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A STUDY USING SINGLE-DEGREE-OF-FREEDOM MODELS TO DETERMINE TYPES OF BUILDINGS INFLUENCED BY VERTICAL ACCELERATIONS

Paul W. Spears[1] and Finley A. Charney, Ph.D., P.E.[2]

1. – Martin/Martin Consulting Engineers
4501 Madison Ave., Kansas City, MO 64111 USA
email: paulwspears@yahoo.com

2. – Depart of Civil and Environmental Engineering, Virginia Tech
200 Patton Hall, Blacksburg, VA 24061 USA
email: fcharney@vt.edu

ABSTRACT

There is disagreement regarding the influence of vertical accelerations on buildings. Some say that vertical accelerations significantly affect the response of structures others disagree. Furthermore, the types of structures that might be sensitive to vertical accelerations are not well understood. This study aimed to move the “influence of vertical accelerations on buildings” discussion toward a resolution and provide ranges of buildings that might be sensitive to vertical accelerations. Generally speaking, vertical accelerations did not influence the lateral response of models examined in this study. It should be noted, however, that some models were influenced by vertical accelerations. These models tended to be inherently unstable, but may not necessarily be outside of the bounds of code acceptability.

Keywords: Vertical Accelerations, Parametric Study, P-Delta, nonlinear, Single-Degree-of-Freedom

1. INTRODUCTION AND BACKGROUND

Based on current research, it would seem that there is some disagreement as to whether vertical accelerations have a significant impact on the lateral response of a structure in a seismic event. Some research indicates that vertical accelerations are significant and other research indicates the opposite. Furthermore, most research on vertical accelerations has focused on specific buildings and so it is also not clear which types of structures would be influenced by vertical accelerations. The authors sought to determine the types of structures that might be sensitive to vertical accelerations using Single Degree of Freedom (SDOF) models with bilinear force-deformation relationships. The SDOF model parameters varied were stiffness (period of vibration), post yield stiffness ratio, yield strength, and geometric stiffness (P-Delta effects). Additional parameters allowed to vary in the study related to the ground motions were earthquake ground motion, lateral ground acceleration intensity, and vertical ground acceleration intensity. Considering both structural parameters and earthquake parameters, seven parameters were varied.

Of the existing literature on vertical accelerations, many studies have shown that vertical acceleration magnitudes can be as great or greater than the horizontal accelerations. Abrahamson and Litchiser (1989) showed that the ratio of vertical acceleration to horizontal acceleration (V/H) was dependent on the magnitude of the event and the distance from the site to the source. Bozorgnia et al. (1995) also noted that the V/H ratio was dependent on the site distance and earthquake magnitude. They also pointed out that the V/H acceleration response spectra ratio is dependent on the period of the structure in question. For short period structures, the ratio can be much greater than one, but for long period structures, the ratio is typically much less than one. Not only does site distance and period affect vertical accelerations, but soil conditions also affect them. Amirbekian and Bolt (1998) showed that the vertical acceleration component tends to be higher than the horizontal in alluvial basins.

There have also been many studies investigating the effects of vertical accelerations on structures. Some have concluded that vertical accelerations do not significantly affect structural response. Maison and Kasai (1997), for example, analytically tested a thirteen story steel moment frame which was damaged in the Northridge earthquake and showed that the lateral displacement of the simulated structure was very similar with and without vertical accelerations included. Furthermore, the major emphasis of their research was to examine connection failures that occurred in the building during the earthquake. They noted that increased gravity loads actually reduced damage to the connections. They reasoned that this was because the gravity loads caused a compressive preloading of the bottom flanges of the beams. They further reasoned that perhaps the lack of gravity loads in buildings under construction during the Northridge earthquake explained why more of those buildings experienced connection damage. So, it would seem that real structures with gravity loads are less likely to be adversely affected by vertical accelerations.

Hjelmstad and Williamson (1998) examined the dynamic stability of hysteretic single-degree-of-freedom inverted pendulum systems subjected to harmonic base excitation, not actual ground motion records, and noted that vertical accelerations increased lateral displacements in some situations if the vertical forcing frequency was twice the lateral frequency of the structure. However, they found that the vertical accelerations did not significantly increase lateral deflections of the structures they studied. Jennings and Husid (1968) and Takizawa and Jennings (1980), using a small and focused range of models and parameters, argued that vertical accelerations have a negligible effect on the lateral response of structures.

While there are many studies indicating that vertical accelerations do not significantly affect structural response, there are also many which show that vertical accelerations do affect structural response. First, there are many studies that indicate that vertical accelerations increase the damage to various structural components in a building. Anderson and Bertero (1973), for example, analyzed a ten story steel moment frame subjected to horizontal and vertical ground motions from the San Fernando earthquake of 1971. From the study, they concluded that the ductility requirements at critical regions would be inaccurate if vertical accelerations were not included. In the upper stories, for example, the ductility requirements were increased by fifty per cent by including vertical accelerations. Saadeghvaziri (1988) and Saadeghvaziri and Foutch (1991) examined the effects of vertical and horizontal accelerations on bridges. They noted that vertical accelerations cause severe fluctuations in the axial loads in bridge piers and that these fluctuations result in highly erratic hysteresis loops. Moreover, the vertical accelerations tend to increase the shear demand and reduce the shear capacity in bridge piers. The increased axial loads stiffened the columns, which in turn caused the columns to attract more force. At the same time, the increased axial load reduced the shear capacity of the piers. Also, the increased axial loads reduced the ductility of the columns. Saadeghvaziri and Foutch speculated that the severe damage to bridges in some earthquakes may be primarily caused by vertical accelerations. Hart et al. (1995) examined a six story special steel moment resisting frame subjected to both vertical and horizontal accelerations as part of the SAC Steel Project. Hart's research focused on the effects of tributary mass, both in the vertical and horizontal directions. The mass distribution caused a great deal of scatter in the axial forces in the columns. Hart noted that the effect of vertical accelerations was most pronounced in the interior columns, which are primarily designed to withstand axial loads. In some situations, Hart found that seismic axial forces could be twice the dead load. Higazy et al. (1996) examined reinforced concrete (RC) beam-column connections under vertical accelerations. They found that should vertical accelerations cause a RC connection to go into tension, the shear capacity of the joint was reduced by eighteen to fifty percent. Also, the confinement in the joint core was rendered ineffective in the event of significant tension. Como et al. (2003) examined the likelihood that vertical accelerations would cause axial shortening in some columns and therefore increased rotations at beam ends.

A second category of vertical acceleration research has focused on how vertical accelerations contribute to lateral displacements. Iyengar and Shinozuka (1972) examined the effects of vertical accelerations on tall buildings. They modeled tall buildings as distributed mass cantilevers. They concluded that vertical accelerations considerably increased the tip deflection, base shear, and base moment for the cantilever models studied. Lin and Shin (1980) examined the effects of vertical accelerations on single degree of freedom inverted pendulum elastic structures. They noted that the vertical accelerations had the potential to amplify lateral response. Then, Shin and Lin (1982) performed a similar study involving single-degree-of-freedom-inverted pendulums except the systems were hysteretic rather than elastic. They examined systems with both 0.1 and 0.5 post-yield stiffness ratios, and found that for hysteretic models, the vertical accelerations had more of an effect on the lateral displacement than in the elastic models. Moreover, the lower post-yield stiffness ratio model was more influenced by vertical accelerations than the higher. Shin and Lin also noted that the vertical accelerations had a significant impact on the amount of residual deformations in their models. Ariaratnam and Leung (1990) analyzed a six story multiple degree of freedom building using random vibrations in both the vertical and horizontal directions. They concluded that both gravity loads and vertical inertial loads can increase the lateral displacement when structures are subjected to earthquakes.

2. SDOF PARAMETER STUDY DESCRIPTION

Of all of the studies conducted, it did not appear that any of them were specifically interested in determining types of structures and situations where vertical accelerations would be particularly damaging or contribute significantly to lateral deflection. Consequently, the research program described in this paper was undertaken. The models used in this study were essentially simple single-degree-of-freedom inverted pendulum structures. They were constructed as portal frames, but acted similar to inverted pendulums. Fig. 2.1 illustrates the form and function of the models used in this study and their relation to inverted pendulums.

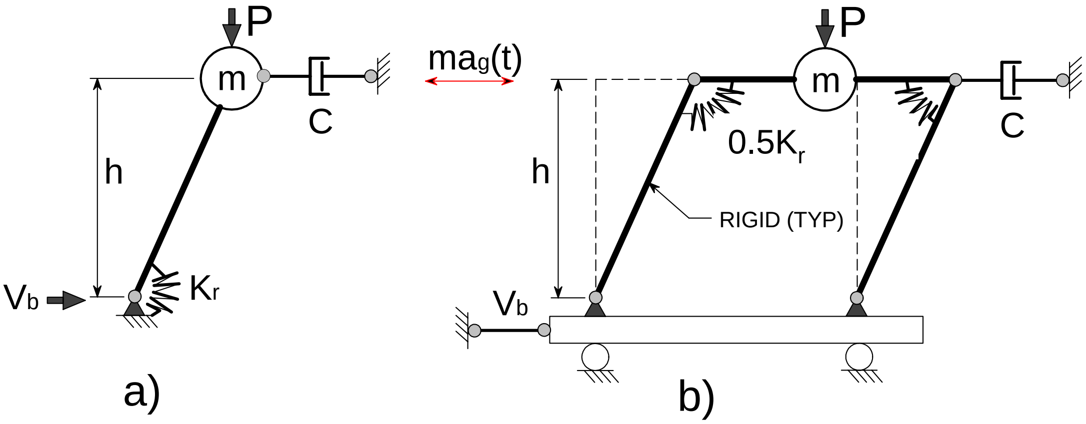

Figure 2.1 – a) Inverted Pendulm and b) Similar Portal Frame Structure

Seven parameters were identified and varied in an attempt to determine types of structures that might be sensitive to vertical accelerations. The first four parameters were related to the structural model and are shown in Fig. 2.2. The other three are related to ground motions and are described later.

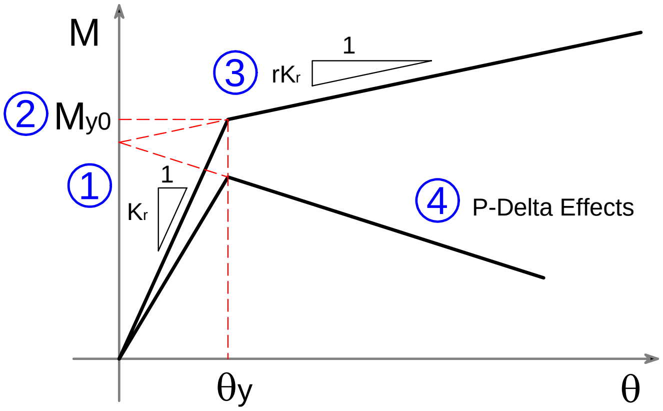

Figure 2.2 – OpenSees Model Parameters

The curves shown in Fig. 2.2 represent the force deformation relationship of the rotational springs used in this study. The horizontal axis plots rotation, and the vertical axis plots moment. The parameters in numerical order from the above figure are rotational stiffness, yield moment, post-yield stiffness ratio, and P-Delta effects. A unit mass was used for each of the models and hence, the initial stiffness was the only variable used to determine periods of vibration. Then, the yield moment corresponded to the base shear required for design. Also, the initial stiffness was modified by the post yield ratio (r), to determine the secondary stiffness of the systems. Finally, the actual force deformation relationship followed the lower curve when P-Delta effects were considered. P-Delta effects were gauged in terms of stability ratio (Q) . The lower curve was determined by subtracting K_rQ from the upper curve. It is important to note that the stability ratios are not the same thing as the rotations plotted in Fig. 2.2 above.

The ranges of the first two parameters, stiffness and yield strength, were determined according to the Equivalent Lateral Force (ELF) method as prescribed by the 2000 NEHRP Provisions (FEMA 2000). The ELF method was used because it allowed for a relatively quick determination of structural parameters without having to perform many calculations. The third and fourth parameters were merely chosen such that the ranges would mirror realistic structures.

Earthquake parameters were also varied. Eight earthquakes were used and various lateral and vertical scale factors were also used. The earthquakes, along with pertinent information about them, are listed in Table 2.1. Table 2.2 summarizes both the structural and earthquake parameters used in the study.

The total number of ground motion and model combinations used in the analyses was 7 x 7 x 6 x 5 x 8 x 6 x 2 = 141,120.

Each of the structural parameter combinations and earthquake combinations used in the parameter study were analyzed once excluding vertical accelerations and once including vertical accelerations. As one might imagine, this generated a significant amount of data. As such, only a few representative figures are presented in this article.

Table 2.1 – Earthquakes Used for the Parameter Study

	Earthquake		PGA (%g)		Duration		
Number	**Location**	**Date**	**Horizontal**	**Vertical**	**(sec)**	**Magnitude**	**Soil**
1	Aqaba, Egypt	22-Nov-95	0.097	0.109	60	7.1	--
2	Cape Mendocino, CA	25-Apr-92	0.59	0.163	35.98	7.2	D
3	Chalfant Valley, CA	21-Jul-86	0.177	0.127	39.925	6.2	D
4	Kern County (Taft)	21-Aug-52	0.156	0.109	54.15	7.4	D
5	Hollister, CA	26-Jan-86	0.114	0.172	40	5.4	D
6	Izmit, Turkey	17-Aug-99	0.152	0.146	30	7.4	A
7	Kobe, Japan	16-Jan-95	0.251	0.158	40.96	6.9	E
8	Tabas, Iran	16-Sep-78	0.328	0.183	23.84	7.4	B

Table 2.1 – Summary of Structural and Ground Motion Parameters

	Period (sec)	Yield Force (% of Wt)	Post-Yield Stiffness Ratio	Stability Ratio	Earthquakes	Lat. Scale (g)	Vertical Multiplier	Vertical Included?
	0.705	3	0	0.04	Aqaba, Egypt	0.1	1.5	Yes
	0.903	6	0.05	0.08	Cape Mendocino, CA	0.2	2	No
	1.093	9	0.1	0.12	Chalfant Valley, CA	0.3	2.5	
	1.278	12	0.15	0.16	Kern County (Taft)	0.4		
	1.459	16.5	0.2	0.2	Hollister, CA			
	1.635	22.5	0.25		Izmit, Turkey	*total number of vert. and hor. combinations used*		
	1.808	28.5			Kobe, Japan			
					Tabas, Iran			
Total:	7	7	6	5	8	6		2

3. SDOF PARAMETER STUDY RESULTS

Before describing the results of the study, it is important to define a quantity that proved to be useful in predicting potential collapse parameter combinations. As is turned out, the most important factor for determining whether a model would collapse or remain stable was the relationship between ductility demand and the post yield stiffness ratio augmented by P-Delta effects (r_p). The r_p factor is essentially the factor that the elastic stiffness K_r must be multiplied by to get the secondary stiffness of the model including P-Delta effects, that is the lower line of Fig. 2.2. A derivation of the r_p value may be found in Spears (2003). It is given as

$$r_p = \frac{r - Q}{1 - Q} \tag{2.1}$$

Figures 3.1 and 3.2 are graphs which show the changes in ductility demands that resulted from including vertical accelerations.

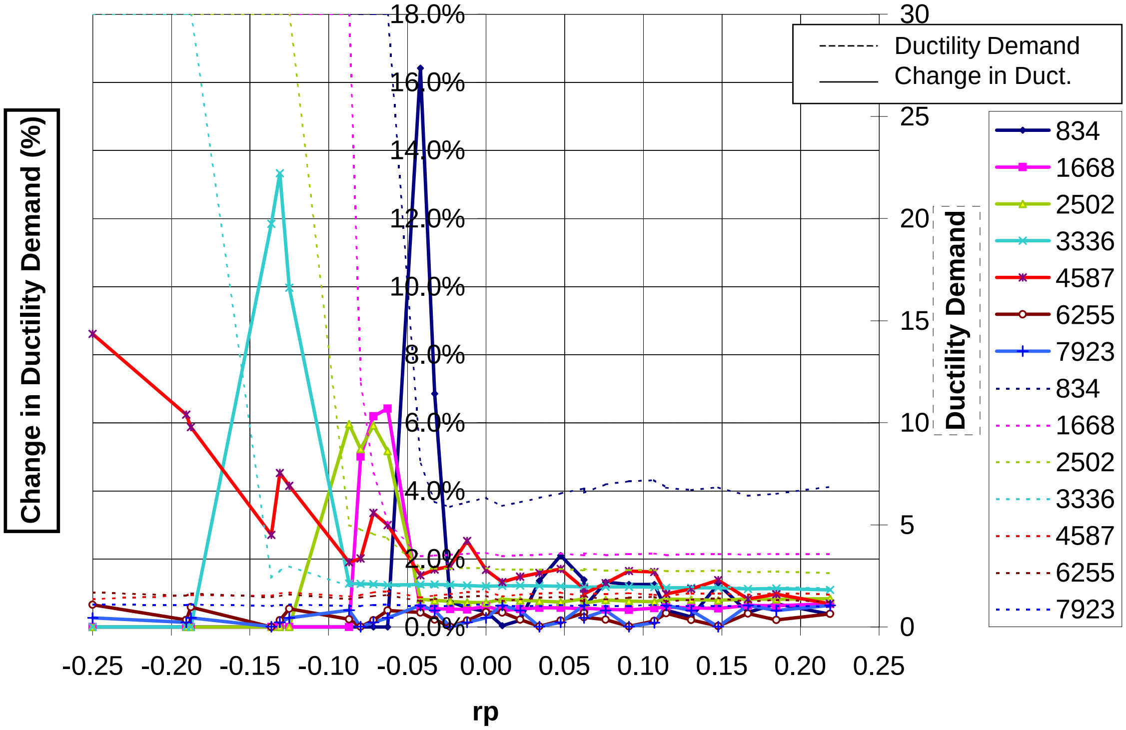

Figure 3.1 – EQ6, Lateral Scale = 0.2 g, Change in Ductility Demand Due to the Inclusion of Vertical Accelerations for Models with a Period of 1.459 Seconds.

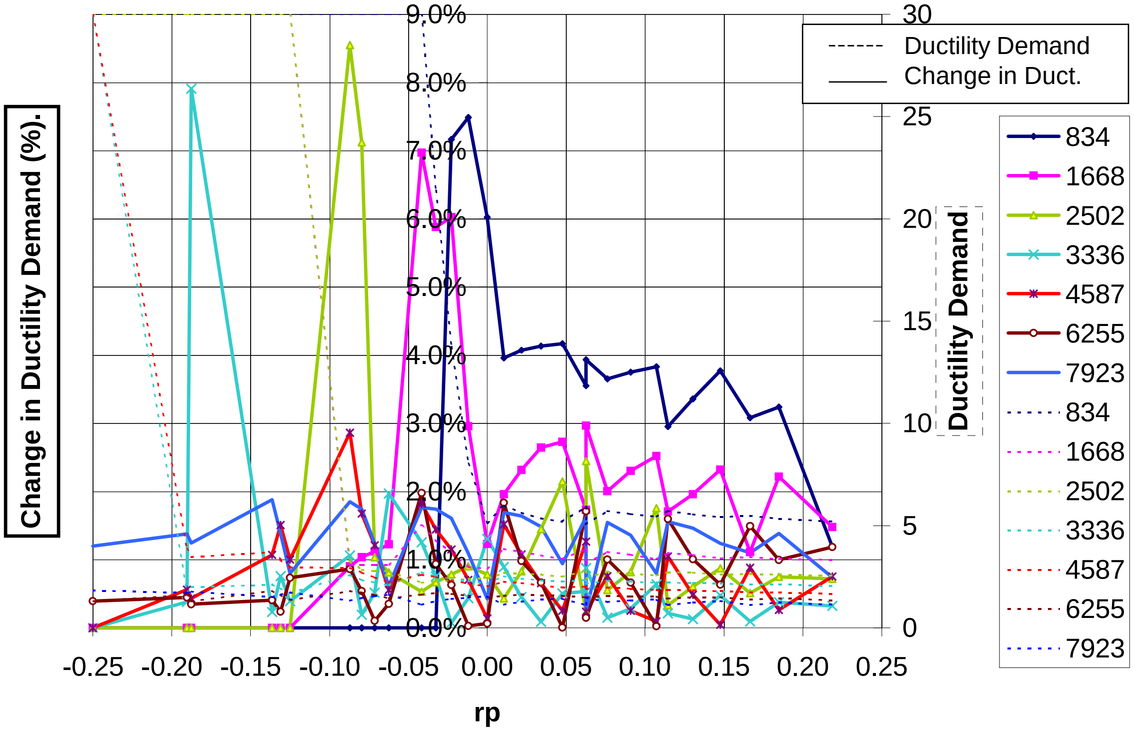

Figure 3.2 – EQ7, Lateral Scale = 0.4 g, Change in Ductility Demand Due to the Inclusion of Vertical Accelerations for Models with a Period of 1.635 Seconds.

In Figures 3.1 and 3.2, there are two vertical axes corresponding to two different sets of data. The vertical axis toward the right of each figure is the ductility demand of the structures. The title of this axis has been boxed with a dashed line to indicate that it applies to the data that also has dashed lines. The thin dashed line data represents the maximum ductility demands excluding vertical accelerations. Notice that r_p is plotted on the horizontal axis and that there is a point, if moving from the right, where the ductility demands increases rapidly, indicating collapse.

The middle axis plots the change in ductility demand that comes from adding vertical accelerations. The title for this axis is on the left-hand side of the figures and is boxed with a heavy solid line to indicate that it applies to the data represented with heavy solid lines. It is important to note that the absolute value of the change in ductility demand has been plotted in the Figs. 3.1 and 3.2. This is because it is impossible beforehand to know whether vertical accelerations will increase or decrease lateral displacements, and if a structure shows sensitivity to vertical accelerations, then it is possible that vertical accelerations would be detrimental to particular structures subjected to particular earthquakes. As will be shown later, vertical accelerations increased the lateral displacements about half the time and about half the time it decreased them.

Notice that Figs. 3.1 and 3.2 represent an entire set of data for a model with a given period of vibration subjected to particular earthquake with a given intensity. The sets of data in each figure were then subdivided in yield forces or design base shears. Each line in Figs. 3.1 and 3.2 represents a yield moment for a corresponding yield force found in table 2.1. The lower yield moments correspond to lower yield force percentages and vise versa.

Figures 3.1 and 3.2 were chosen as a representative sample of all of the parameter study data. Several observations can be made from the data created. First, given a particular structure with a given period and yield force, subjected to a particular earthquake, vertical accelerations did not significantly affect the lateral displacements until the r_p values were reduced to a particular level. For the purposes of this discussion, if the change in ductility demand was greater than about five percent, then it will be considered significant. In both the figures above, a spike in the change in ductility demand curves, if there was a spike, occurred in the negative r_p range just before collapse. Secondly, when the vertical accelerations did significantly affect the lateral displacements, it was generally not by more than about ten percent. There were a few cases where the change in ductility demands was much greater, but that was not the norm. Third, it seems that the increase in lateral displacements is only dependent on the r_p value near the point of collapse. The individual earthquakes did not have much effect on overall lateral displacement increases due to vertical accelerations. Also, the lateral scale did not influence the amount of change that vertical accelerations caused. In Fig. 3.1, the models were subjected to earthquake six at a lateral PGA scale of 0.2g and the average ductility demand increases caused by vertical accelerations were around ten or twelve percent, which is relatively close to the results from the other figures which had PGAs of 0.4 g. From the data of this study, it would seem that vertical accelerations do not significantly affect the lateral displacements unless the combination of structural and earthquake parameters was inherently unstable. Even then, the change in lateral displacements due to vertical accelerations was on average around ten percent.

While the change in lateral displacements was relatively small, a small change could be enough to cause collapse in a building. The additional ductility demand may be just enough to fracture a critical steel connection or crush the concrete at a critical joint. To address this possibility, another set of figures was created that defined collapse as an increase in ductility demand greater than a specified value. Moreover, in the same figures, a structure was considered “saved” if the ductility demands decreased by a specified amount. In the figures, the number of collapses was plotted in the positive direction and the number of saves was plotted in the negative direction. This was done merely to create a visually distinguishing effect for the collapses and saves. Furthermore, three changes in ductility demand values were used to determine collapses and saves, 0.5, 1.0, and 2.0. A representative plot of the aforementioned collapse criteria is shown in Figure 3.3.

One thing to keep in mind about the collapse and save figures is that the total numbers of collapses and saves represent 1470 analyses. Thus, if 20 collapses occurred for a given set of models, then 1.4 percent of the models in the set would have collapsed. This is not said to minimize the potential hazards of vertical accelerations, but to provide some perspective for the data shown in the figures.

Also, each collapse or save category considers the entire set of data. As one might expect, there are fewer models in each category as the absolute value of the ductility demand range is increased. Force example, when the change in ductility demand was greater than 0.5, 11 models of 1470 fell into that category. However, when the change in ductility demand was increased to greater than 1.0, only 5 models of 1470 fell into that category.

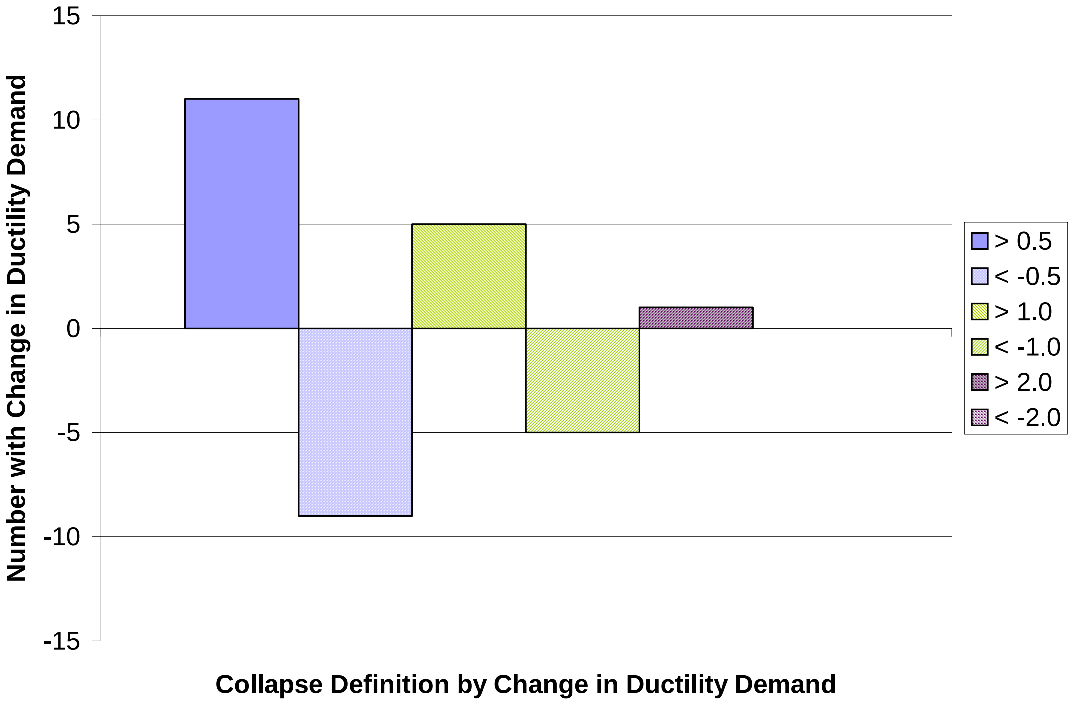

Figure 3.3 – Collapses and Saves Due to the Inclusion of Vertical Accelerations for the Set of Models Subjected to EQ 3 with a Lateral Scale of 0.2 g and a Vertical Multiplier of 2.5.

From Fig. 3.3, it is clear that vertical accelerations would both cause structures to collapse as well as save structures from collapse in the sets of data considered. However, the numbers of collapses and saves varies from earthquake to earthquake. This is why it was said earlier that it is impossible to predict beforehand the effects that vertical accelerations will have on a particular structure. There are too many factors that influence whether or not a structure's response will be altered by such accelerations. While it may not be clear before an analysis if the inclusion of vertical accelerations will cause a structure to collapse or be saved, it is clear that increasing the vertical acceleration magnitudes increases the probability that a collapse or save could occur.

An important point to make about the effect of vertical accelerations on maximum displacements is that structures should ideally be designed so that they would not come close to the point of collapse. Consequently, a well-designed structure should rarely, and possibly never, be significantly affected by vertical accelerations. This assumes, however, that the hazards and risks of a given region are known exactly and that there are no shortcomings in the current design philosophies. While the hazards and risks may be known with a fairly high degree of certainty in the Western United States (WUS) and the current design philosophies have been incrementally improved with each new earthquake, the same cannot be said about structures in the Central and Eastern United States (CEUS). There is much more uncertainty regarding the definition of a well designed CEUS structure.

Furthermore, the above statements have been based on SDOF models with bilinear stiffness. The results may be very different for MDOF models incorporating degrading strength and stiffness.

In addition to including vertical accelerations, several sets of models were analyzed for varying vertical earthquake scales. Recall that multipliers ranging from 1.5 to 2.5 were used to scale the vertical accelerations. In Figures 3.4 through 3.6, the effect of increasing vertical multipliers on lateral displacement is seen. All six of the figures involve earthquake three, which had an unscaled vertical PGA of 0.127 g. After multiplying the vertical PGA by 1.5, 2.0, and 2.5, the resulting values were 0.191, 0.254, and 0.318, respectively.

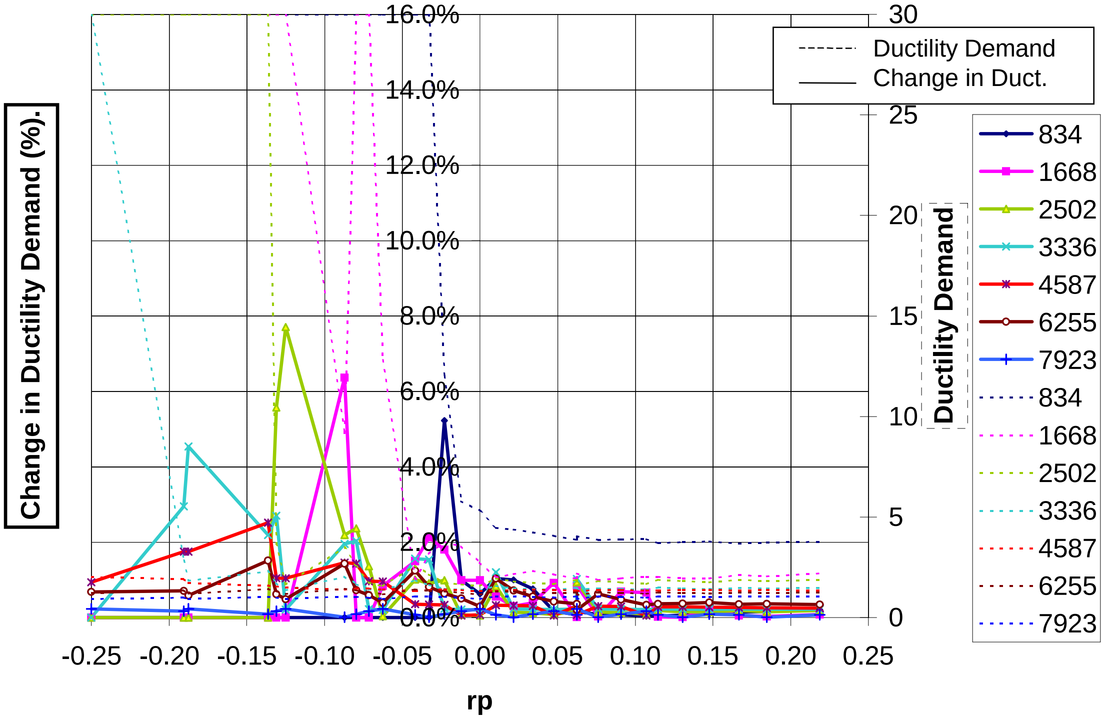

Figure 3.4 – EQ3, Lateral Scale = 0.2 g, Vertical Multiplier = 1.5, Change in Ductility Demand Due to the Inclusion of Vertical Accelerations for Models with a Period of 1.093 Seconds

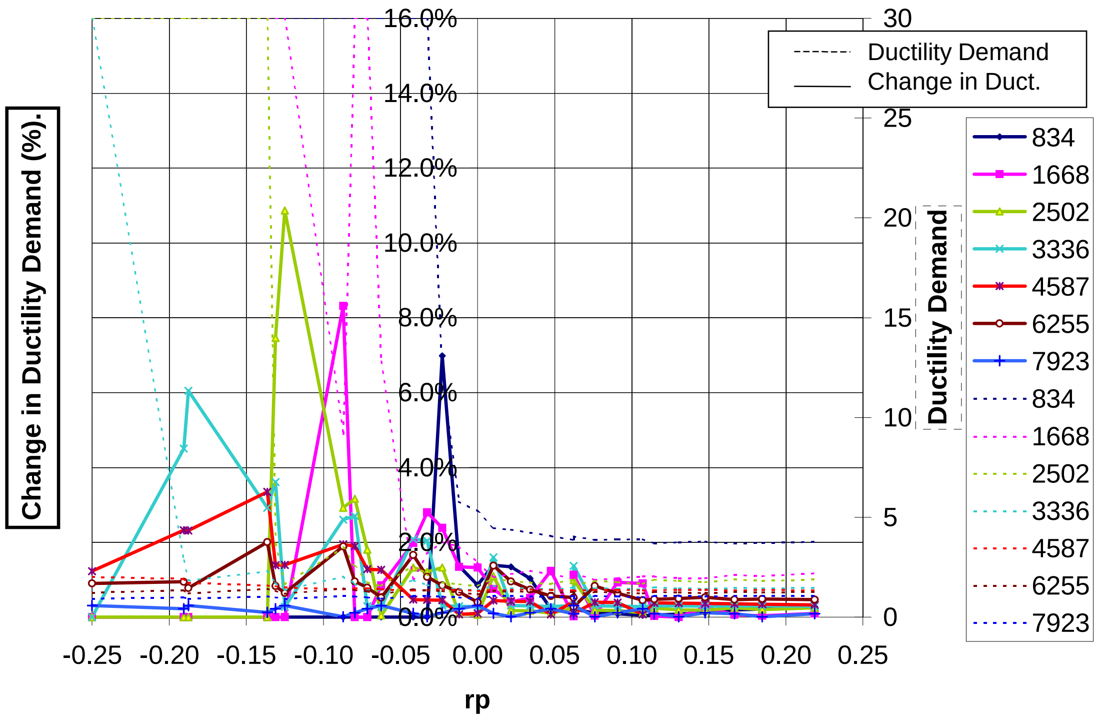

Figure 3.5 – EQ3, Lateral Scale = 0.2 g, Vertical Multiplier = 2.0, Change in Ductility Demand Due to the Inclusion of Vertical Accelerations for Models with a Period of 1.093 Seconds.

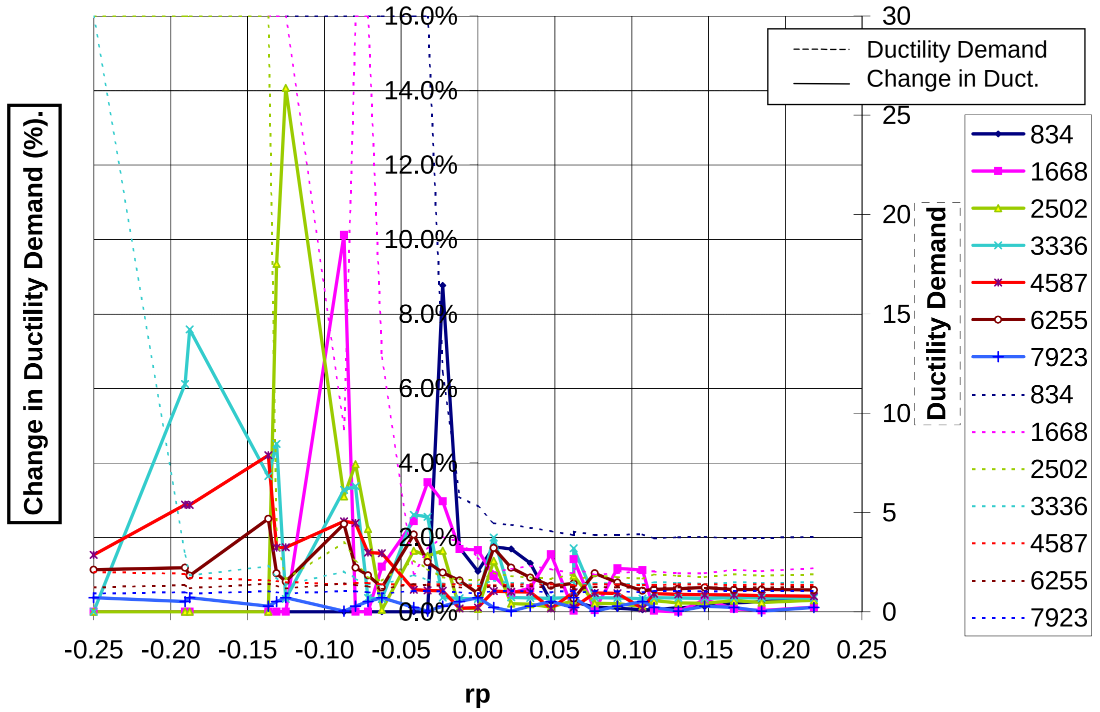

Figure 3.6 – EQ3, Lateral Scale = 0.2 g, Vertical Multiplier = 2.5, Change in Ductility Demand Due to the Inclusion of Vertical Accelerations for Models with a Period of 1.093 Seconds.

Notice that in all three of the figures, the maximum values increase as the vertical accelerations increase. Also notice, though, that as the vertical accelerations increase from 0.191 g to 0.318 g the change in ductility demand increases from an average of about six percent to an average of about ten percent. Thus, while the ductility demands increase by about 50 percent, the values are still relatively small whereas the vertical ground motion has increased significantly in magnitude.

4. CONCLUSIONS AND NEEDS FOR FUTURE RESEARCH

Vertical accelerations typically did not significantly influence lateral displacements except for certain combinations of structural parameters and earthquake parameters. From this study, it seems that the vertical accelerations could likely increase or decrease the maximum lateral displacements by approximately 10 percent if the structure's ductility demand were close to the combination of ductility demand and r_p that would result in collapse for model with a given period and yield force subjected to a given earthquake without vertical accelerations. The 10 percent increase is, however, approximate. There were many models in the study that were never influenced by vertical accelerations and several where the vertical accelerations changed lateral displacements by up to 60 percent. It is important to note that the intensity of the vertical accelerations influenced the amount of change in lateral displacements for those structures affected by them. As the vertical intensity increased, the change in lateral displacements generally increased.

This study may be particularly applicable to CEUS structures. As mentioned earlier, WUS structures have been incrementally improved with each new earthquake. This is not the case with CEUS structures. Also, there have been enough earthquakes in the WUS to speculate what sorts of earthquakes might occur in the future. Since no major earthquake has occurred in the CEUS for over a century, and since ground motion records are not available for these earthquakes, the characteristics of large CEUS earthquakes are not clear. Thus, it is the authors' opinion that considerable time be invested in both determining relevant ground motions for the CEUS and scrutinizing the current design codes to determine if it is possible to design a code compliant structure that would have a propensity to collapse. It would also be beneficial to scrutinize the previous design codes to determine if those structures would be inclined to collapse.

This study should also be expanded to MDOF models with degrading strength and stiffness to gain further insight into the influence of vertical accelerations on realistic structures.

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