

## CAPACITY SPECTRUM METHOD BASED ON INELASTIC DEMAND SPECTRA

PETER FAJFAR\*

*Faculty of Civil and Geodetic Engineering, University of Ljubljana, Ljubljana, Slovenia*

### SUMMARY

By means of a graphical procedure, the capacity spectrum method compares the capacity of a structure with the demands of earthquake ground motion on it. In the present version of the method, highly damped elastic spectra have been used to determine seismic demand. A more straightforward approach for the determination of seismic demand is based on the use of the inelastic strength and displacement spectra which can be obtained directly by time-history analyses of inelastic SDOF systems, or indirectly from elastic spectra. The advantages of the two approaches (i.e. the visual representation of the capacity spectrum method and the superior physical basis of inelastic demand spectra) can be combined. In this paper, the idea of using inelastic demand spectra within the capacity spectrum method has been elaborated and is presented in an easy to use format. The approach represents the so-called N2 method formulated in the format of the capacity spectrum method. By reversing the procedure, a direct displacement-based design can be performed. The application of the modified capacity spectrum method is illustrated by means of two examples. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: capacity spectrum method; inelastic spectra; displacement-based design; deformation-controlled design; cumulative damage

### INTRODUCTION

After recent earthquakes, especially the Northridge and Hyogoken–Nanbu (Kobe) earthquakes, a consensus has been reached that present codes need significant improvements and expansion.

In the U.S.A., the structural engineering community is in the process of developing a new generation of design and rehabilitation procedures that will incorporate performance-based engineering concepts. The most prominent examples are Vision 2000,<sup>1</sup> ATC-40,<sup>2</sup> and FEMA-273.<sup>3</sup> In Japan, the Building Standard will be revised into a performance-based design format. In all documents, non-linear static analysis procedures play a central role. Among them, the popularity of the so-called capacity spectrum method is increasing rapidly.

The capacity spectrum method was developed by Freeman.<sup>4,5</sup> By means of a graphical procedure, it compares the capacity of a structure with the demands of earthquake ground

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\* Correspondence to: P. Fajfar, FGG, Jamova 2, SI-1000 Ljubljana, Slovenia, e-mail: peter.fajfar@ikpir.fagg.uni-lj.si

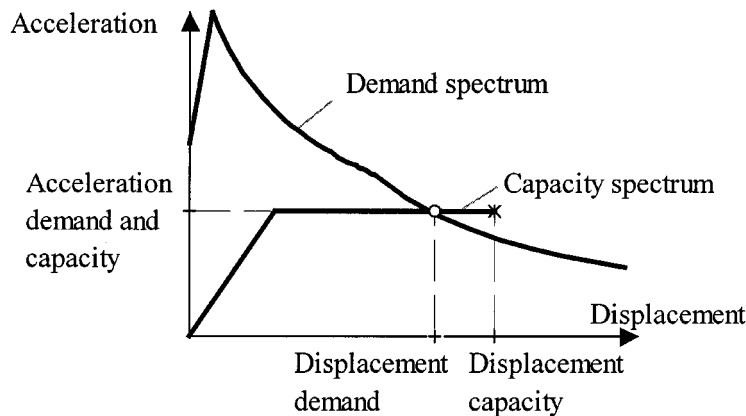


Figure 1. Capacity spectrum method

motion on the structure (Figure 1). The graphical presentation makes possible a visual evaluation of how the structure will perform when subjected to earthquake ground motion. The method is easy to understand. The capacity of the structure is represented by a force–displacement curve, obtained by non-linear static (pushover) analysis. The base shear forces and roof displacements are converted to the spectral accelerations and spectral displacements of an equivalent Single-Degree-Of-Freedom (SDOF) system, respectively. These spectral values define the capacity spectrum. The demands of the earthquake ground motion are defined by highly damped elastic spectra. The Acceleration–Displacement Response Spectrum (ADRS) format<sup>6</sup> is used, in which spectral accelerations are plotted against spectral displacements, with the periods  $T$  represented by radial lines. The intersection of the capacity spectrum and the demand spectrum provides an estimate of the inelastic acceleration (strength) and displacement demand.

A controversial part of the described method is the use of highly damped elastic spectra for the determination of seismic demand. According to Krawinkler<sup>7</sup> ‘there are two fundamental flaws that render the quantitative use of the capacity spectrum method questionable. First, there is no physical principle that justifies the existence of a stable relationship between the hysteretic energy dissipation of the maximum excursion and equivalent viscous damping, particularly for highly inelastic systems. The second flaw is that the period associated with the intersection of the capacity curve with the highly damped spectrum may have little to do with the dynamic response of the inelastic system’. Due to these deficiencies it was stated in Vision 2000 that ‘the theoretical foundations of the method are open to question’. Freeman, the author of the capacity spectrum method, wrote:<sup>5</sup> ‘One controversial item of the capacity spectrum method, as well as for other inelastic methods, is the relationship between inelastic response spectra and equivalent linear elastic response spectra’. As an alternative to the ATC-40 recommendations, he presented effective damping ratios based on Newmark–Hall studies.<sup>8</sup>

The lack of consensus on the definition of seismic demand is reflected in the two different approaches used in recent US documents. Whereas the capacity spectrum method is the main method used in ATC-40, it is the so-called displacement coefficient method which is used in FEMA-273. In this latter method, the displacement demand is, essentially, determined from the

inelastic displacement spectra which are obtained from the elastic displacement spectra by using a number of correction factors based on statistical analyses. Inelastic spectra are, in principle, expected to be more accurate than elastic spectra with equivalent damping, especially in the short-period range and in the case of high ductilities.

In order to overcome the deficiencies of the original version of the capacity spectrum method, Bertero<sup>9</sup> recommended to use 'smoothed inelastic design response spectra' as demand spectra. However, to realise this recommendation, the classical acceleration–period format was applied. Reinhorn<sup>10</sup> demonstrated that, as an alternative to the use of elastic spectra with equivalent damping, inelastic demand spectra in the ADRS format can be applied within the capacity spectrum method. So, the advantages of the two approaches (i.e. the visual representation of the capacity spectrum method and the superior physical basis of inelastic demand spectra) can be combined. The procedure eliminates the controversial part of the original procedure. Neither equivalent viscous damping nor the period associated with the intersection of the capacity curve with the highly damped spectrum is used.

Using the Reinhorn's idea, the author of this paper has formulated the so-called N2 method, developed at the University of Ljubljana,<sup>11,12</sup> in the format of the capacity spectrum method. All equations given in Reference 12 remain valid. Looking from another perspective, the N2 method in the new format is, in fact, a variant of the capacity spectrum method based on inelastic spectra. Inelastic demand spectra are determined from a typical smooth elastic design spectrum. Reduction factors, that relate inelastic spectra to the basic elastic spectrum, are consistent with the elastic spectrum. A simple transformation from a Multi-Degree-Of-Freedom (MDOF) to an equivalent SDOF system is used. It turns out that, if the simple alternative for the reduction factor spectrum (equation 6(a)) is applied, the proposed method is equivalent to the displacement coefficient method presented in FEMA-273.

This paper contains no basic original developments. It just synthesises existing information and presents it in an easy to use format, which might be acceptable for practical design and for the development of future design guidelines.

The paper is mainly restricted to the proposed modification of the existing capacity spectrum method. The problems connected with the pushover analysis are not discussed, because they remain the same as in the original approach. Among them, the treatment of the contribution of higher modes is especially important and difficult. The reader may wish to consult, for instance, the paper by Krawinkler and Seneviratna,<sup>13</sup> in which pushover analysis is discussed in detail.

In the paper, it is also shown that different design and performance evaluation approaches, including deformation-controlled design, can be applied within the framework of the capacity spectrum method. Four quantities define structural behaviour: strength, displacement, ductility and stiffness. Design and/or performance evaluation begins by fixing two of them. The other two are determined by calculations. Different approaches differ in the quantities that are chosen at the beginning of the design or evaluation.

The force–displacement relationship, obtained by pushover analysis, corresponds to monotonically increasing loading. In order to use it for seismic design, ultimate capacity should be reduced due to cumulative damage effects. For this purpose the concept of equivalent ductility factors<sup>14</sup> can be employed. The procedure is briefly summarized in this paper.

Seismic demand can, in principle, be defined by any inelastic spectra. The specific demand spectra applied in this paper are based on statistical analyses, in which the near-fault, impulsive type of ground motion has not been included. Thus, the proposed spectra may not be adequate for this type of ground motion. Furthermore, care should be taken in the long-period

range, where actual displacements are typically constant, and in the very-long-period range, where the spectral displacements decrease to the level of the peak ground displacement. Displacement spectra are discussed in two recent papers.<sup>15,16</sup> According to Whittaker *et al.*,<sup>17</sup> the equal displacement rule is not conservative for systems with low strength (i.e. with a yield strength to required elastic strength ratio of less than 0.2). Additional research on these topics is needed.

The application of the modified capacity spectrum method is illustrated by means of two examples. It is shown that the capacity spectrum method can be used both for the seismic performance evaluation of newly designed or existing structures, and as a tool for the implementation of the direct displacement-based design approach, in which design starts from a predetermined target displacement.

### SEISMIC DEMAND IN ADRS FORMAT

For an elastic SDOF system the following relation applies:

$$S_{de} = \frac{T^2}{4\pi^2} S_{ae} \quad (1)$$

where  $S_{ae}$  and  $S_{de}$  are values in the elastic spectrum of (pseudo)-acceleration ('pseudo' will be omitted in the following text) and displacement, respectively, corresponding to the period  $T$  and a fixed viscous damping ratio. A typical smooth elastic acceleration spectrum for 5 per cent damping, normalised to a peak ground acceleration of 1.0 g, and the corresponding elastic displacement spectrum, are shown in Figure 2. For an inelastic SDOF system with a bilinear force-deformation relationship, the acceleration spectrum ( $S_a$ ) and displacement spectrum ( $S_d$ )

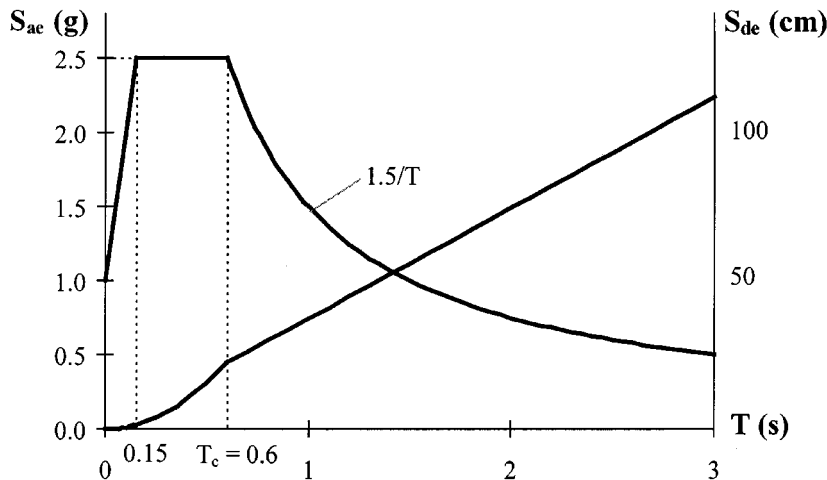


Figure 2. Typical elastic acceleration ( $S_{ae}$ ) and displacement spectrum ( $S_{de}$ ) for 5 per cent damping normalised to 1.0 g peak ground acceleration

can be determined<sup>18</sup> as

$$S_a = \frac{S_{ae}}{R_\mu} \quad (2)$$

$$S_d = \frac{\mu}{R_\mu} S_{de} = \frac{\mu}{R_\mu} \frac{T^2}{4\pi^2} S_{ae} = \mu \frac{T^2}{4\pi^2} S_a \quad (3)$$

where  $\mu$  is the ductility factor defined as the ratio between the maximum displacement and the yield displacement, and  $R_\mu$  is the reduction factor due to ductility, i.e. due to the hysteretic energy dissipation of ductile structures. Note that  $R_\mu$  is not equivalent to the reduction (behaviour, response modification) factor  $R$  used in seismic codes. The code reduction factor  $R$  takes into account both energy dissipation and the so-called overstrength  $R_s$ . It can be defined as  $R = R_\mu R_s$ .

Several proposals have been made for the reduction factor  $R_\mu$ . An excellent overview was made by Miranda and Bertero.<sup>19</sup> In this paper, we will make use of the formulae proposed by Vidic *et al.*<sup>18</sup> in slightly modified form. They not only provide reasonably accurate results, but are also very simple and suited for the use in the capacity spectrum method format.

$$R_\mu = (\mu - 1) \frac{T}{T_0} + 1, \quad T \leq T_0 \quad (4)$$

$$R_\mu = \mu, \quad T \geq T_0 \quad (5)$$

$$T_0 = 0.65 \mu^{0.3} T_c \leq T_c \quad (6)$$

$T_c$  is the characteristic period of the ground motion. It is typically defined as the transition period where the constant acceleration segment of the response spectrum passes to the constant velocity segment of the spectrum (Figure 2). It roughly corresponds to the period at which the largest energy is imparted to the structure.

Equations (4) and (5) define a bilinear  $R_\mu$  spectrum (Figure 3). In the medium- and long-period range the equal displacement rule applies (equations (3) and (5)), i.e. the displacement of the inelastic system is equal to the displacement of the corresponding elastic system with the same period. The transition period  $T_0$  depends on the ductility (equation (6)).  $T_0$  should not be larger than  $T_c$ . This limitation represents only a small modification regarding the original proposal in Reference 18.

Equations (4)–(6) were derived from a statistical study of a stiffness-degrading system with 10 per cent strain hardening and 5 per cent mass-proportional damping. A bilinear hysteretic model and damping proportional to instantaneous stiffness were also studied. From more detailed results presented by Vidic *et al.*<sup>18</sup> it can be concluded that equations (4)–(6) represent the approximate mean values of the different combinations studied. Studies performed at Stanford University (e.g. Reference 20) indicate only minor influence of the magnitude of the strain hardening ratio on  $R_\mu$ , provided that this ratio is positive.

An even simpler version of  $R_\mu$  spectra can be obtained by fixing the transition period to  $T_c$

$$T_0 = T_c \quad (6a)$$

This approach is, in the case of low ductility demand, conservative for short-period structures. However, it further simplifies the analysis procedure in the short-period range. Thus it may be

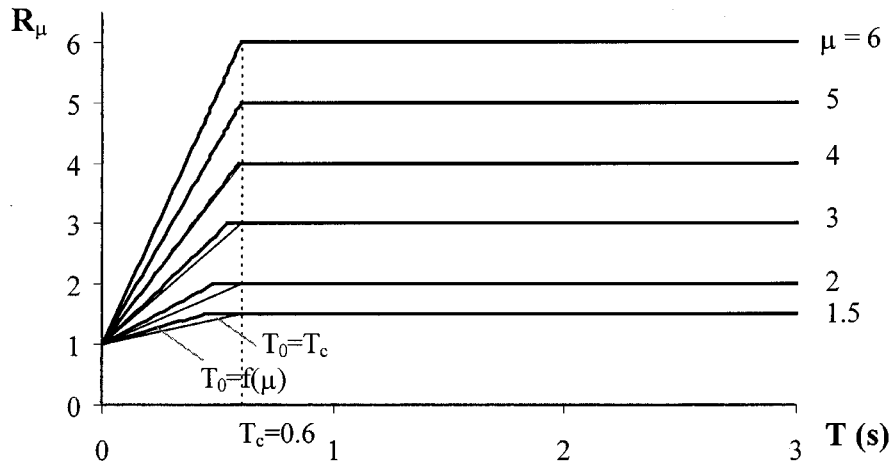


Figure 3. Ductility-dependent reduction factor  $R_\mu$  compatible with elastic acceleration spectrum in Figure 2

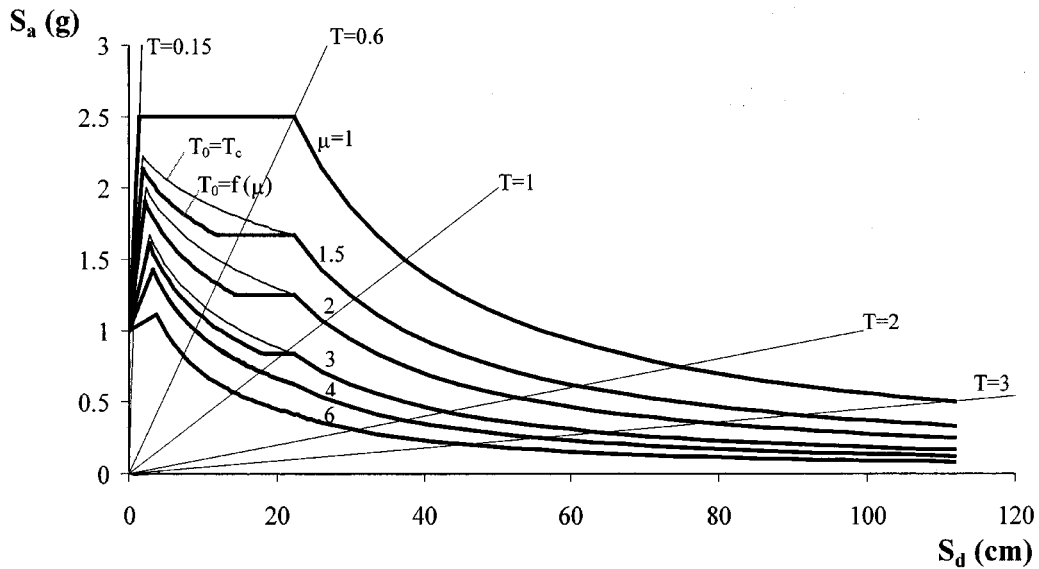


Figure 4. Demand spectra for constant ductilities in ADRS format

an attractive alternative for designers. Note that the formula for the modification factor  $C_1$  in FEMA-273 is based on the  $R_\mu$  spectra defined by equations (4), (5) and (6a).

Starting from the typical elastic design spectrum in Figure 1, and using equations (2)–(6) or (6a), the demand spectra for the constant ductility factors  $\mu$  in ADRS format can be obtained (Figure 4).

In principle, any realistic elastic and corresponding (compatible) inelastic spectrum can be applied. For example, for a specific acceleration time history, the elastic acceleration spectrum as well as the inelastic spectra which take into account specific hysteretic behaviour can be computed and used in the capacity spectrum method. However, elastic spectra for specific accelerograms should not be combined with smooth  $R_{\mu}$  spectra because they are not compatible. Furthermore, spectra which for different reasons deviate from the actual spectral shape should not be used. For example, the elastic acceleration spectra used as design spectra in codes are usually increased in the medium- and long-period ranges. These spectra are not appropriate as a basis for the determination of displacement spectra according to equation (1). They significantly overestimate displacements at longer periods.

The spectrum in Figure 2 has been intentionally cut off at the period  $T = 3$  s. At longer periods the displacement spectrum is typically constant. Consequently, the acceleration spectrum in the long-period range typically decreases with the square of the period  $T$ . Depending on the earthquake and site characteristics, the constant displacement range of the spectrum may begin even at shorter periods, e.g. at about 2 s.<sup>16</sup> In the very-long-period range, spectral displacements decrease to the value of the peak ground displacement. The actual features of general purpose elastic spectra in the long- and very-long-period ranges are still under investigation.

## THE MODIFIED METHOD

The modified capacity spectrum method consists of the following steps:

1. *Determine the base shear–top displacement relationship by a pushover analysis.* It is assumed that the lateral force in the  $i$ th storey is proportional to the component of the assumed displacement shape  $\Phi_i$  weighted by the storey mass  $m_i$

$$P_i = m_i \Phi_i \quad (7)$$

Such a distribution has a physical basis (inertia forces), and yields the simplest transformation from MDOF to SDOF systems (see Reference 12). However, any other reasonable distribution can also be used. The distribution remains constant during the pushover analysis.

2. *Transform the force–deformation relationship of the MDOF into that of an equivalent SDOF system.* Provided that the distribution of the lateral loading is defined by equation (7), the transformation of all quantities is performed by means of the equation

$$Q = \Gamma Q^* \quad (8)$$

where  $Q^*$  represents the quantities in the equivalent SDOF system (force  $F^*$ , displacement  $D^*$ , and hysteretic energy  $E_H^*$ , if needed), and  $Q$  represents the corresponding quantities in the MDOF system (base shear  $V$ , top displacement  $D_t$ , and hysteretic energy  $E_H$ ). The constant  $\Gamma$  is defined as

$$\Gamma = \frac{\sum m_i \Phi_i}{\sum m_i \Phi_i^2} \quad (9)$$

It is usually called modal participation factor. Note that the assumed displacement shape  $\Phi$  is normalized—the value at the top is equal to 1. Note also that any reasonable shape can be used for  $\Phi$ . Only in a special case  $\Phi$  represents the first mode shape.  $\Gamma$  is equivalent to  $PF_1$  in capacity spectrum method and to  $C_0$  in displacement coefficient method (ATC-40 and FEMA-273). The value in the numerator represents the mass of the equivalent SDOF system

$$m^* = \sum m_i \Phi_i \quad (10)$$

Since the same transformation is used both for forces and displacements, the initial stiffness of the equivalent SDOF system remains the same as that of the MDOF system.

3. *Idealize the force-displacement relationship of the equivalent SDOF system into an elastic – perfectly plastic form.* In this step, engineering judgement has to be used. In a regulatory document, some guidelines may be given. For example, it may be required that the effective lateral stiffness shall be taken as the secant stiffness calculated at a force equal to 60 per cent of the yield strength (FEMA-273).

Note that the graphical procedure requires the post-yield stiffness equal to zero. This is because the reduction factor  $R_\mu$  is defined as the ratio of the required elastic strength to the yield strength. The influence of moderate strain hardening is incorporated in the demand spectra. Once again it should be emphasized that moderate strain hardening does not have a significant influence on displacement demand. In the case of strain softening, however, the maximum displacements are increased.<sup>20</sup> Based on this observation, a multiplication factor ( $C_3$ ) was introduced in FEMA-273 into the method for determining the maximum displacements. The value of this factor is 1.0 in the case of a positive post-yield stiffness, whereas in the case of a negative post-yield stiffness  $C_3$  is greater than 1.0. Such a factor can be easily included in the formulae for the determination of demand spectra.

The elastic period of the idealised bilinear system  $T^*$  can be determined as

$$T^* = 2\pi \sqrt{\frac{m^* D_y^*}{F_y^*}} \quad (11)$$

where  $F_y^*$  and  $D_y^*$  are the yield strength and displacement, respectively.

If the forces in the force-deformation curve for the equivalent SDOF system are divided by the equivalent mass  $m^*$ , the acceleration-displacement relation (capacity curve) is obtained.

4. *Determine the seismic demand for the equivalent SDOF system.* The intersection of the radial line corresponding to the elastic stiffness of the idealised bilinear system and the elastic demand spectrum defines the strength required for elastic behaviour and the corresponding elastic displacement demand.

If the elastic period  $T^*$  is larger than  $T_0$ , determined according to equation (6) or equation (6a) depending on the representation of seismic demand, the inelastic displacement demand is equal to the elastic one. The ductility demand is equal to the reduction factor and it can be obtained from the graph. If  $T^*$  is smaller than  $T_0$  and the target ductility is low, iteration is required unless a simple conservative assumption (6a) is used.

If the elastic period of the system is smaller than  $T_0$ , the reduction factor  $R_\mu$  is first determined, as the ratio between the elastic acceleration  $S_{ae}$  and the yield acceleration  $S_{ay}$ , representing the acceleration demand of the inelastic system. The ductility demand is then



calculated from the rearranged equation (4)

$$\mu = (R_\mu - 1) \frac{T_0}{T^*} + 1 \quad (4a)$$

The displacement demand is determined as

$$D^* = S_d = \mu D_y^* \quad (12)$$

In both cases ( $T^* \leq T_0$  and  $T^* \geq T_0$ ) the inelastic demand in terms of accelerations and displacements corresponds to the intersection point of the capacity spectrum and the demand spectrum corresponds to the ductility demand.

5. *Check performance at the expected maximum displacement.* The procedure is the same as in the original method. First, the displacement has to be transformed back from the SDOF to the MDOF system (equation (8)). Then the performance at maximum displacement is evaluated on the global and local level.

If equation (6a) is used, the displacement demand according to the proposed procedure corresponds to the displacement demand determined by the displacement coefficient method (ATC-40 and FEMA-273). This is because the constant  $\Gamma$  (equation (9)) corresponds to the coefficient  $C_0$ , whereas the ratio  $\mu/R_\mu$  corresponds to the coefficient  $C_1$ . The coefficients  $C_2$  and  $C_3$  are not included in the proposed procedure, i.e. they are set equal to 1. However, if so desired, they can easily be incorporated into the procedure.

## DEFORMATION CONTROLLED DESIGN

At the International Workshop on Seismic Design Methodologies for the Next Generation of Codes,<sup>21</sup> the following recommendations, inter alia, were made:

- (1) 'Future seismic engineering practice should be based on explicit and quantifiable performance criteria, considering multiple performance and hazard levels'.
- (2) 'The most suitable approach for seismic design to achieve the objectives of performance based engineering appears to be deformation controlled design. It is recommended that deformation controlled design be implemented in future codes, both by enhancing force-based design through verification of deformation targets and by the development of direct deformation based design approaches'.

Both force-based and deformation-based approaches can be easily implemented within the capacity spectrum method. There are four important parameters which define structural behaviour: strength, displacement, ductility, and elastic stiffness. In the capacity spectrum method these characteristics are represented by the acceleration  $S_a$ , the displacement  $S_d$ , the ductility factor  $\mu$ , and the elastic period  $T$ . The four quantities are related by equation (3). An additional relationship defines seismic acceleration demand as a function of  $\mu$  and  $T$  (equations (2), (4)–(6)). Seismic demand in terms of the elastic acceleration spectrum  $S_{ae}$  is defined in the input data. Consequently, there are two equations (relations) for four unknown quantities. It follows that two quantities need to be chosen, and the other two computed. In different approaches, different quantities are chosen at the beginning (Table I). The usual force-based design typically starts by assuming the period and the ductility capacity. Then the seismic forces (related to strength)

Table I. Assumed and determined structural characteristics in different approaches

	Force-based design	Displacement-based design	Performance evaluation
Assumed	Period (stiffness) Ductility	Displacement Ductility	Stiffness Strength
Determined	Strength Displacement	Period (stiffness) Strength	Displacement Ductility

are determined, and finally the displacement demand is calculated. In a performance evaluation procedure, the strength and the period are known, whereas the displacement and ductility demands are calculated. In direct displacement-based design, the starting points are typically displacement and ductility demands. The quantities to be determined are period and strength.

It should be noted that in all cases the strength corresponds to the actual strength and not to the design-base shear in seismic codes. Design-base shear is, for a number of reasons, in all practical cases less than the actual strength. The ratio between the actual and design strength is called the overstrength factor  $R_s$ .

### CUMULATIVE DAMAGE CONSIDERATIONS

In some cases, especially for existing structures with poor detailing subjected to long-duration ground motions, cumulative damage effects caused by several inelastic cycles may be very important. These effects can be easily incorporated into the described procedures by using the so-called 'equivalent ductility factors'.<sup>14</sup> The idea behind the equivalent ductility factor is to reduce the monotonic deformation capacity of an element and/or a structure as a consequence of cumulative damage due to the dissipation of hysteretic energy. The relation between the ultimate monotonic and cyclic deformation capacity depends on the damage model used. In the case of the Park–Ang damage model,<sup>22</sup> the following relation applies:<sup>14</sup>

$$\frac{\mu_c}{\mu_m} = \frac{1}{1 + \beta\gamma^2\mu_c} \quad (13)$$

where  $\mu_c$  and  $\mu_m$  represent the ductility capacity under cyclic (earthquake) and monotonic loading, respectively. The same ratio applies to displacements. The ratio depends on the amplitude of vibrations in terms of the ultimate ductility  $\mu_c$  and on two additional non-dimensional parameters. The first of these is the structure-dependent parameter  $\beta$ , which defines the strength degradation due to cyclic loading in the Park–Ang model. The second one is the parameter  $\gamma$ , which is related to the ratio between the dissipated hysteretic energy and the maximum displacement. It depends mainly on the characteristic period of the ground motion and on the duration of the strong part of the ground motion. In addition, it depends slightly on the hysteretic behaviour and on the period of the ground motion. In an average case ( $\beta = 0.15$ ,  $\gamma = 1.0$ ,  $\mu_c = 4$ )

the ratio between the two ductilities  $\mu_c$  and  $\mu_m$  amounts to 0.625. Consequently, in order to assure a ductility capacity under earthquake loading equal to  $\mu = \mu_c = 4$ , a monotonic ductility capacity of  $\mu_m = 6.4$  should be provided.

Following the concept of equivalent deformation capacity the effect of cumulative damage can be easily taken into account in the capacity spectrum method by appropriately reducing the capacity determined by pushover analysis at both the global (structure) and local (element) level.

## ILLUSTRATIVE EXAMPLES

In order to demonstrate the applicability of the proposed method, a four-storey reinforced concrete frame building has been analysed. Full-scale pseudo-dynamic tests were conducted on the building in the European Laboratory for Structural Assessment (ELSA) of the Joint Research Centre of the European Commission in Ispra. The bare frame building was designed according to Eurocodes 2 and 8 as a high ductility class structure for a peak ground acceleration of 0.3 g and medium soil conditions. The design-base shear coefficient amounted to 0.15. The storey heights are 3.5 m in the first storey and 3.0 m in the storeys above. The storey masses from the bottom to the top amount to 87, 86, 86, and 83 t.

By using the proposed method we will estimate the top displacement. The ground motion is defined by a peak ground acceleration twice as large as the design peak ground acceleration, i.e.  $a_g = 0.6$  g, and an elastic acceleration response spectrum according to Eurocode 8, for subsoil class B ( $T_c = 0.6$  s) and 5 per cent damping. The normalized spectrum is shown in Figure 2.

A linear displacement shape is assumed:  $\Phi^T = [0.28, 0.52, 0.76, 1.00]$

The lateral force pattern is obtained from equation (1), and normalized so that the force at the top is equal to 1.0:  $\mathbf{P}^T = [0.293, 0.539, 0.787, 1.000]$

With this force pattern, pushover analysis yields the base shear–top displacement relationship shown in Figure 5.

The MDOF system is transformed to an equivalent SDOF system using equation (8). The equivalent mass amounts to  $m^* = 217$  t (equation (10)), and the transformation constant is  $\Gamma = 1.34$  (equation (9)).

A bilinear idealization of the pushover curve (Figure 5) is based on the following considerations:

- (1) The post-yield stiffness is equal to zero.
- (2) The areas under the original and idealized curve within the range of interest are approximately equal.
- (3) The two curves cross at a force equal to 60 per cent of the yield strength.
- (4) The resulting yield strength and displacement amount to  $F_y^* = 830$  kN and  $D_y^* = 6.1$  cm. The elastic period is  $T^* = 0.79$  s.

The capacity curve (Figure 5) is obtained by dividing the forces in the idealized pushover curve by the equivalent mass. The acceleration at the yield point amounts to  $S_{ay} = F_y^*/m^* = 830/217 = 3.82$  m/s<sup>2</sup> = 0.39 g.

The capacity curve and demand spectra are compared in Figure 6. Equations (4)–(6) were used to obtain the inelastic demand spectra.

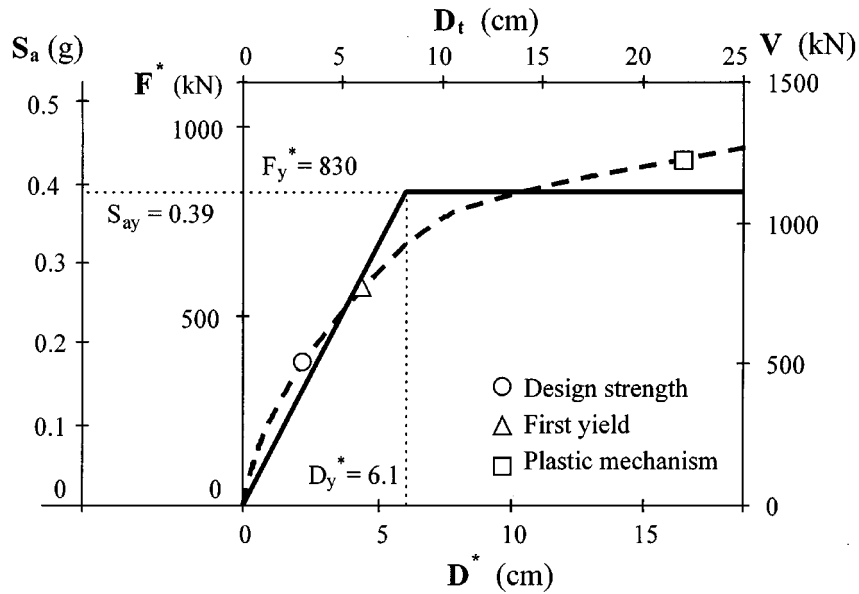


Figure 5. Pushover curve and corresponding capacity spectrum for the 4-storey RC frame

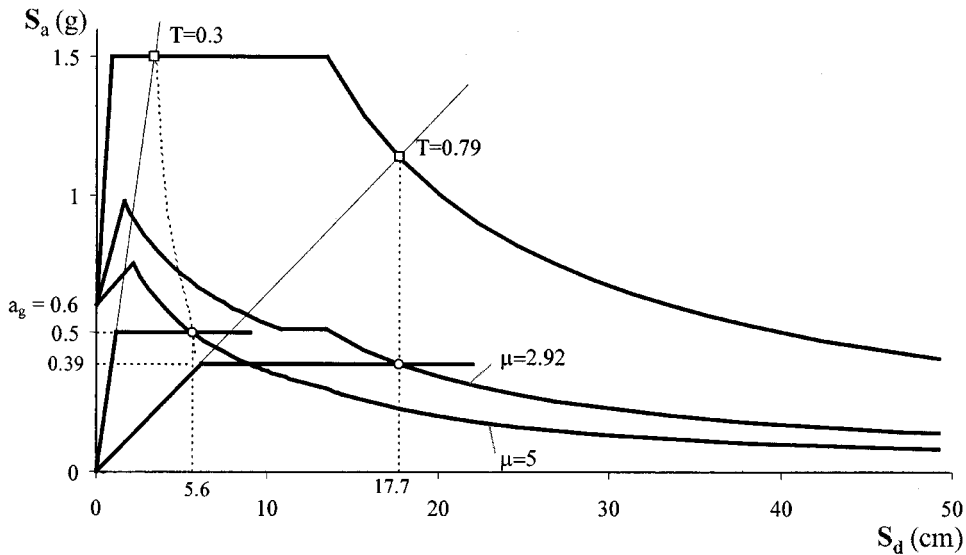


Figure 6. Demand and capacity spectra for two illustrative examples

In the case of unlimited elastic behaviour of the structure, seismic demand is represented by the intersection of the elastic demand spectrum and the line corresponding to the elastic period ( $T^* = 0.79$  s) of the equivalent SDOF system. The values  $S_{ae} = 1.14$  g and  $S_{de} = 17.7$  cm are obtained. The reduction factor  $R_\mu$  amounts to  $R_\mu = S_{ae}/S_{ay} = 1.14 \text{ g}/0.39 \text{ g} = 2.92$ .

The period of the system  $T^* = 0.79$  is larger than  $T_c = 0.6$ . For the inelastic spectra used,  $T_c$  will always be larger than  $T_0$ . Thus the equal displacement rule (equations (3) and (5)) applies:  $\mu = R_\mu = 2.92$ ,  $S_d = S_{de} = 17.7$  cm.

The seismic demand for the equivalent SDOF system is graphically represented by the intersection of the capacity curve and the demand spectrum for  $\mu = 2.92$ . Note, however, that the inelastic seismic demand can be determined without constructing the demand spectra.

In the next step the displacement demand of the equivalent SDOF system is transformed back to the top displacement of the MDOF system (equation (8)):  $D_t = 1.34 \times 17.7 = 23.7$  cm.

The next steps of analysis include determination of local seismic demands, comparisons with available capacity (here reductions due to cumulative damage can be taken into account) and estimation of local and global damping indices. Discussion of these steps is, however, outside the scope of this paper.

The computed top displacement agrees well with both the experimental top displacement (21 cm) and the top displacements determined by non-linear time-history analyses (22–30 cm).<sup>2,3</sup> These results were obtained with a smaller damping (1 per cent) and a smaller intensity of excitation ( $a_g = 0.45$  g).

In the example, a performance evaluation of an existing (already designed) building was made. In direct displacement-based design the procedure is reversed (Figure 7(a)). Let us start from a target displacement of  $D_t = 23.7$  cm. The corresponding displacement of the equivalent SDOF system is  $S_d = 23.7/1.34 = 17.7$  cm. From the spectrum in Figure 6 it is evident that the period of the structure is in the medium-period range. Thus, the equal displacement rule applies. The period of the structure corresponds to the radial line through the intersection of the elastic spectrum and the vertical line corresponding to the constant displacement  $S_d = 17.7$  cm. The values  $S_{ae} = 1.14$  g and  $T^* = 2\pi\sqrt{(S_d/S_{ae})} = 0.79$  s are obtained. An infinite number of structures exist which comply with the target displacement. In the next step, either ductility or strength has to be assumed. For example, if we assume ductility  $\mu = 2.92$ , the acceleration

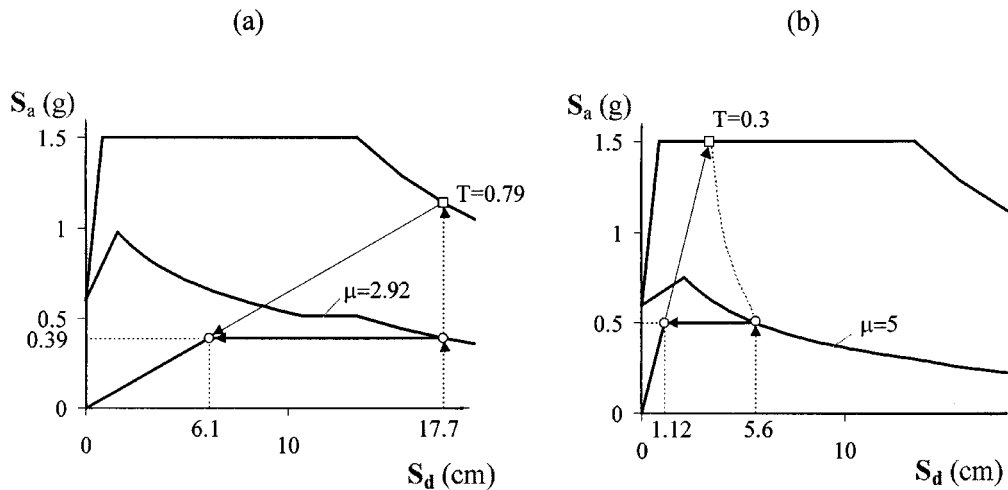


Figure 7. Direct displacement-based method in capacity spectrum method format (a) 4-storey RC frame, (b) single-storey structure

demand is  $S_a = S_{ae}/2.92 = 0.39$  g. The required base shear of the building is equal to  $V = \Gamma m^* S_a = 1112$  kN.

The next example is intended to demonstrate the procedure in the case of a short-period structure. We will analyse a SDOF system with a yield strength of  $F_y = F_y^* = 0.5 mg$  (where  $m$  is the mass and  $g$  is the acceleration of gravity), and a yield displacement of  $D_y = D_y^* = 1.12$ . The yield acceleration  $S_{ay}$  amounts to 0.5 g. Seismic demand is represented by the same demand spectra as in the first example.

The elastic period of the system amounts to  $T^* = 0.3$  s. The capacity curve is plotted in Figure 6. The seismic demand in terms of the acceleration and displacement of an elastic structure with the same period amounts to 1.5 g and 3.35 cm, respectively. The reduction factor  $R_\mu$  is equal to 3.0. We assume that the ductility demand is greater than about 4.0 and, as a consequence,  $T_o = T_c = 0.6$  (equation (6)). The ductility demand is obtained using equation (4a) ( $\mu = 5.0$ ). The assumption  $T_o = T_c$  turns out to be correct. The displacement demand is computed from equation (12) ( $D = D^* = S_d = 5.0 \times 1.12 = 5.6$  cm). Again, the seismic demand is graphically represented by the intersection of the capacity curve and the demand spectrum for  $\mu = 5.0$  (Figure 6).

In the direct displacement-based design the procedure is reversed, and we start with the target displacement  $S_d = 5.6$  cm (Figure 7b). The spectra indicate that the period of the structure is within the short-period range. Thus the equal displacement rule does not apply. In order to determine the period, either the ductility or acceleration demand has to be assumed. Let us assume a ductility of  $\mu = 5$ . The acceleration demand ( $S_a = 0.5$  g) corresponds to the intersection of the vertical line ( $S_d = 5.6$  cm) and the inelastic demand spectrum for  $\mu = 5$ . The yield displacement is equal to  $5.6/5 = 1.12$  cm and the period is  $T^* = 2\pi\sqrt{(1.12/981/0.5)} = 0.3$  s.

## CONCLUSIONS

The seismic demand in the capacity spectrum method can be represented by inelastic spectra. In principle, any realistic inelastic spectra can be used. However, they should be compatible with the basic elastic spectrum. The specific demand spectra applied in this paper are simple and reasonably accurate for a broad range of design situations. It has been shown that the performance evaluation procedure, called the N2 method, can be formulated in the format of the capacity spectrum method. Furthermore, by reversing the procedure, a direct deformation-based design can be performed.

## ACKNOWLEDGEMENTS

The author is grateful to Professors M. Fischinger and H. Krawinkler for stimulating discussion and for providing very useful comments and suggestions. The results presented in this paper are based on work continuously supported by the Ministry for Science and Technology of the Republic of Slovenia. This support is gratefully acknowledged.

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