Computers and Geotechnics **10** (1990) 59-79

THREE DIMENSIONAL FINITE ELEMENT MODEL OF LATERALLY LOADED PILES

Dan A. Brown and Chine-Feng Shie
Department of Civil Engineering
Harbert Engineering Center
Auburn University, AL 36849-5337 U.S.A.

ABSTRACT

The behavior of a pile subjected to lateral loading has been analyzed using a three dimensional finite element model. The study represents an attempt to develop a reasonably realistic model of the problem, including gap formation and plastic deformations in the soil around the pile, so as to provide a basis for parametric studies of the effects of pile spacing, pile head fixity, and soil stiffness on pile response. Constitutive models for soil include a simple elastic-plastic model with a Mises yield surface and associated flow and an extended Drucker-Prager model with nonassociated flow. Frictional interface elements were used to provide for slippage at the pile/soil interface and to allow gapping in the space behind the pile.

The results of the analyses provide insight into the deformation patterns and development of areas of plastic deformation around the pile. Such data also provide a basis for evaluation of more simple two dimensional approximations of the problem, such as described by Kooijman (1). In addition, bending moment data from the pile were reduced to obtain *p-y* curves in a manner similar to that used to produce *p-y* curves from physical experiments. These data provide another level of comparison of the finite element results with the empirical design procedures currently in use.

INTRODUCTION

Design of piles for lateral loading is most often performed using an approach which relies upon either the Winkler beam-on-elastic foundation model with empirically derived nonlinear springs to represent the soil (*p-y* curves) or upon a model of the soil as a linear elastic continuum. Both of these approaches have limitations with respect to developing an appropriate model for problems such as a pile in sloping ground or on batter, pile groups, or piles subjected to combined loadings. The former does not model the soil as a continuum and thus provides no direct means of accounting for variables which were not present in the experiments used as a basis for the *p-y* curves. The latter does not capture the nonlinearity inherent in the problem except through empirical modification.

Computers and Geotechnics 0266-352X/90/$03·50

Previous attempts to examine this problem using a three dimensional finite element model have been limited to either linear elastic soil or a nonlinear elastic soil model such as the Duncan-Chang model widely used for earth dams (2,3). While the Duncan-Chang model provides some material nonlinearity, it was not developed for the complex stress changes which occur in the soil around a laterally loaded pile and is not likely to do a good job of modelling the stresses and deformations in the soil around the pile. In particular, the soil near the ground surface behind the pile (i.e., the side which is unloaded) is subject to a reduction in horizontal stress as gap formation occurs.

This paper describes a three dimensional finite element model of a laterally loaded pile which includes the provision for plastic yield in the soil as well as gapping and slippage at the pile/soil interface. The three dimensional nature of the problem and the high degree of nonlinearity which is present require an enormous computational effort; a supercomputer was used to perform the computations described in this paper. Although the great expense of performing the analyses described in this paper precludes the routine use of such techniques in design, it is felt that this model can address some of the limitations present in currently used analytical techniques and provide a basis for parametric studies of the limitations of available design procedures. The computational model is quite economical compared to full scale field experiments on pile groups, for example. The model can also provide a basis for evaluation of more simple analytical procedures, such as the two dimensional model described by Kooijman (1).

Results of analyses using two simple plasticity models for soil are presented. The ability of the model to capture many of the essential elements (as per experimental measurements and observations from large scale load tests) of the problem is demonstrated. In addition, the bending moment data derived from the pile stresses are used to compute experimental *p-y* curves following procedures commonly used for well instrumented field tests. The *p-y* curves derived from this model provide a basis for comparison of the results with well established design procedures for single piles. The *p-y* curves derived from the compuational model may also provide a mechanism for implementing future recommendations for design from additional parametric studies.

DESCRIPTION OF MODEL

General

The model consists of 27 node brick elements arranged as shown on Figure 1 to produce a mesh with approximately 10,000 degrees of freedom. Figure 1 shows the deformed mesh with the pile pushed laterally into the soil. Analyses performed with somewhat coarser and finer meshes indicated that the arrangement shown provides a reasonable degree of both sensitivity and computational efficiency.

Two symmetric boundaries are used, so that the problem analyzed really consists of a widely spaced row of piles with planes of symmetry through the pile centerline and through the soil midway between piles. This geometric arrangement provides efficiency and allows investigation of the effect of pile spacing by simply adjusting the mesh to vary the distance between the two planes of symmetry. With a suitably wide spacing between these boundaries, the mesh essentially models the behavior of a single isolated pile.

The pile is a 3 by 7 pattern of linear elastic solid elements which form a circular shape. In order to reproduce the bending stiffness (EI) of a 10.75 inch (273mm) diameter pile with 3/8 inch (9.5mm) wall thickness, the value of Young's modulus, E, was taken as 7000ksi (48000mPa) instead of 29000ksi (200000mPa). Because the pile model was used to derive experimental *p-y* curves, the fineness of the mesh in the vertical direction was critical. Analyses of a simple cantilever beam using 27 node brick elements indicated that a minimum of 7 elements were necessary to provide bending moments of sufficient reliability to allow determination of *p-y* curves. In addition, one analysis was performed using a mesh with 10 elements in the vertical direction to verify the suitability of the 7 element thick mesh; the results of this analysis were very close to the results obtained using a similar model with a 7 element thick mesh. In all cases the base of the pile was fixed against vertical translation, with the center node fixed against displacement in any direction.

Besides the routine checks using linear elastic soil elements, two types of plasticity models have been used for soil. The first is a simple elastic-plastic (VM) model using a constant yield strength Von Mises envelope. This model is considered to provide an approximation of undrained clay behavior. The model parameters used for this case consist of a uniaxial yield strength of 8psi (55kPa), a Young's modulus of 1600psi (11000kPa), and Poisson's ratio of 0.45.

The second constitutive model for soil used is an extended Drucker-Prager model with nonassociated flow. This model is available in the code ABAQUS by Hibbit, Karlsson & Sorensen, Inc. which is used in this study. The Extended Drucker-Prager (EDP) yield surface is defined as a straight line in the *p-t* plane (Figure 2), where p is the equivalent pressure stress (first stress invariant) and t is the Mises equivalent stress (square root of the second deviatoric stress invariant). When plotted in the *p-t* plane, this surface has a slope, β, and t intercept d, which can be related to conventional Mohr-

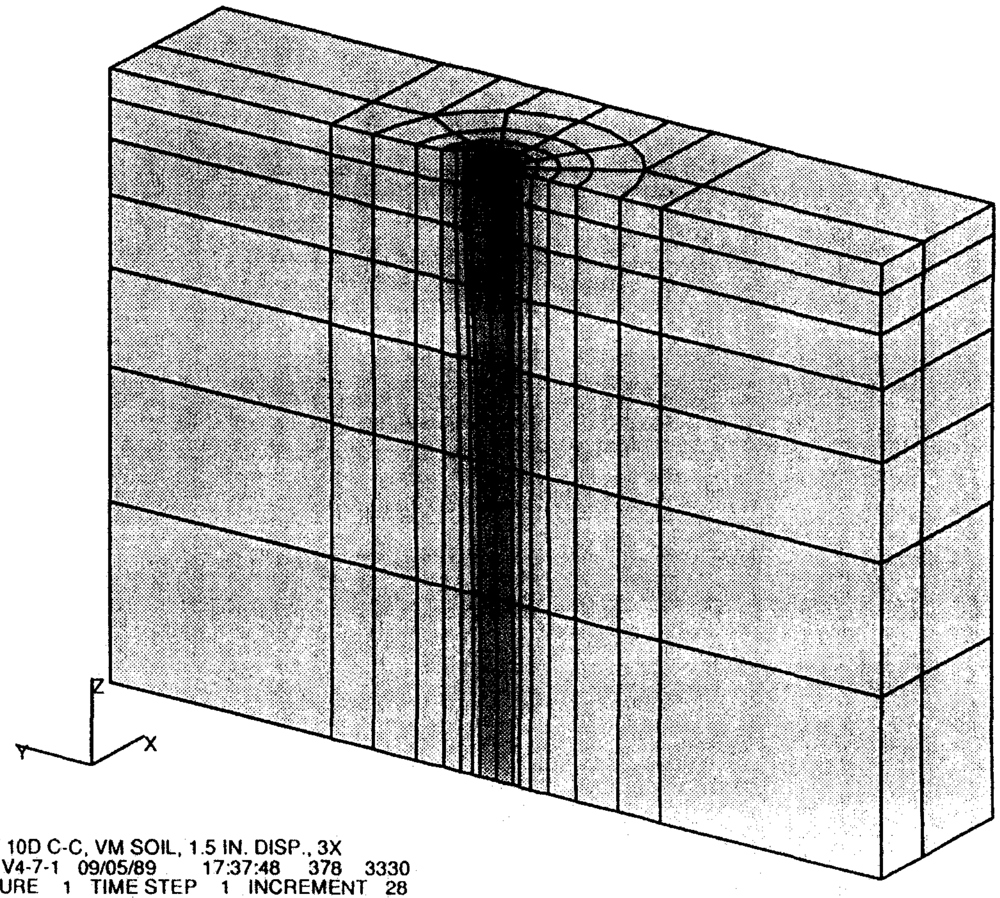


Figure 1 View of the Three Dimensional Mesh with Pile Displaced Laterally

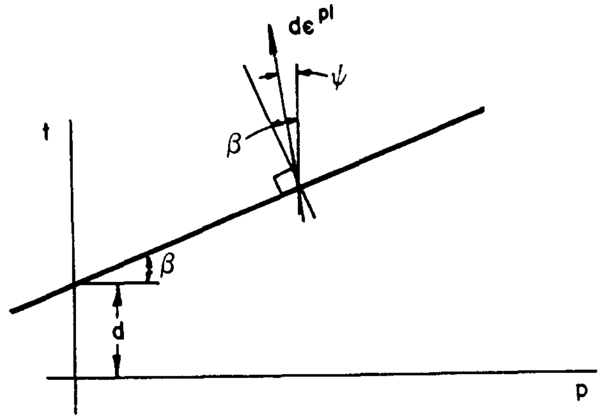


Figure 2 Schematic diagram of the *p-t* plane, for the Extended Drucker-Prager model

Coulomb parameters c and ϕ as:

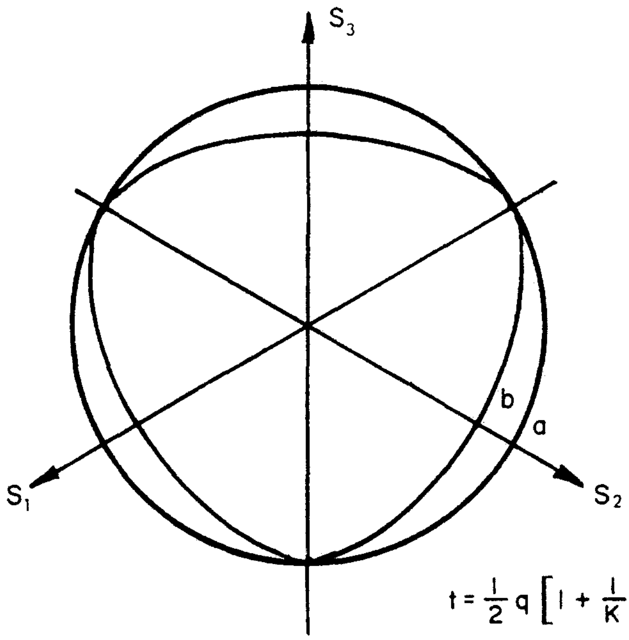
$$\tan\beta = \frac{6\sin\phi}{3-\sin\phi} \tag{1}$$

$$d = c\frac{6\cos\phi}{3-\sin\phi} \tag{2}$$

The results presented in this paper are based on values of either c=2psi (13.8kPa) and $\phi=23^{o}$ or c=1psi (6.9kPa) and $\phi=30^{o}$. For nonassociated flow, an angle ψ of 0^{o} has been used in this work; this value results in constant volume plastic deformation. Another extension of the Drucker-Prager model implemented in ABAQUS involves a modification of the yield surface in the deviatoric plane as shown on Figure 3. The parameter K controls the dependence of the yield surface on the intermediate principal stress; K of less than one results in a yield surface which more nearly represents the behavior of soil in triaxial extension. for this study, K was taken as:

$$K = \frac{3-\sin\phi}{3+\sin\phi} \tag{3}$$

which provides a smooth approximation to the Mohr-Coulomb surface.



$$t = \frac{1}{2} q \left[1 + \frac{1}{K} - \left(1 - \frac{1}{K}\right)\left(\frac{r}{q}\right)^3 \right]$$

Curve	K
a	1.0
b	0.8

where:

q = Mises equivalent stress = $[3/2(S:S)]^{1/2}$
r = third invariant of deviatoric stress = $[9/2(S\ S:S)]^{1/3}$
S = stress deviator

Figure 3 Yield surfaces in the deviatoric plane for the Extended Drucker-Prager model

The pile/soil interface was modelled with 18 node interface elements which provide for no forces transmitted across the interface upon separation and frictional behavior when the surfaces are in contact. The friction coefficient in this study is taken as equal to tan 23°. To more realistically model the interface and to add numerical stability, an elastic stiffness is included which allows a limited amount of shear deformation before slippage takes place.

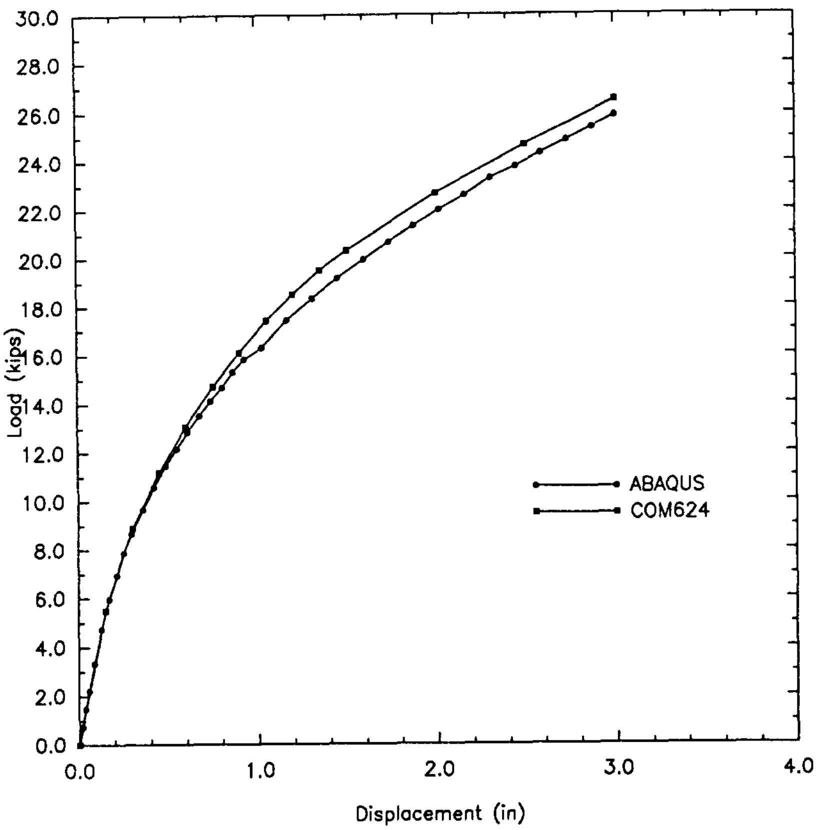


Figure 4 Load vs Displacement, VM model (1in.=2.54cm, 1kip=4.45kN)

RESULTS OF ANALYSES - VON MISES MODEL

General

A plot of pilehead load vs deflection for the Von Mises (VM) soil model is provided on Figure 4, along with the results of computations with the widely used beam-on-elastic subgrade approach as implemented in the code COM624 (5). The *p-y* curves for the COM624 analyses were generated using the criteria for statically loaded piles in soft clay of Matlock and Reese, with a soil undrained shear strength of 8 psi (55kPa). This undrained shear strength does not correspond with the Von Mises yield

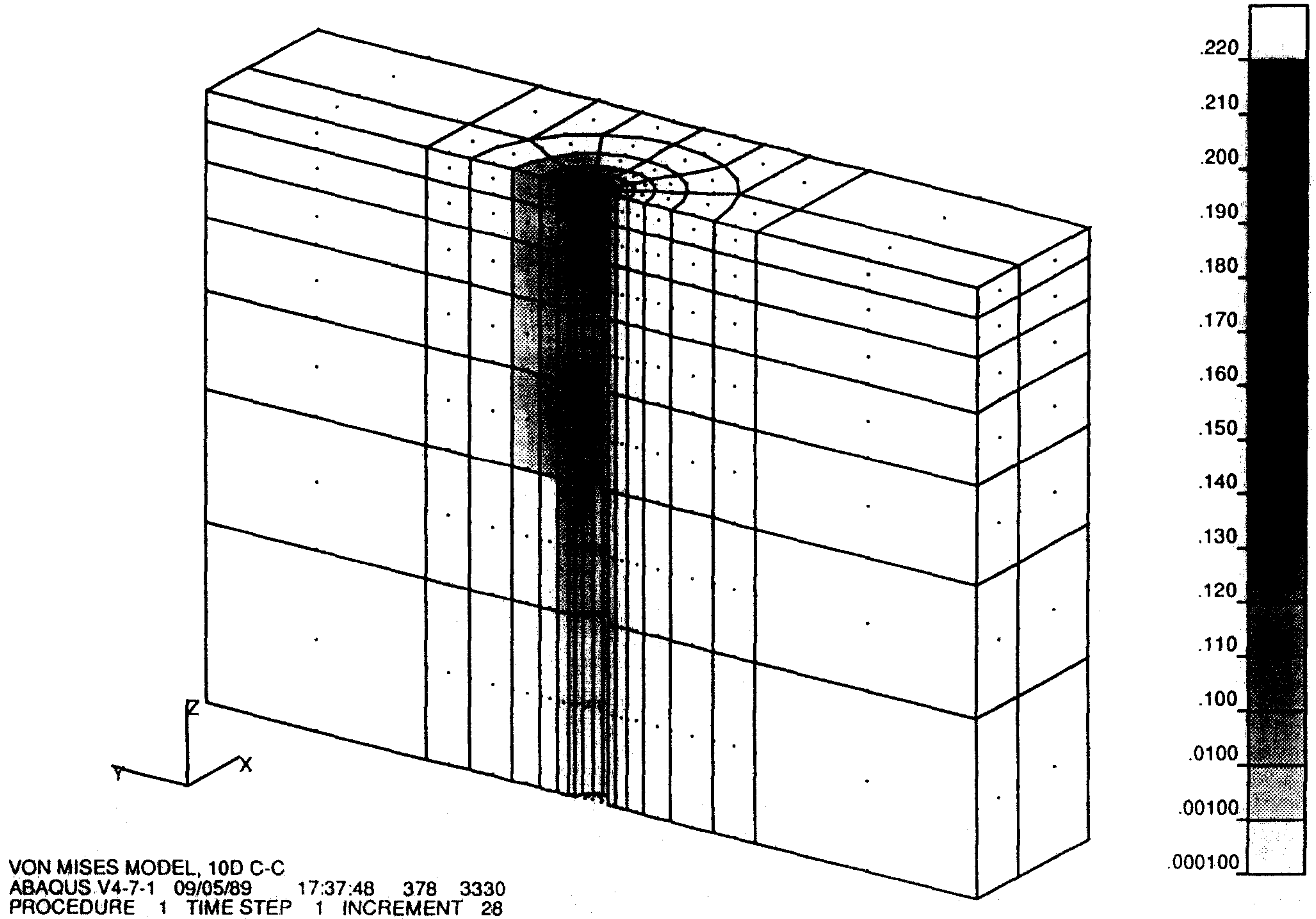


Figure 5 Accumulated Plastic Strains, VM model at 1.5 in. pilehead deflection (1in.=2.54cm)

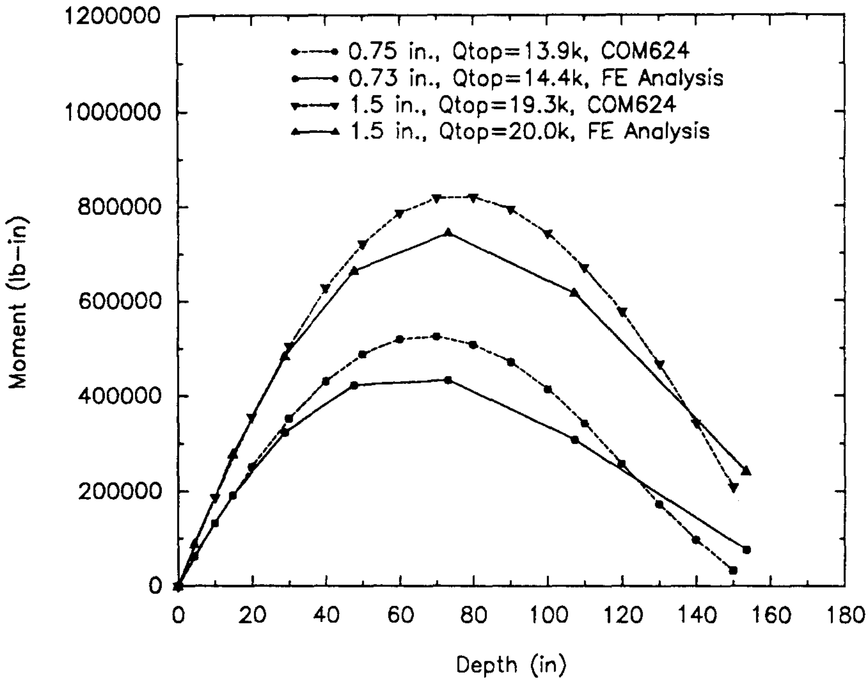


Figure 6 Bending Moment vs Depth, VM model (1in.=2.54cm, 1kip=4.45kN, 1lb.=4.45N)

surface used in the finite element analyses, but rather was selected as the value producing the best fit to the load vs pilehead displacement data. Note that the Von Mises yield strength is based upon a uniaxial stress at failure of 8 psi (55kPa).

Shown on Figure 5 is a contour plot of accumulated plastic strains in the soil at a pilehead deflection of 1.5 inches. Note that the pile elements have been removed to reveal soil elements with more clarity. The elements which are deforming plastically are seen to be confined to an area in front of the pile and within about 2 diameters of the pile. As expected, the aerial extent of yielding diminishes with depth.

Plotted on Figure 6 are curves of bending moment vs depth for the finite element model and for the COM624 comparative result at two similar values of pilehead load and deflection. Bending

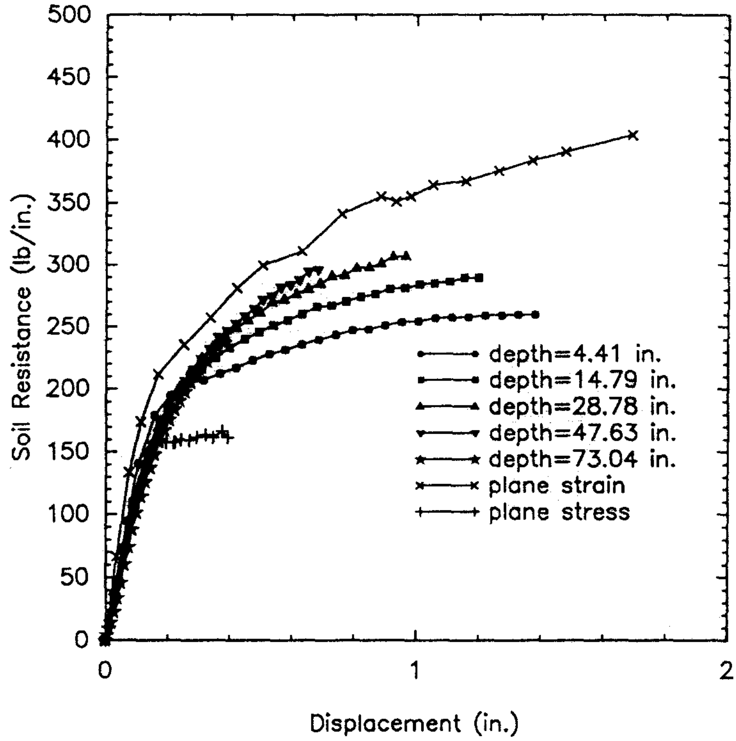


Figure 7 *P-y* Curves, VM model (1in.=2.54cm, 1lb.=4.45N)

moment data were derived from the values of stress in the pile elements at the element centroids. Although the patterns are quite similar, the bending moments from COM624 are seen to be somewhat higher and peak at a slightly lower depth. This trend reflects a somewhat lesser distribution of load to the soil in the near surface.

Load Transfer (*p-y*) Curves

The bending moment data from the finite element model were used to derive *p-y* curves for the pile in a manner similar to that used on physical experiments. The seven values of bending moment from the element centroids along with the known moment of zero at the top of the pile were fitted with a fifth degree polynomial using the least squares technique. From the differential equation for a beam on a Winkler-type subgrade (or with any distributed load):

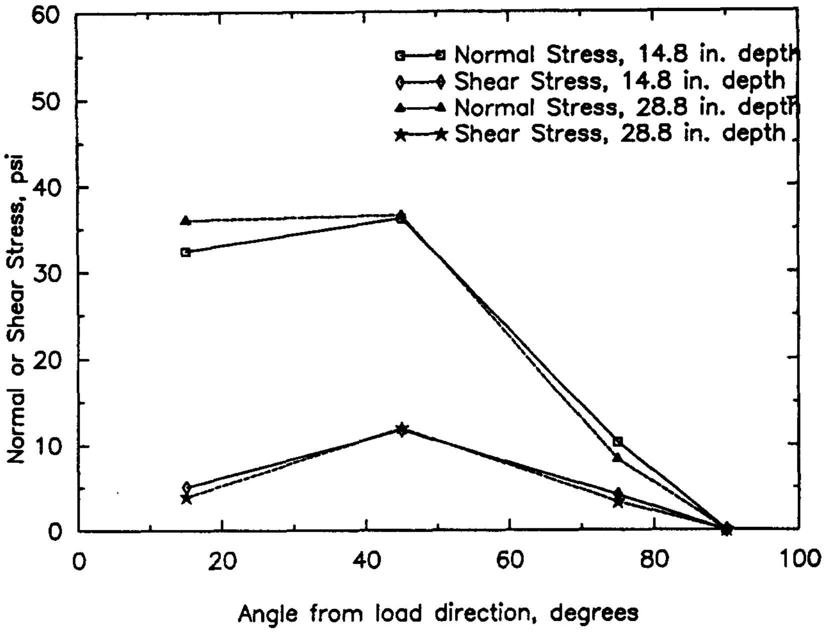


Figure 8 Interface Stresses, VM model (1psi=6.9kPa, 1in.=2.54cm)

$$\frac{d^2M}{dz^2} + p = 0 \qquad (4)$$

the polynomial curve representing the bending moment data was twice differentiated to obtain the soil resistance, *p*. Values of deflection, *y*, were obtained directly from the computational results at the centroid of each pile element.

Plotted on Figure 7 are the *p-y* curves at various depths which were derived from the finite element results. As is evident from these plots, there exists some reduction due to surface effects in the maximum soil resistance which can be mobilized. Also plotted on Figure 7 are the results from plane strain and plane stress analyses using a two dimensional mesh which was identical to the three dimensional mesh in plan. As would be expected, the plane strain and plane stress results bound the *p-y* curves derived from the three dimensional model. The plane strain case provides a reasonable upper bound, as a "flow around" type of failure must occur in this model. The plane stress seems to provide a reasonable lower bound to the p-y curves near the ground surface as the plane stress model allows out of plane distortion.

The stresses at the pile-soil interface were obtained at the centroid of the interface elements and are shown as a function of circumferential distance around the pile on Figure 8. A circumferential direction of 0^0 corresponds to the front face of the pile in the direction of loading. Since gapping occurred behind the pile, all of the stresses beyond 90^0 are zero. These results appear to demonstrate the pattern of stress which is expected at that interface and, in principle, could be used to obtain the soil resistance at a given depth directly by integrating the area under each curve times the direction cosine function. However, the relatively few number of points at a given depth do not permit a reliable integration of stress around the pile circumference. Analyses using different friction coefficients indicated that the pile response was not greatly influenced by the interface, so long as the gapping and slippage was possible, and so the mesh is thought to be sufficiently fine to provide representative results in the pile and soil elements. However, a considerably finer mesh with respect to the pile-soil interface is felt to be necessary to reliably compute the distribution of stress around the pile circumference.

As a comparison with the finite element results, the p-y curves obtained from the COM624 output using the soft clay criteria are presented on Figure 9. As mentioned previously, the design rules used to generate these p-y curves were developed empirically from experimental results. These curves clearly indicate a greater reduction in soil resistance near the ground surface than was obtained in the finite element results. One can only speculate at this point as to some of the reasons for this pattern; however, the authors offer the following thoughts:

1) The undrained shear strengths derived from unconfined and UU triaxial tests used to develop the empirical p-y curves provide a simplified representation of the shear stress in the soil at failure. In particular, the soil near the ground surface may be significantly affected by the fact that the loading is more nearly that of triaxial extension than triaxial compression. If there is any secondary structure present in the clay soil, very little confinement exists near the ground surface.
2) The simple elastic-plastic Von Mises constitutive model is not likely to represent undrained loading in a saturated clay in a fundamental way, in that the shear strength mobilized in reality is influenced by the loading path. Although this plasticity model captures the large degree of nonlinearity present in the laterally loaded pile problem, the soil constitutive model used in this study thus must be considered to provide only a crude idealization of true soil behavior.

In spite of the limitations of the constitutive model for clay soil, the model is observed to capture the essential elements of the laterally loaded pile problem, including soil nonlinearity, pile-soil

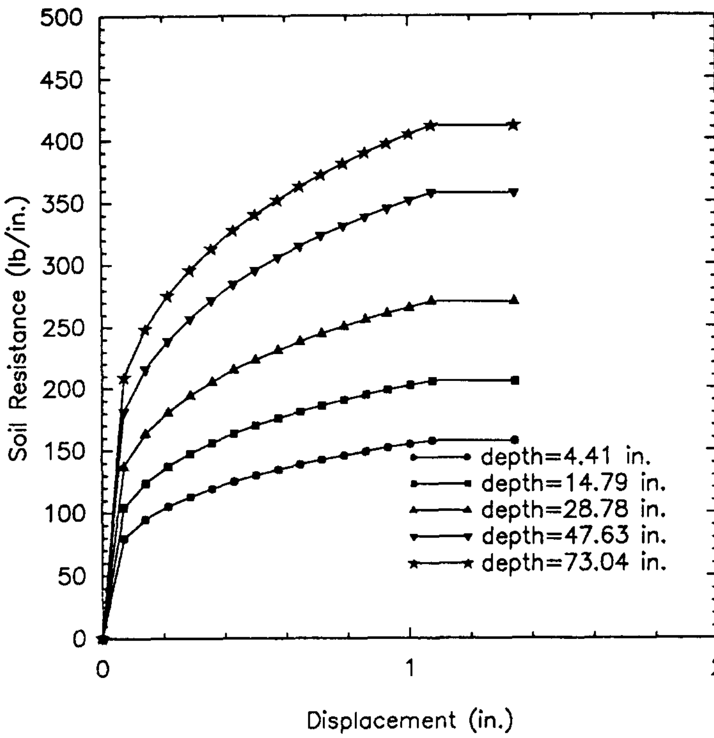


Figure 9 *P-y* Curves from COM624, Clay Soil (1in.=2.54cm, 1lb.=4.45N, 1psi=6.9kPa)

separation behind the pile, and frictional interface behavior. With some "calibration" of soil strengths to expected field conditions, this model is expected to provide a reasonable basis for additional study of three dimensional geometric effects on pile response. Particular topics of interest include the behavior of pile groups, the effect of sloping ground, and the effect of pile head fixity on pile response to lateral loading.

RESULTS OF ANALYSES - EXTENDED DRUCKER-PRAGER MODEL

General

The Extended Drucker-Prager (EDP) model for soil was used to provide an elastic-plastic

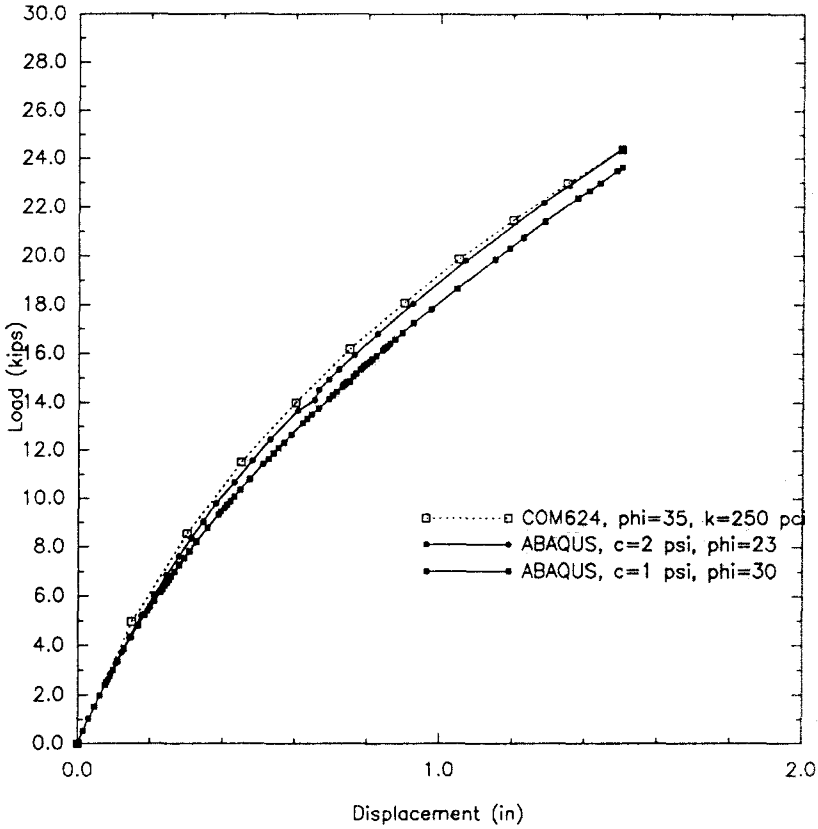


Figure 10 Load vs Displacement, EDP models (1in.=2.54cm, 1kip=4.45kN)

constitutive model of a frictional material, i.e. sand. Attempts to model a sand for the laterally loaded pile problem face difficulties in the zone immediately behind the pile where gap formation is initiated. Unless a small amount of cohesion is present, the soil elements in this zone begin to fail immediately and result in convergence problems; the computer expends enormous effort on these elements without achieving significant pile displacements. As a result, the two sets of soil strength values used to represent sand each included a small amount of cohesion. The design rules included in COM624 for generating *p-y* curves for sand include no provision for cohesion, and thus a direct comparison with the *p-y* curves derived from the finite element model is difficult.

Presented on Figure 10 are the pilehead load vs deflection curves for the two EDP finite element analyses as well as for COM624 results. The EDP models included soil with differing relative

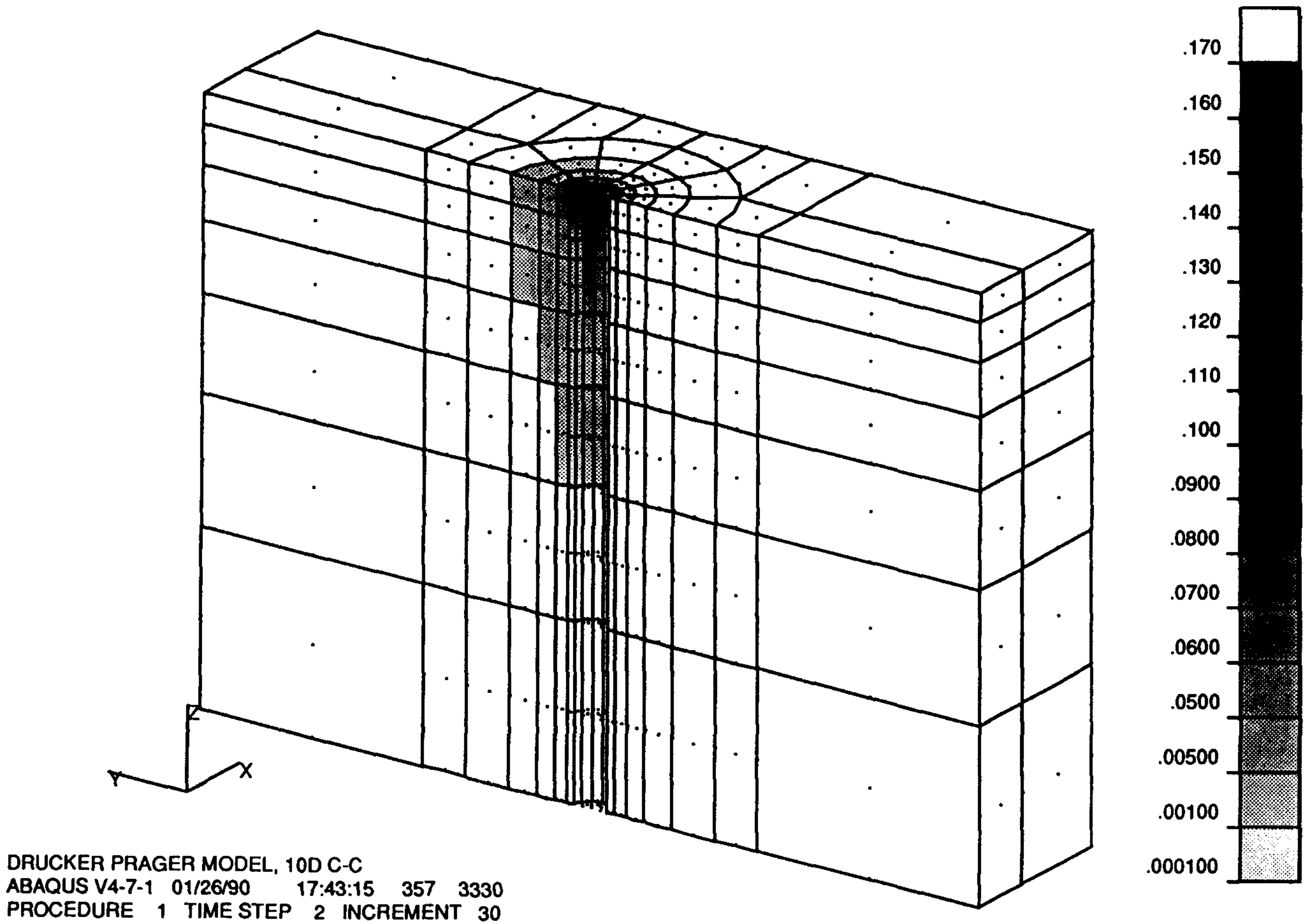


Figure 11 Accumulated Plastic Strains, EDP model (c=2psi, $\phi=23^0$) at 1.5 in. pilehead deflection (1in.=2.54cm)

amounts of cohesion (1 to 2 psi, 6.9 to 13.8 kPa) and internal friction (30^0 to 23^0). Both of these soil models included a unit weight of 0.07pci ($18.9kN/m^3$) for computation of gravity stresses. The elastic modulus in both cases was specified to increase with depth, where E was taken as 600 psi+ 9psi/inch of depth (87kPa + 51kPa/m of depth). Note that the pile response for each of these two cases is quite similar, indicating that an increase in ϕ of about 7^0 was seen to approximately equal a decrease in cohesion of about 1psi (6.9kPa) as far as pilehead response was concerned. The COM624 output indicates the values of ϕ and k used with the design guidelines for *p-y* curves in sand which provided reasonably close agreement with the pilehead load-deflection response.

Note that the run with the lesser cohesion produced a great many more data points because of the greater number of increments (i.e. steps toward the final 1.5 inch (3.81cm) displacement) necessary for convergence. This job required about twice as much CPU time; these jobs took about 8 and 15 hours of CPU time on a Cray XMP/24 for the 2psi (13.8kPa) and 1psi (6.9kPa) cohesion models, respectively.

The zone of plastic deformation at a pilehead deflection of 1.5 inches (3.81cm) for the c=2psi (13.8kPa), $\phi=23^0$ case is shown on Figure 11. Note that the soil yielding is not as extensive at depth as is the case for the Von Mises material shown on Figure 5. This trend seems reasonable because the frictional material will have a significant increase in strength and stiffness with depth.

Load Transfer (*p-y*) Curves

The bending moment data were used to derive *p-y* curves in a manner similar to that described previously; *p-y* curves for the two EDP models are shown on Figures 12 and 13. The variation of soil resistance with depth is clearly much more significant than for the VM model. The EDP model with the greater cohesion is also seen to exhibit somewhat more soil resistance near the ground surface, but slightly less resistance at depth because of the lower ϕ value. The *p-y* curves from the COM624 analysis shown on Figure 14 are seen to have much lower soil resistance at the ground surface; as stated previously, there is no provision for inclusion of any cohesion with the COM624 *p-y* criteria for sand.

The frictional interface used for the EDP models was identical to that used for the VM model, with a similar pattern of results (although the interface stresses increased with depth more significantly). Stresses at the pile-soil interface at selected depths are shown on Figure 15. As mentioned previously, the mesh used at the pile-soil interface is not particularly refined and so the distribution of stress around the circumference is defined at only a relatively few points. The cohesion used with the EDP models allowed gapping behind the pile to occur, as evident from the interface stresses.

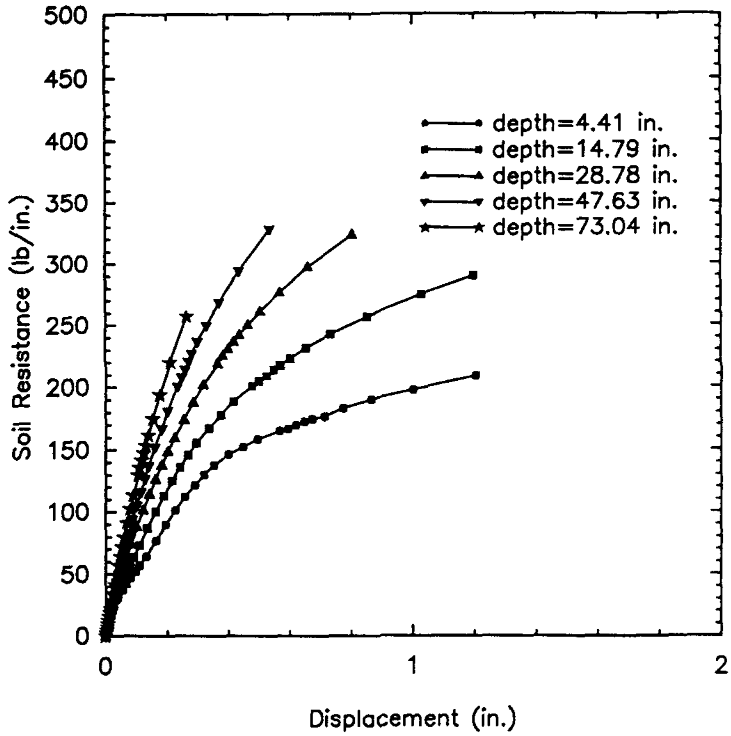


Figure 12 *P-y* Curve, EDP model with $\phi=23^0$, c=2psi (1in.=2.54cm, 1lb.=4.45N, 1psi=6.9kPa)

SUMMARY

Three dimensional finite element analyses have been performed of a laterally loaded pile using two types of plasticity models for soil. Bending stress data from the pile has been analyzed to obtain the distribution of overall soil resistance (*p-y* curves) in a manner similar to that used for physical experiments on laterally loaded piles. The results offer insight into the three dimensional and nonlinear nature of the problem as well as some of the difficulties in modelling this problem using conventional plasticity constitutive models. The finite element models developed appear to capture the most essential elements of the problem which are:

1) the three-dimensional nature of the problem, with the influence of the ground surface on the

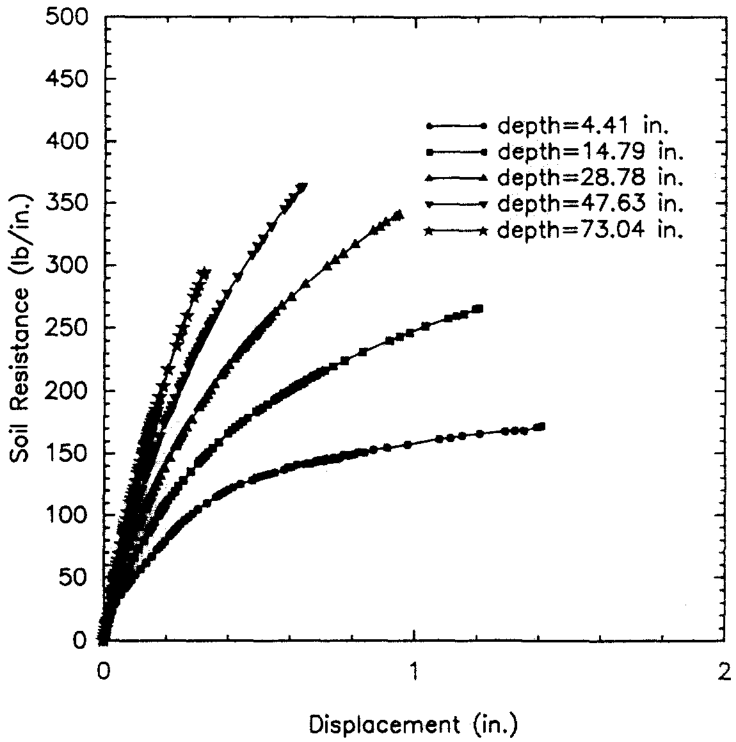


Figure 13 *P-y* Curve, EDP model with $\phi=30^0$, c=1psi (1in.=2.54cm, 1lb.=4.45N, 1psi=6.9kPa)

development of yield zones in the soil and the distribution of soil resistance with depth,

2) the nonlinearity of the soil response, in which yield occurs near the ground surface at relatively small displacements and the zone of yielding soil propagates laterally and to greater depths with increasing pile displacement,
3) the slippage and gapping at the pile-soil interface, which in turn affects the distribution of stress from the pile to the soil, and
4) the pattern of soil shear strength with depth in which the mobilized soil resistance from undrained loading in clay is influenced primarily by geometric effects and the soil resistance in sand is influenced by the confining pressure as well as the geometry of the problem.

The three dimensional finite element model is felt to be a useful tool for parametric studies of factors influencing lateral load response of piles, rather than as a routinely used analytical technique. Particular topics of interest include the behavior of pile groups, the effect of sloping ground, and the

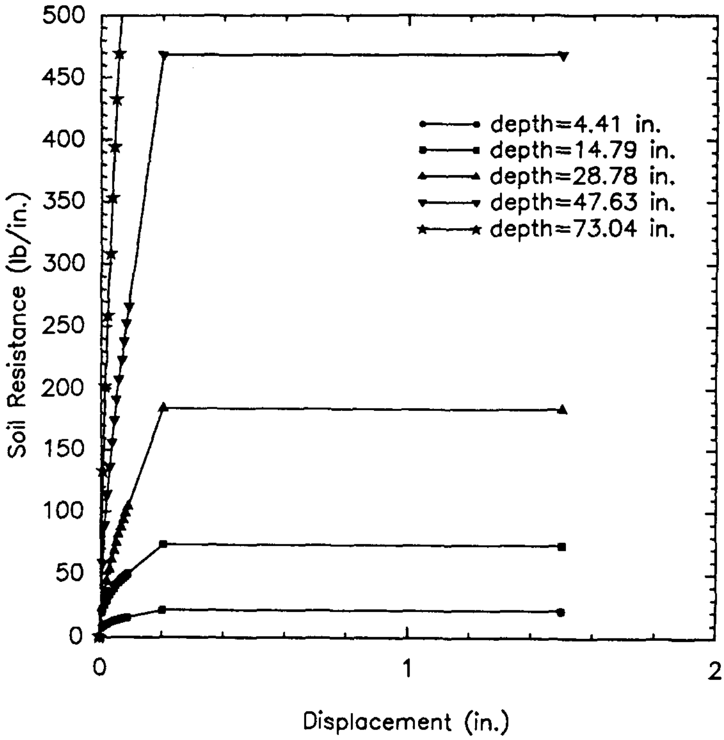


Figure 14 *P-y* Curve, COM624 with $\phi=35^0$, k=250pci (1in.=2.54cm, 1lb.=4.45N)

effect of pile head fixity on pile response to lateral loading. In addition, the three dimensional model provides a basis for evaluation of more simple approximations of the laterally loaded pile problem.

ACKNOWLEDGEMENTS

The research described in this paper was supported by a grant from the National Science Foundation, for which the authors are most appreciative. In addition, the authors gratefully acknowledge the support through computational resources which were provided by the Alabama Supercomputer Network and the Auburn University Engineering Experiment Station.

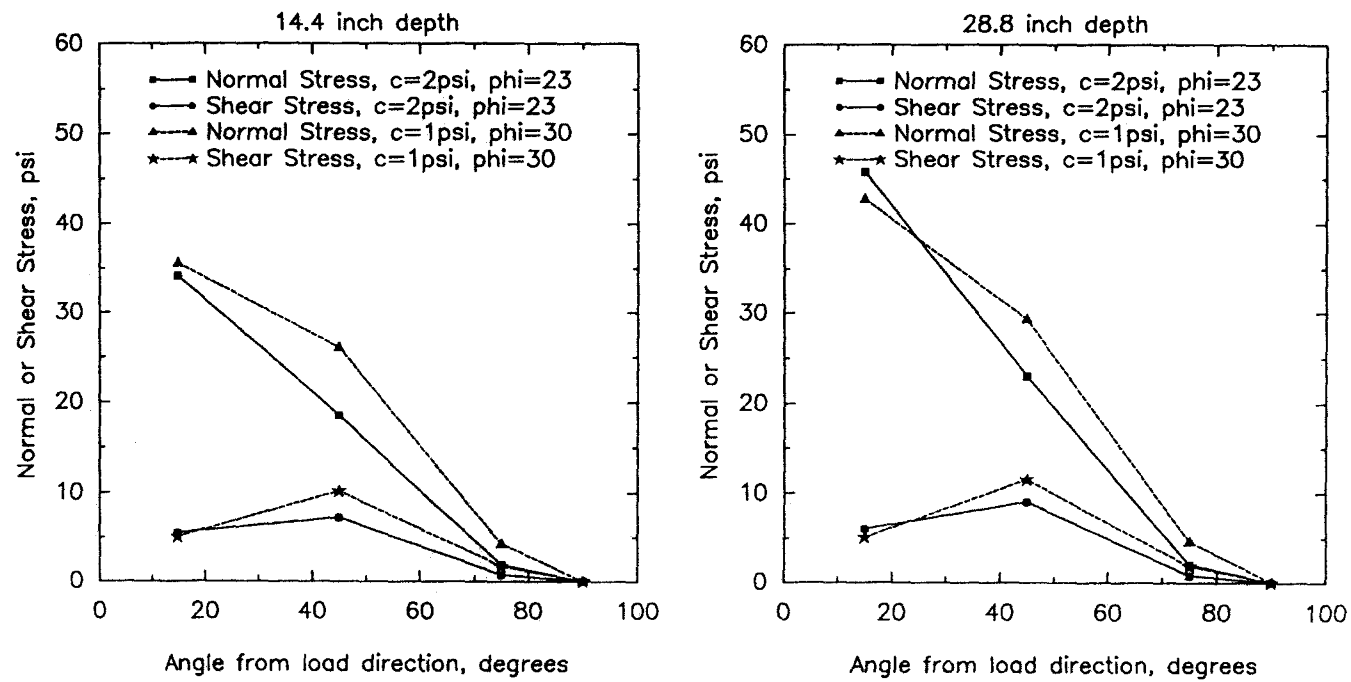


Figure 15 Interface Stresses, EDP models at 1.5 in. pilehead deflection (1in.=2.54cm, 1psi=6.9kPa)

REFERENCES

1. Kooijman, A.P., Comparison of an Elastoplastic Quasi Three-Dimensional Model for Laterally Loaded Piles with Field Tests. Proc. 3rd Int. Conf. on Numerical Methods in Geomechanics (NUMOGIII), Niagra Falls, Canada (1989) 675-682.
2. Muqtadir, A. and Desai, C.S., Three-Dimensional Analysis of a Pile-Group Foundation. Intl J. for Num. and Anal. Meth. in Geomechanics. 10 (1986) 41-58.
3. Tamura, A., Ozawa, Y., Sunami, S., and Murakami, S., Reduction in Horizontal Bearing Capacity of Pile Group. Proc., 4th Intl Conf. on Num. Meth. in Geomechanics, Edmonton, Can. (1982) 865-874.
4. Hibbitt, Karlsson and Sorensen, Inc. ABAQUS Theory Manual, Version 4.6. (1987).
5. Reese, L.C. and Sullivan, W.R., Documentation of Computer Program COM624, Geotechnical Engineering Center, The Univ. of Texas at Austin, (1980).
6. Reese, L.C., Cox, W.R., and Koop, F.D., Analysis of Laterally Loaded Piles in Sand, Paper No. OTC 2080, Proc., Fifth Annual Offshore Tech. Conf., Houston, Texas, (1974).

Received 9 July 1990; revised version received 3 October 1990; accepted 18 October 1990