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TECHNICAL NOTE

THREE-DIMENSIONAL FINITE ELEMENT ANALYSIS OF PILE GROUPS UNDER LATERAL LOADING

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Abstract—The analysis of a group of foundation piles under horizontal load is difficult because the system is strongly three-dimensional in nature. Also the materials of the piles and of the soil have very different stiffness properties and the piles and pile cap respond primarily in bending deformation while the soil acts initially as an elastic half space. The rapid expansion in mainframe CPU and memory capacities now allows full 3-D analysis of small groups of piles connected by a stiff pile cap. The types of elements used to represent the pile shafts and the pile cap need to be selected carefully to ensure compatibility with the surrounding soil elements. Meshes were constructed to model a single pile, and two- and three-pile groups, all subject to horizontal loads. After preliminary verification, the meshes were used to calculate deflections, pile axial forces and moments, and pile/soil pressures, for the two- and three-pile groups. The two main variable parameters were the spacing of the piles, and the overhang height of the pile cap above ground. The soil modulus values were then modified in the case of a two-pile group in order to evaluate the effects of strain-softening and of pile/soil separation. It was found that the overall stiffness of the group decreased, and the bending moment at the head of the front pile became greater than in the rear pile.

INTRODUCTION

Bearing piles have been developed over many years, in steel, concrete and timber to transmit primarily vertical loads from foundation level down to soil or rock strata capable of carrying the high loads without excessive settlement. Piles are normally constrained to work in groups by very stiff pile caps. Design is often based upon achievement of an adequate factor of safety upon pile capacity as estimated by relatively simple soil mechanics analysis later verified by load tests. Limiting settlement may also be considered.

Some situations arise in which lateral or horizontal loads upon the pile cap are significant or even dominant. Examples of such loading upon groups of bearing piles include wind loading on tall buildings, vehicle braking or acceleration forces upon bridge abutments, wave forces on platforms, and ship berthing forces on jetties. The designer of such structures may wish to limit lateral deflection of the loaded pile cap; he/she will need to ensure that the piles are capable of carrying the bending moments, shear and axial forces induced into the piles.

Methods of analysis of a single pile under horizontal loading include estimates of ultimate lateral resistance based upon limiting equilibrium [1, 2]. Such analyses may be carried out for free or fixed head conditions and for cohesive or granular soils [3].

As computer-based solutions have improved, computational analyses of single piles and of pile groups have been made. Significant advances include the boundary element methods of Poulos [4, 5], for single piles in an elastic medium, with soil slip, and interaction factors for pile groups and by Banerjee and Driscoll [6] for analysis of pile groups under general loading.

Finite element analyses were presented by Ottaviani [7], Randolph [8], and more recently by Justo *et al.* [9]. A useful summarizing guide for the design of laterally loaded piles was produced on behalf of CIRIA by Elson [10]. However, fully three-dimensional finite element analysis of pile groups carrying non-vertical loads are few.

The difficulties inherent in a 3-D FE model are the need to achieve compatibility between pile elements and the soil, the large difference in stiffness properties between pile and soil, and the large matrix rank for a model containing 3-D elements. The approach adopted in this work is now described.

A PILE/SOIL FE MODEL

The FE model was developed for comparisons with a series of field tests on pile groups under lateral loading which are reported elsewhere [11]. Basically, the 3.35 m long piles in these tests were steel square-box section 154 × 154 mm driven into a 2.1 m deep layer of sand and into the firm brown clay beneath and connected by a steel pile cap. Thus the dominant effect upon lateral movement of the piles was the horizontal resistance of the sand, although the induced axial load transfer was also a function of the pile toe resistance.

In the FE model, the shaft of each pile was modelled by 3-D prism elements occupying the full cross-section of the box section, but of reduced modulus such that the prism element was of equivalent web stiffness to the webs of the box section, see Fig. 1. The flanges of the box section were represented by plane-stress elements linked to the web prism elements at the corner nodes. This slightly complex model for the pile shaft was developed to be compatible with the adjacent 3-D prism elements. The surrounding soil was modelled by a mesh of 3-D prisms, of increasing modulus with depth, where $E = 0$–7 MPa, to a depth of 2.1 m, and then of uniform modulus $E = 14$ MPa in the clay layer. The pile cap was represented by steel plates using plane stress elements 'rigidly' connected to the pile heads. A half-symmetric model was used to reduce the high rank of the stiffness matrix. Boundary conditions were applied around the soil mesh of total x, y, z restraint. Horizontal 'loading' was applied to the model by an imposed horizontal displacement of 20 mm.

MODEL VERIFICATION

The bending behaviour of the pile shaft model was first tested by constructing a FE analysis of a cantilever, of a 154 × 154 mm box section, 1000 mm long, subjected to a transverse end load. The results of both beam deflection and

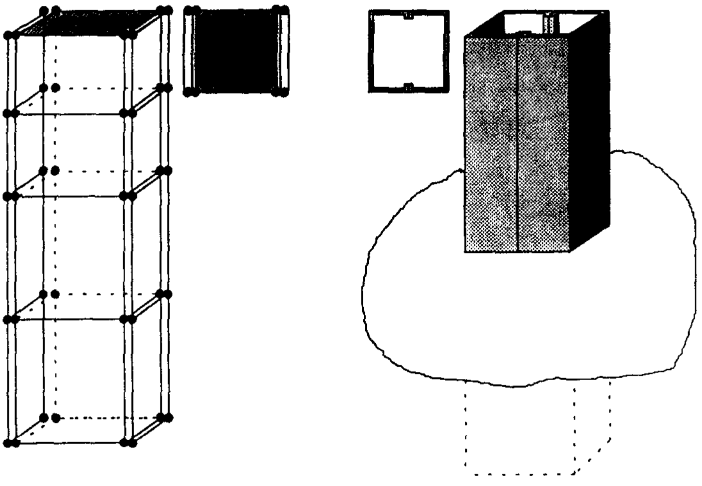

Fig. 1. FE model of pile shaft.

Table 1. (kN/20 mm)

	Poisson's ratio	Free	Fixed
FE	0.3	20.7	32.22
FE	0.49	25.05	41.55
Poulos	0.5	20.4	62.4

surface bending stresses were compared with those calculated for a simple Euler beam (neglecting shear lag effects in the flanges). Results correlated to within 8%, and were considered to be acceptable.

Convergence tests were undertaken for the number of soil layers used to represent the Gibson soil, of 2.1 m depth. Convergence of horizontal head displacement was monotonic, and appeared to have approached within 3% of an asymptotic value when four layers were used. In the main analyses, six layers were used.

A more thorough test of the modelling was achieved by comparisons of results for a single pile under horizontal load with published deflections and pile shaft moments by Poulos [4]. Comparisons were made for both free-head and head rotation restraint. The soil conditions were for uniform elastic modulus $E = 7 \times 10^6$ Pa, and Poisson's ratio of 0.3 and 0.49. Results of the comparisons are given in Table 1, and fair agreement is shown, the maximum difference being 33%.

An approximate comparison was made for two-pile groups against results by Poulos (see Table 2). Agreement was generally better than 20%. However, the FE solution indicated larger group stiffness. Differences could have arisen from any of three sources. Poulos' solution using interaction factors assumed fixed head piles, while the FE solution incorporates pile cap tilting. Also there was a difference in section shape in that although the pile flexural stiffnesses and pile widths were the same, the FE was for a square box, while Poulos' solution was for a circular cross-section. Finally, the use of only six bands of elements between the pile and the restrained boundary would tend to result in an over-stiff model.

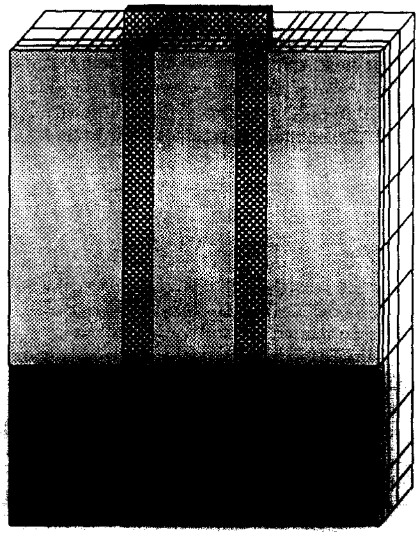

Fig. 2. 3-D FE model of two-pile group plus soil.

ELASTIC RESPONSE OF TWO-PILE GROUPS TO HORIZONTAL LOADING

The FE mesh used to model the two-pile groups was as shown in Fig. 2. Half symmetry was utilized and external soil boundaries were fully restrained. The model used up to 3674 degrees of freedom, dependent upon pile spacing.

The cases analysed included pile spacings of 3, 5, 8 and 12 pile-widths, centre to centre, each with overhang heights of the pile cap (and loaded point) above ground surface of 150, 300 and 400 mm.

Table 2. Lateral loads to cause 20 mm deflection: two-pile groups (kN)

	Poisson's ratio	Overhang (mm)	Pile spacing (diameters) 3	5	8	12
FE	0.3	150	66.81	81.3	95.6	99.9
FE	0.49	150	83.1	102.1	120.5	125.1
Poulos	0.5	0	83.1	90.3	97.3	103.8

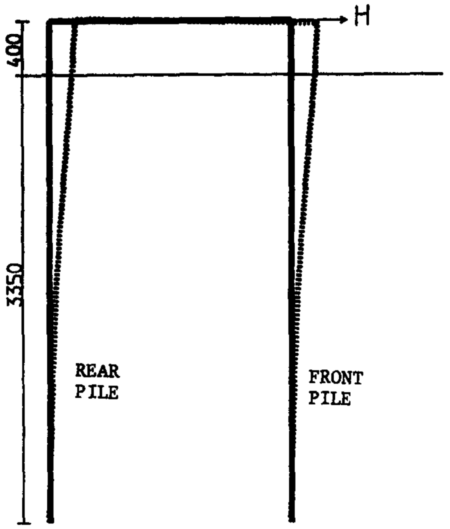

Fig. 3. Deflected shape of a two-pile group at 12-D spacing elastic finite element analysis.

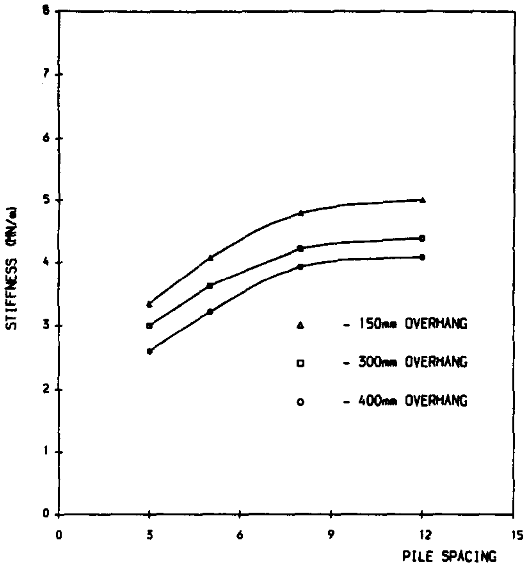

Fig. 4. Stiffness of two-pile group against pile spacing.

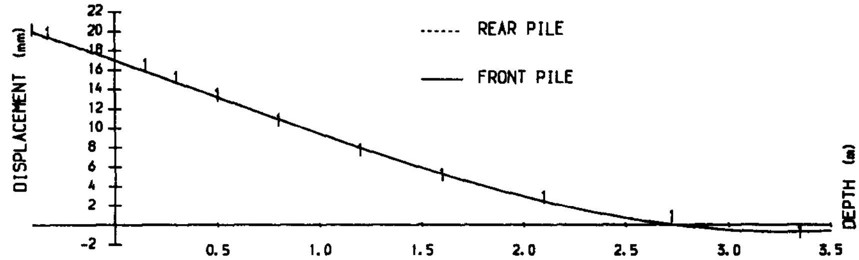

Fig. 5. Pile deflection diagram for two-pile group for 20 mm pile-cap displacement, piles at 8-D-spacing 400 mm overhang.

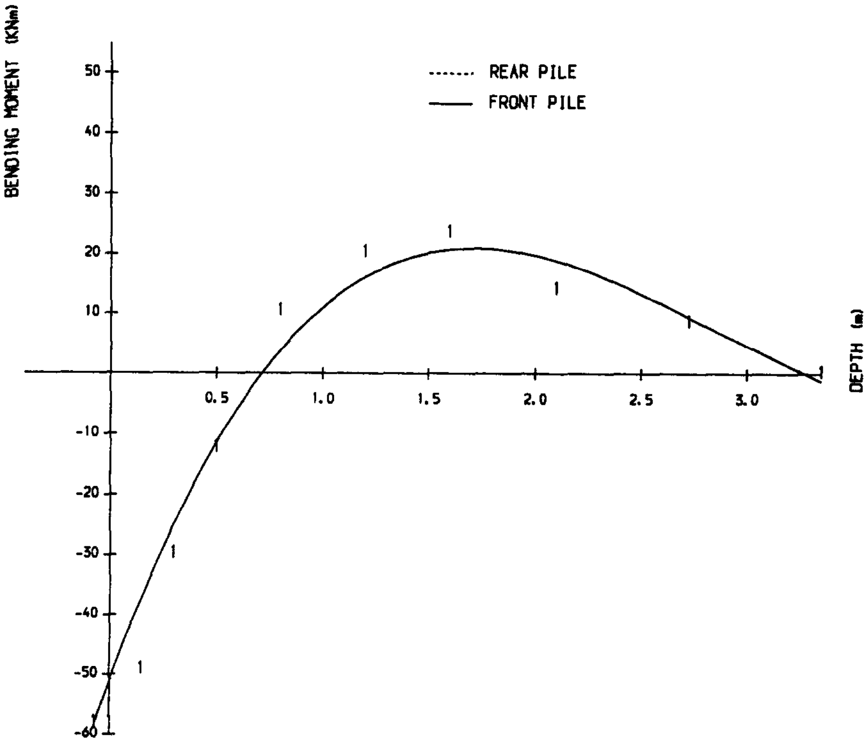

Fig. 6. Bending moment diagram for two-pile group for 20 mm pile-cap displacement, piles at 12-D-spacing 150 mm overhang.

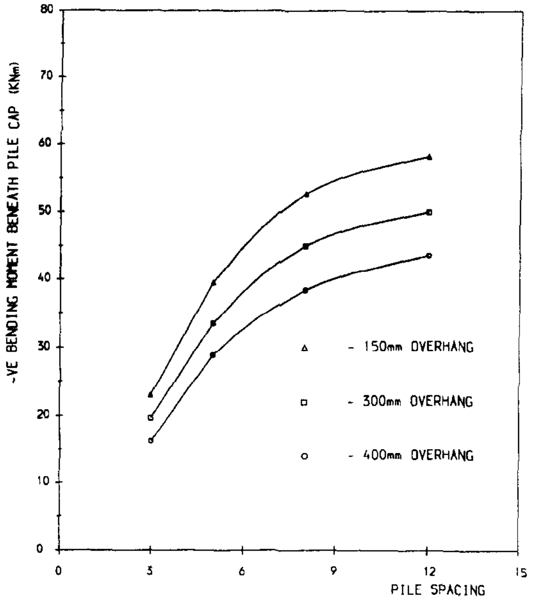

Fig. 7a. Plot of negative bending moment against pile spacing, $u = 20$ mm.

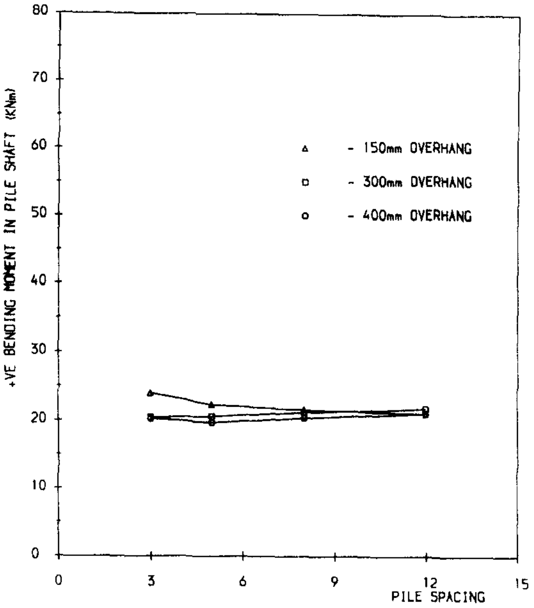

Fig. 7b. Plot of positive bending moment against pile spacing, $u = 20$ mm.

'Loading' was applied as an imposed horizontal pile cap displacement of a standardized 20 mm, for which load was computed. This facilitated comparisons of shaft loads, moments and soil pressures.

1. *Load–deflection characteristics*

The primary response of the two-pile group to an in-line horizontal load was of lateral sway, although in addition the front pile settled under the induced downward force while the rear pile lifted (Fig. 3). For an elastic analysis the settlement and lift were equal, and the load/deflection relation was linear, and is simply described as a pile group lateral stiffness, *Kg*, where

$$Kg = \frac{\text{horizontal force}}{\text{horizontal displacement of the pile cap}}.$$

The effect of increasing the pile spacing was to increase group stiffness, *Kg*, while increasing pile cap overhang reduced *Kg* (see Fig. 4). The lateral deflections of the pile shafts are plotted in Fig. 5.

2. *Pile bending moments*

When a two-pile group is displaced laterally, there is a small rotation of the pile cap (Fig. 3) such that the pile restraint is part way between the free-head and fixed-head condition. Thus the bending moment diagram for each pile shaft showed a maximum at around 1/3 depth, and a maximum negative moment at the pile cap (see for example Fig. 6). Moments in the front and back piles were computed to be equal by an elastic analysis.

The effects of pile spacing and of cap overhang are shown in Figs 7a and b. The moment in the pile shaft hardly varied,

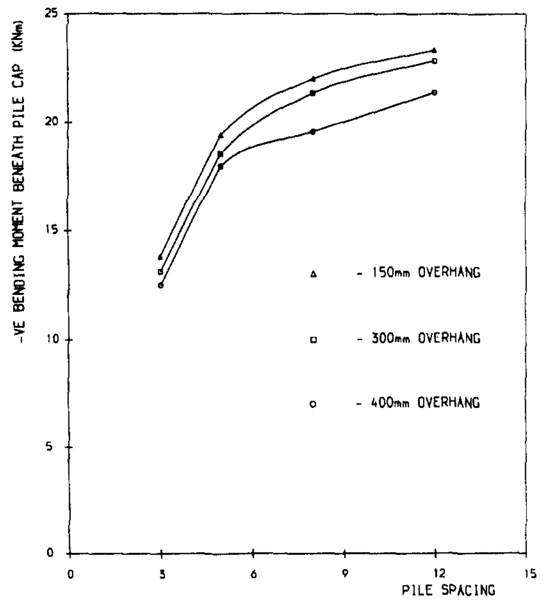

Fig. 8a. Plot of negative bending moment against pile spacing, $H = 40$ kN.

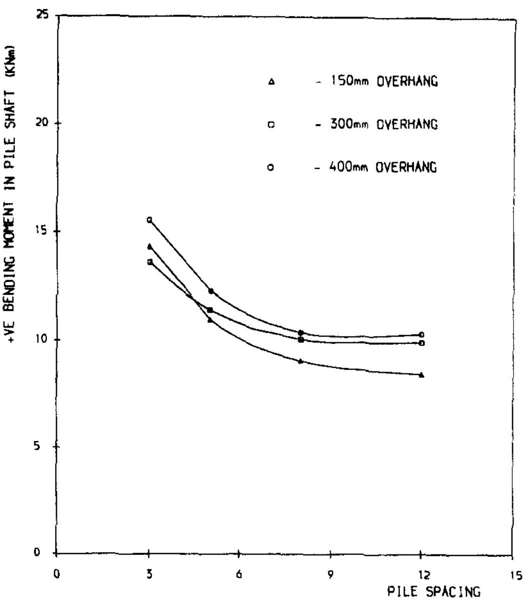

Fig. 8b. Plot of positive bending moment against pile spacing, $H = 40$ kN.

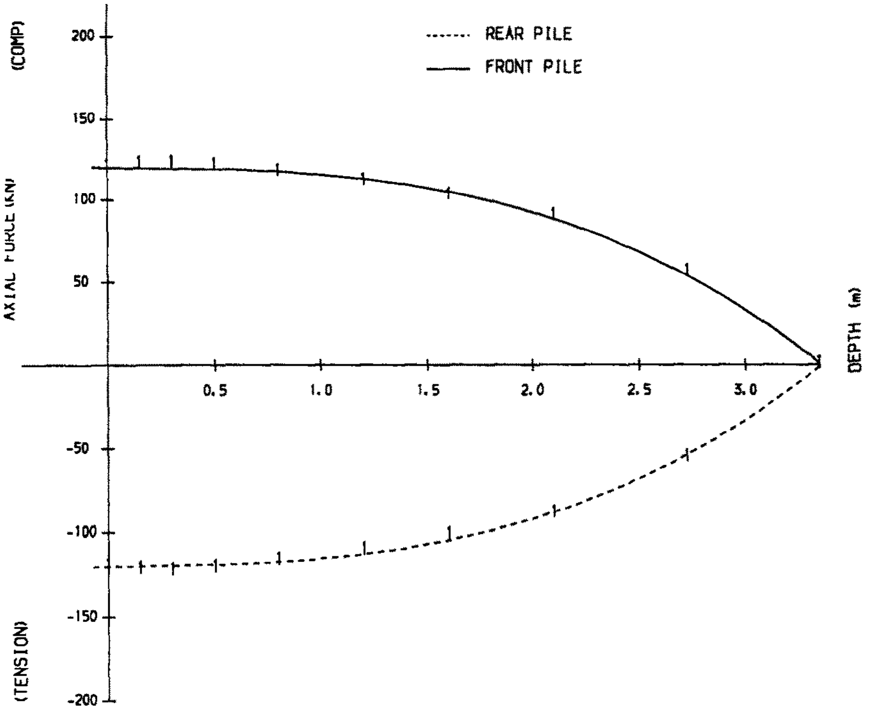

Fig. 9. Axial force diagram for two-pile group for 20 mm pile-cap displacement, piles at 8-D-spacing 150 mm overhang.

with either variable, for a cap head displacement of 20 mm. The negative moment in the piles just beneath cap-level, however, increased rapidly with pile spacing and with decreasing overhang, for the 20 mm displacement. A rather different picture emerged if the plots were made for a constant *force*, as shown in Figs 8a and b. In this case, the pile shaft moment decreased with pile spacing but increased with pile overhang, while the negative moment below the pile cap increased with pile spacing and decreased with cap overhang, for constant force.

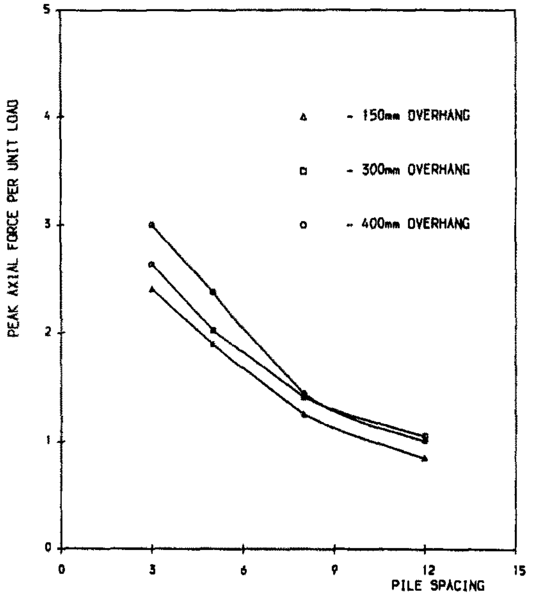

Fig. 10. Plot of peak axial pile force per unit load against pile spacing.

3. *Pile axial forces*

The horizontal applied force (or displacement) caused axial downward load in the front pile and uplift in the rear pile (see Fig. 9). The load was shed into the soil by shaft friction and by some end bearing in the elastic analysis. Vertical equilibrium of the pile cap was satisfied. For a given force, the axial loads reduced with pile spacing and increased slightly with pile cap overhang (Fig. 10).

4. *Pile/soil contact stresses*

The primary resistance to pile group movement is caused by horizontal bearing stresses against the pile shafts. In an elastic analysis compression on the front face of the front pile was equal to tension on the rear face of the rear pile, and similarly for the inner faces. In a granular (Gibson) soil, these values reached a maximum at some 1.5 m below ground surface (see Fig. 11).

RESULTS FROM ANALYSES OF THREE-PILE GROUPS

Analyses of three-pile groups under horizontal in-line loading were broadly of the same pattern as for the two-pile groups. The extended mesh of elements required up to 5226 degrees of freedom. As previously, the pile spacing was varied to include 3, 5, 8 and 12 pile-width spacings, centre to centre, and overhang heights of 150, 300 and 400 mm were used.

As before, the cap load–deflection relationship is described by the pile group lateral stiffness, *Kg*, and values are plotted in Fig. 12. The stiffness values were considerably higher than for the two-pile groups (Fig. 4), but the trends of increasing stiffness with increased pile spacing and with reduced overhang were similar.

The pile shaft bending moments for an imposed 20 mm displacement were broadly similar to those for the two-pile group, but the centre pile attracted a smaller moment than the front or rear piles (see Fig. 13). The reason for this behaviour may be explained in terms of the interactions of

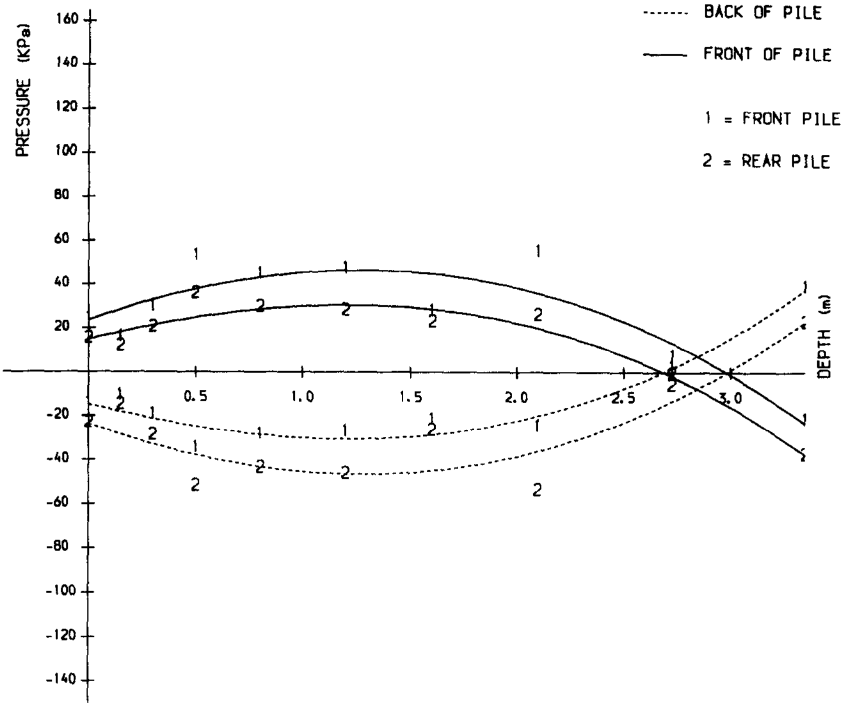

Fig. 11. Pressure distribution diagram for two-pile group for 20 mm pile-cap displacement, piles at 8-D-spacing 400 mm overhang.

the piles upon each other; the centre pile is strongly affected by the close proximity of the front and rear piles; an outer pile is affected strongly by the centre pile but only weakly by the more distant third pile. The moments caused by an imposed force were considerably lower for three-pile groups than for two-pile groups because of the greater group stiffness.

Induced axial forces were downward in the front pile, nearly zero in the centre pile and uplift in the rear pile. The shape of the force diagrams was similar to that for the two-pile groups, but the values were smaller (see Figs 14a and b).

Finally, pile/soil contact stresses showed values very similar to those of the two-pile groups, for a cap displacement of 20 mm, except that the centre pile saw lower contact stresses than the outer piles.

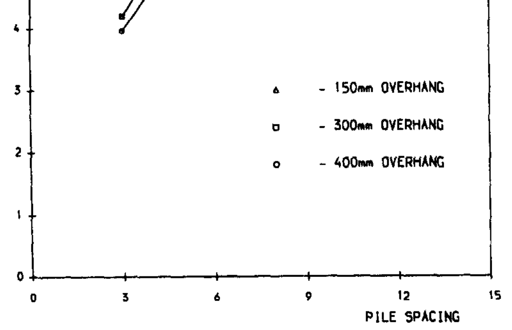

Fig. 12. Stiffness of three-pile group against pile spacing.

NON-LINEAR ANALYSIS OF A TWO-PILE GROUP

While the previous linear–elastic models are appropriate to small loads or displacements, the soil behaviour at higher loads will be strongly non-linear, and separation may occur between the back of the piles and the soil.

An elastic/plastic/separation 3-D analysis was not available, and so a semi-iterative procedure was followed. Firstly, the stresses from an elastic analysis were studied in detail. Elements were identified in which the horizontal tensile stresses due to horizontal cap loading exceeded the K_0, at-rest earth pressures, and these elements were effectively removed by ascribing a negligible modulus. The strain-softening characteristics of the near-surface soil layers were incorporated by a reduction in modulus based upon triaxial test results. Basically, an expression for modulus reduction was derived from the ratios of tangent modulus to secant modulus for increasing strains.

The results of this rather laborious approach can be described in terms of two major effects:

1. Reduction in pile group stiffness. There was a progressive reduction in stiffness as deflection (or load) increased. This was caused by an increasing zone of tensile separation behind the piles, by spreading and increasing strain softening of the soil in front of the piles, and by a reduction in axial stiffness of the piles. This is demonstrated by a load–deflection curve for the pile cap (Fig. 15) in which the stiffness reduced from about 3.34 to some 2.3 MN/m for a deflection of 20 mm.

2. Redistribution of bending moments. When non-linear soil properties and especially pile/soil separation were incorporated, the moments in the front and back piles were no longer equal (Fig. 16). This effect has been demonstrated empirically by Selby and Poulos [12] and by Kim and Brungraber [13].

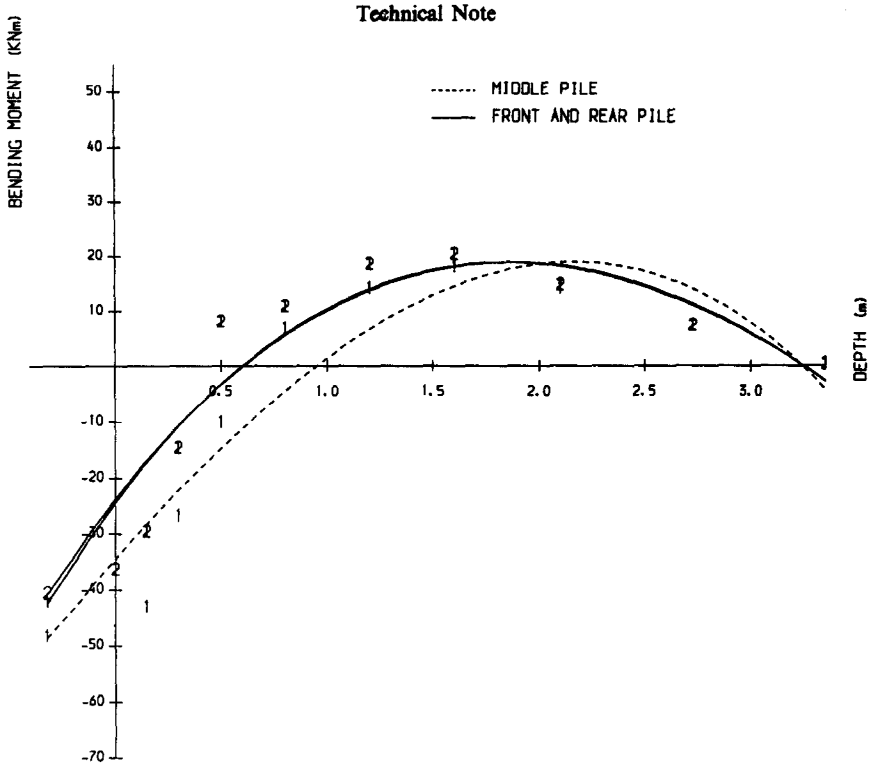

Fig. 13. Bending moment diagram for three-pile group for 20 mm pile-cap displacement, piles at 8-D-spacing 400 mm overhang.

It may be explained by the reduction in soil pressure upon the front face of the rear pile because of the low or zero horizontal stresses in the soil just behind the front pile.

Results from this FE model showed ratios of front to rear pile moments in the shaft of around 1.2:1, and values between 1.5:1 and 1.1:1 for the reversed pile head moments.

Whilst this effect has been demonstrated empirically, and now computationally, it should not be given undue importance. The major difficulty in analysis of a pile group carrying lateral loads is in identifying a realistic soil modulus profile. Take for example one elastic analysis which showed a two-pile group carrying a 40 kN load deflecting by 20 mm, and with

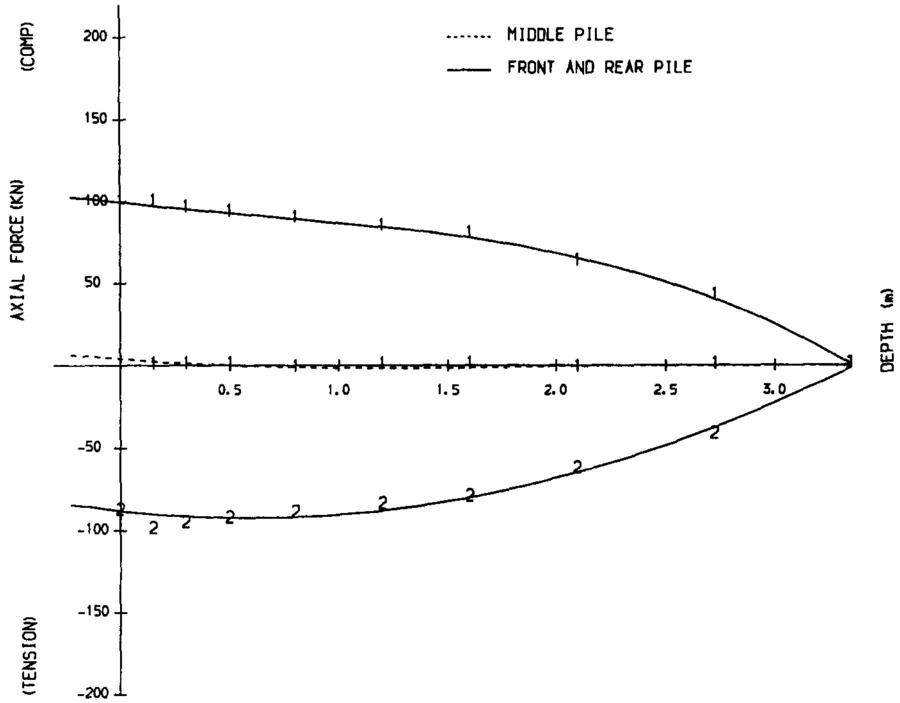

Fig. 14a. Axial force diagram for three-pile group for 20 mm pile-cap displacement, piles at 8-D-spacing 300 mm overhang.

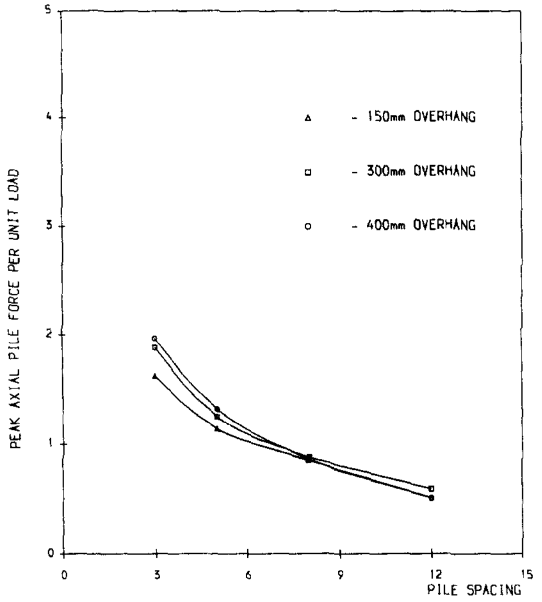

Fig. 14b. Plot of peak axial pile force per unit load against pile spacing.

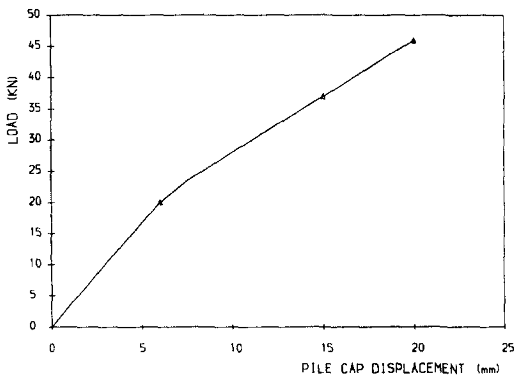

Fig. 15. Load–deflection curve for 3-D spacing 400 mm overhang.

maximum pile head and pile shaft moments of −13.8 and 14.3 kN m, respectively, for a Gibson soil modulus profile of 0–7 MPa at 2.1 m; if the elastic soil modulus is changed to 0–14 MPa, then for the same load, deflection decreases to 7.8 mm and moments change to −17.2 and 12.8 kN m. These changes are at least as significant as the 20% imbalance of moments between front and rear piles estimated in a non-linear analysis.

CONCLUSION

It has been shown that the complex problem of a pile group under horizontal load can now realistically be solved using a fully three-dimensional linear finite element analysis. Careful selection of element combinations is required.

Detailed analysis of FE results showed that wider spacing and smaller overhang increased the pile group stiffness. This has been quantified. The bending moments, axial forces and pile/soil pressures have been deduced for a range of pile group geometries.

The elastic analyses were successfully extended to cover the three-pile case, and it is feasible that slightly larger pile groups could be analysed in this way. The three-pile analyses showed that the centre pile carried no vertical load and attracted smaller moments and soil pressures than the outer piles.

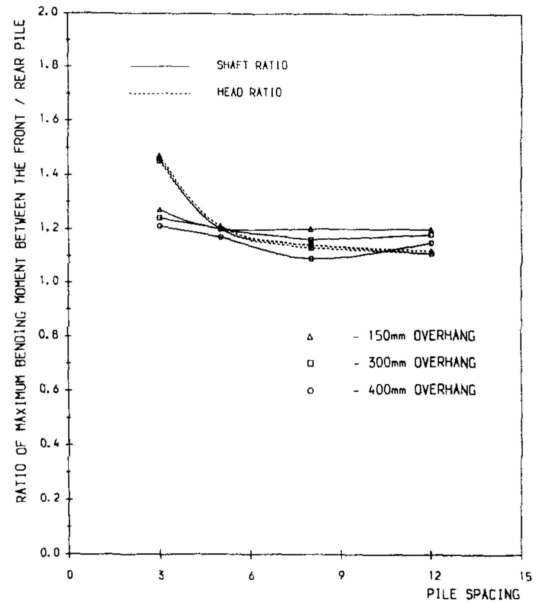

Fig. 16. Plot of ratio of maximum bending moment against pile spacing.

It is possible to evaluate, by relatively straightforward means, the effects of soil non-linearities, comprising both separation and soil strain-softening, for larger pile group displacements. However, the reliability of these analyses is relatively low. The non-linear analyses of a two-pile group showed an unequal load distribution between the front and rear piles.

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