compared with a continuum solution based on the work of Tajimi (1969). As for the case of static lateral loading, the analysis was conducted for a homogeneous soil, modeled as a viscoelastic material. The comparison was made for the lateral translation stiffness, as presented in Fig. 10 for the case of $L/r_0=38.5$, $\rho_s/\rho_p=0.625$, $\nu=0.4$, $V_S/V_L=0.044$ (or $E_p/E_s=295.159$) with material damping $\beta_s=0$ and 5%. It can be seen from Fig. 10 that both the real and imaginary parts of the complex stiffness are in excellent agreement with the solution based on Tajimi's work. It is seen from Fig. 10 that the coupled model captures the phenomenon of decreased stiffness around the natural fundamental frequencies and the overall shape. The radiation damping cutoff frequency is also well modeled. Since Tajimi's (1969) formulation imperfectly captures the material damping behavior (it is taken into account only for the shear waves traveling along the depth and not for shear waves traveling horizontally), it was necessary to modify his formulation for more accurate consideration of material damping. A frequency independent viscosity was introduced into the equations of the elastic continuum by complementing Lamé's constants with their imaginary (out-of-phase) components.

Coupled Model: Single-Pile Seismic Response

Unlike Winkler based models, the coupled model inherently captures the development of shear stresses (τ_{xz} and τ_{yz}); thus no separate modeling of free fields is required for superposition of forces. However, in order to allow dissipation of pile vibration energy (radiation damping) to the infinite soil, nonreflecting boundaries are used together with the free field calculation. These boundaries are not mandatory; an alternative approach is to locate the plane strain problem boundaries at a sufficient distance from the pile. In this case, waves emitted from the pile will dissipate due to material damping. For soils with low material damping the latter approach is impractical, since a large number of soil elements is required. Again, the formulation of the nonreflecting boundary (free field boundary) is similar to that used in *FLAC* for plane strain problems, based on the viscous boundary developed by Lysmer and Kuhlemeyer (1969). *FLAC*'s built-in free field boundary cannot be applied to the proposed model since its formulation is limited to a single plane strain problem where the grid represents a vertical plane. Fig. 11 demonstrates the mechanism of the free field boundary. It comprises a column of concentrated masses connected by springs (detail A) with each mass connected to a plane strain system through a viscous elements (detail B). A one-dimensional free field is modeled by discrete masses connected by the coupling springs simultaneously with analysis of the plane strain problems. The free field motion may also be modeled by a plane strain problem (a vertical bar) with a suitable constitutive law, but using the same springs for the free field calculation as for the coupling is numerically more accurate. At each time step an additional force is applied to the boundaries via the viscous element, according to the expression

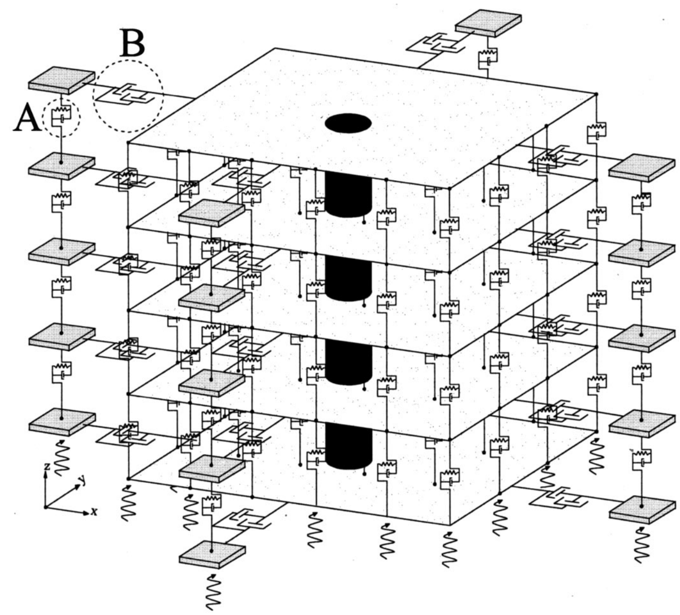

Fig. 11. Mechanism of free field boundary condition

$$\Delta F=C(v_{ff}-v)s \tag{13}$$

where the terms are as defined in Eq. (7). It can be noted from Eq. (13) that if the main grid motion is identical to the free field motion (i.e., $v=v_{ff}$) the dashpots (illustrated in Fig. 11) are not exercised. However, if the main grid motion differs from that of the free field, then the dashpots act to absorb energy. Evaluation of the free field motion can also be conducted with an external program such as *SHAKE*. However, since the vertical propagation of the waves would be different, even slightly, from those of the main grid, the dashpots would be unnecessarily exercised and might cause unreasonable results. The verification problem con-

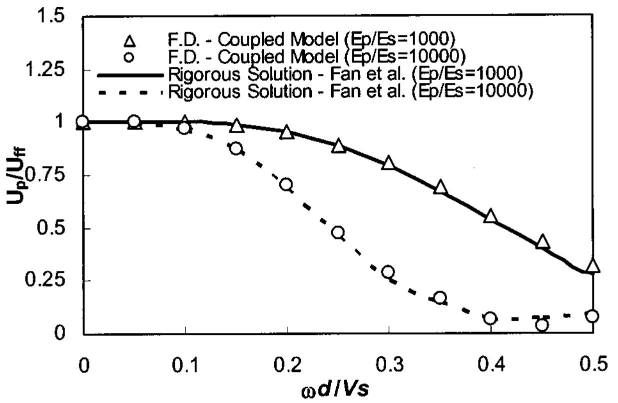

Fig. 12. Kinematic seismic response of single, fixed-head pile in homogeneous soil layer ($\rho_s/\rho_p = 0.7$, $\beta_s = 0.05$, $\nu = 0.4$)

sidered was identical to that of the uncoupled model. Again, results were compared with the rigorous solution of Fan et al. (1991), and are presented in Fig. 12. Excellent agreement is noted between the results.

Scope of Method

The presented technique and models may be used for solving a vast variety of soil-pile interaction problems; they are not restricted to linear elastic materials nor to homogeneous soil layers. For the uncoupled model, any desired constitutive law may be invoked by using either one of the FD code's library of constitutive relations, or by introducing a new relation through an internal subroutine. However, for the coupled model, an additional subroutine must be written in order to incorporate a constitutive law, since the coupling option is a modification of the code. Soil-pile gapping may be considered by inserting an interface at the cavity boundary. Moreover, in the case of a soil in which the permeability is significantly higher in the horizontal direction, a fully coupled analysis of pile-soil-groundwater interaction may be solved, according to the assumption that the water pressure will dissipate only in the horizontal direction. For soil with permeability equal in all directions, Nogami and Kazama (1991) showed that analytical expressions for pile lateral stiffness in a fluid saturated porous medium using cylindrical plane strain conditions yield results with accuracy very similar to those obtained for single-phase solids, which are well accepted. The method may also be used to solve loading in two directions, by assuming that the pile is linear elastic and analyzing two unsupported beams which represent the principal axes of the pile.

Conclusions

A general approach, based on the commercial 2D finite difference code *FLAC*, for evaluating the soil-pile interaction of single piles and pile groups under static, seismic, and lateral dynamic loading was presented. Two models were derived using this approach. Good agreement, for both models, was obtained with true 3D models and Winkler models for simple verification problems.

The uncoupled model has an advantage over the true 3D model, since discretization along the vertical axis is determined by the soil-pile characteristics and not only by the soil; thus fewer zones need to be defined. Its main advantage over other Winkler type models is that the behavior of the soil may be defined by a constitutive relation, and not just by spring coefficients or empirical *p-y* curves. The coupled model may be considered an expansion of the uncoupled model, overcoming most of its limitations.

A major advantage of the technique presented is that it can easily be implemented by any practicing engineer, with minor knowledge of numerical methods, using a computer code that is relatively inexpensive compared with true 3D programs.

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Notation

The following symbols are used in this paper:

a_0 = dimensionless frequency;
a_{ff}, v_{ff} = free field acceleration and velocity;
E_p = Young's modulus of pile;
E_s = Young's modulus of soil;
$f_{i,j}$ = stiffness parameters of pile;
I_p = inertia moment of pile cross section;
I_{US}, I_{UR}, $I_{\theta S}$, $I_{\theta R}$ = Poulos's parameters;
k^s = single-pile stiffness;
k_{us} = horizontal soil stiffness;
L = length of pile;
P, M = horizontal force and moment applied to pile's head;
r_0, d = radius and diameter of pile;
S_{u1}, S_{u2} = soil stiffness parameters;
s/d = normalized distance between piles;
U = horizontal translation at head of pile;
u = horizontal displacement along pile;
V_L = longitudinal wave velocity of pile;
V_P, V_S = P wave and S wave velocities of soil;
$\alpha_{i,j}$ = interaction factors;
β_s = material damping of soil;
$\epsilon_{i,j}$ = values of inverse matrix of interaction factors;
λ^s = single-pile flexibility;
μ = mass of pile per unit length;
ν = Poisson's ratio;
ρ_p = density of pile;
ρ_s = density of soil;
Φ = rotation at head of pile; and
ω = circular frequency.

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