EARTHQUAKE ENGINEERING AND STRUCTURAL DYNAMICS
Earthquake Engng Struct. Dyn. 2000; **29**:1759–1778

Critical response of structures to multicomponent earthquake excitation

Oscar A. Lopez[1,*], Anil K. Chopra[2] and Julio J. Hernandez[3]

[1] *IMME, Facultad de Ingenieria, Universidad Central de Venezuela, Urb. Santa Paula, Ave. Circunvalacion del Sol, Res. 18, # 10A, Caracas, Venezuela*
[2] *Department of Civil and Environmental Engineering, University of California, Berkeley, U.S.A.*
[3] *CDCH, Universidad Central de Venezuela, Caracas, Venezuela*

SUMMARY

This paper aims to develop an improved understanding of the critical response of structures to multicomponent seismic motion characterized by three uncorrelated components that are defined along its principal axes: two horizontal and the vertical component. An explicit formula, convenient for code applications, has been derived to calculate the critical value of structural response to the two principal horizontal components acting along any incident angle with respect to the structural axes, and the vertical component of ground motion. The critical response is defined as the largest value of response for all possible incident angles. The ratio r_{cr}/r_{srss} between the critical value of response and the SRSS response—corresponding to the principal components of ground acceleration applied along the structure axes—is shown to depend on three dimensionless parameters: the spectrum intensity ratio γ between the two principal components of horizontal ground motion characterized by design spectra $A(T_n)$ and $\gamma A(T_n)$; the correlation coefficient α of responses r_x and r_y due to design spectrum $A(T_n)$ applied in the x- and y-directions, respectively; and $\beta = r_y/r_x$. It is demonstrated that the ratio r_{cr}/r_{srss} is bounded by 1 and $\sqrt{(2/1+\gamma^2)}$. Thus the largest value of the ratio is $\sqrt{2}$, 1.26, 1.13 and 1.08 for $\gamma = 0$, 0.5, 0.75 and 0.85, respectively. This implies that the critical response never exceeds $\sqrt{2}$ times the result of the SRSS analysis, and this ratio is about 1.13 for typical values of γ, say 0.75. The correlation coefficient α depends on the structural properties but is always bounded between -1 and 1. For a fixed value of γ, the ratio r_{cr}/r_{srss} is largest if $\beta = 1$ and $\alpha = \pm 1$. The parametric variations presented for one-storey buildings indicate that this condition can be satisfied by axial forces in columns of symmetric-plan buildings or can be approximated by lateral displacements in resisting elements of unsymmetrical-plan buildings. Copyright © 2000 John Wiley & Sons, Ltd.

KEY WORDS: critical response; multicomponent motion; critical angles; correlated responses; uncorrelated components; complete quadratic combination rules (CQC3)

1. INTRODUCTION

The response of structures to multicomponent ground motion has been examined in several publications [1–9]. Translational ground motion is decomposed usually into three components: two in the

* Correspondence to: Oscar A. Lopez, IMME, Facultad de Ingenieria, Universidad Central de Venezuela, Urb. Santa Paula, Ave. Circunvalacion del Sol, Res. 18, # 10A, Caracas, Venezuela.

Received 30 September 1999
Revised 5 April 2000
Accepted 7 April 2000

horizontal plane and one in the vertical direction, with rotational ground motion neglected. When defined along a special orthogonal system of axes, the ground motion components are uncorrelated [10, 11]. This system of axes, defined as the principal axes of ground motion, are oriented such that the major principal axis is horizontal and directed toward the epicentre of the earthquake, the intermediate principal axis is in the orthogonal principal direction, and the minor principal axis is vertical [10]. The components of the ground motion along any other orthogonal system of axes are obviously correlated.

Because the location of the epicentre is not known, it is necessary to determine the response of a structure for all possible orientations of the principal axes and design for the largest or critical response. To determine this response, the CQC3 rule has been developed, which describes the structural response as a function of the incident angle (the angle between the principal axes of ground motion and the reference axes of the structure) of seismic components described in terms of response spectra [6–8]. Because the CQC3 equation provides a simple formula for determining the critical angle, it is not necessary to determine the response for various values of the incident angle. The CQC3 equation, evaluated numerically for this critical angle, provides the critical response.

The overall objective of this paper is to develop an improved understanding of the critical response of structures to multicomponent ground motion. The specific objectives are as follows: (1) develop an explicit formula for the critical response in terms of the responses to single components of ground motion applied separately along the three structural axes and the correlation between these responses; (2) develop an upper bound for the critical response; and (3) identify the ground motion and system parameters that influence the critical response and the variation of the response with the incident angle. The first part of the paper presents results applicable to any structure, followed by a parametric study of one-story building systems.

2. CRITICAL RESPONSE OF STRUCTURES

2.1. Earthquake excitation

The excitation is defined in terms of design spectra associated with the principal directions of the translational components of ground motion, which are oriented along the two horizontal axes 1 and 2 and the vertical axis z, as shown in Figure 1. The pseudo-acceleration spectra are denoted as $A(T_n)$ for the major principal axis, $\gamma A(T_n)$ for the intermediate principal axis, and $A_z(T_n)$ for the minor principal axis; T_n is the natural vibration period of a single-degree-of-freedom system.

Note that the design spectra in the two horizontal directions have the same shape and differ by the ratio γ of spectrum intensities where $0 \leqslant \gamma \leqslant 1$. The ground acceleration components along the principal axes (1, 2, and z) are assumed to be uncorrelated [10, 11]. They do correlate, however, if defined along any other set of axes, for example, along x, y, and z (the reference axes of the structure). As shown in Figure 1, θ denotes the orientation of the earthquake's major principal axis relative to structural axis x. Defined as the incident angle of the ground motion, θ in the counter-clockwise direction is taken to be positive.

2.2. Structural response-incident angle relation

The peak response of a structure to a single component of ground motion applied along one of the structure axes is commonly evaluated using the response spectrum method. Accounting for

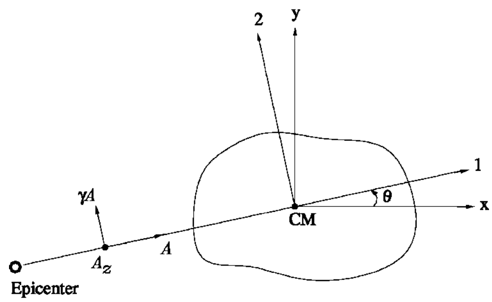

Figure 1. Definition of principal axes of ground motion, structural axes, and θ in the horizontal plane.

the correlation among ground motion components mentioned above, these individual responses are combined using the CQC3 rule to obtain the mean peak value $r(\theta)$ of the total response [6–8], where r is a specific response quantity that can be expressed as a linear function of the nodal displacements of the structure

$$r(\theta) = \{[r_x^2 + (\gamma r_y)^2]\cos^2\theta + [(\gamma r_x)^2 + r_y^2]\sin^2\theta + 2(1-\gamma^2)r_{xy}\sin\theta\cos\theta + r_z^2\}^{1/2} \tag{1}$$

where r_x and r_y are the mean peak values of response quantity r due to a single component of ground motion defined by the spectrum $A(T_n)$ applied first along the x-direction and then along the y-direction, respectively; and r_z is the mean peak value of r due the vertical component of ground motion defined by the spectrum $A_z(T_n)$. The peak response, r_k $(k = x, y, z)$, to these individual components of ground motion is given by the CQC combination rule [12]

$$r_k = \left\{\sum_i \sum_j \rho_{ij} r_{ki} r_{kj}\right\}^{1/2} \tag{2}$$

where r_{ki} is the peak response due to the ith natural mode of vibration, and ρ_{ij} is the modal correlation coefficient for modes i and j. The term r_{xy} in Equation (1) is a cross-term of the modal responses that contribute to r_x and r_y:

$$r_{xy} = \sum_i \sum_j \rho_{ij} r_{xi} r_{yj} \tag{3}$$

Equations (2) and (3) can be written in terms of the modal static responses [13] as follows:

$$r_k = \left\{\sum_i \sum_j \rho_{ij} r_{ki}^{\mathrm{st}} r_{kj}^{\mathrm{st}} A_{ki} A_{kj}\right\}^{1/2} \qquad r_{xy} = \sum_i \sum_j \rho_{ij} r_{xi}^{\mathrm{st}} r_{yj}^{\mathrm{st}} A_{xi} A_{yj} \tag{4}$$

where A_{ki} is the spectral acceleration for the ith mode, $k = x$, y and z, and r_{ki}^{st} is the ith modal static response associated with ground motion in the kth direction.

2.3. SRSS response

If the principal components of ground acceleration are applied along the structural axes, the response is given using Equation (1) with $\theta = 0°$ when the major principal component is oriented in the

x-direction, and Equation (1) with $\theta=90°$ when the major principal component is oriented in the y-direction. Therefore,

$$r(\theta=0°)=\{r_x^2+(\gamma r_y)^2+r_z^2\}^{1/2}, \qquad r(\theta=90°)=\{(\gamma r_x)^2+r_y^2+r_z^2\}^{1/2} \tag{5}$$

These equations represent the SRSS combination of responses to the individual components of ground motion. Here, the larger of these two response values will be defined as the SRSS response, r_{srss}:

$$r_{\text{srss}}=\max[r(\theta=0°),\ r(\theta=90°)] \tag{6}$$

2.4. *The cross-term r_{xy} and correlation coefficient α*

If the principal components of ground motion coincide with the structural axes, i.e. the incident angle is $\theta=0$ or $90°$, the total response is given by Equation (5) (the SRSS rule) by combining uncorrelated responses to the three uncorrelated components of ground motion. For other values of θ, the responses are correlated, as indicated by the term containing r_{xy} in Equation (1). Observing the structure of Equation (3) for r_{xy} and Equation (2) for r_x and r_y, it is apparent that r_{xy} measures the correlation between responses r_x and r_y to ground motions that are perfectly correlated.

The correlation coefficient α for responses r_x and r_y is defined as

$$\alpha=\frac{r_{xy}}{r_x r_y} \tag{7}$$

which is defined for $r_x \neq 0$ and $r_y \neq 0$. For any structure and any spectral shape, α is bounded as follows (see Appendix A):

$$-1 \leqslant \alpha \leqslant 1 \tag{8}$$

The limiting values of α, 0 and ± 1, denote that responses r_x and r_y (to perfectly correlated ground motions) are uncorrelated and perfectly correlated, respectively.

2.5. *Critical angles*

Because the value of θ may not be known, it is prudent to design for that value of θ that gives the largest response. Differentiating Equation (1) with respect to θ and setting the derivative equal to zero gives the critical values of the incident angle [6–8].

$$\theta_{\text{cr}}=\frac{1}{2}\tan^{-1}\left[\frac{2r_{xy}}{r_x^2-r_y^2}\right] \tag{9}$$

Equation (9) leads to two values of θ between 0 and 180°, separated by 90°, which give the maximum ($r_{\max}$) and minimum ($r_{\min}$) response values. Note that the critical values of θ are independent of the intensity ratio γ between the horizontal components of ground motion and are not influenced by the vertical component; in particular, θ_{cr} is the same whether the ground motion contains one horizontal component or two components. For the special case of $\gamma=1$ (i.e. two equally intense horizontal components of ground motion), Equation (1) indicates that the response is independent of the incident angle.

Equation (9) can be rewritten in terms of two dimensionless parameters, α and β:

$$\tan(2\theta_{\mathrm{cr}}) = \frac{2\alpha\beta}{1-\beta^2} \tag{10}$$

where the response correlation coefficient α is given by Equation (7), and β is a positive parameter defined as the ratio of responses r_y and r_x:

$$\beta = \frac{r_y}{r_x} \tag{11}$$

Observe that $\theta_{\mathrm{cr}} = 0°$ if either one of α or β is equal to zero; if $\beta = 1$, $\theta_{\mathrm{cr}} = 45$ or $135°$, depending on the sign + or −, respectively, of the correlation coefficient α.

2.6. *Explicit formula for the critical response*

To determine $r_{\max}$ and $r_{\min}$, the maximum and the minimum values of $r(\theta)$ in Equation (1), usually the two numerical values of θ_{cr} determined from Equation (9) are substituted for θ. We can, however, derive explicit equations for $r_{\max}$ and $r_{\min}$ by recognizing that they represent the combined response to three components of ground motion acting in directions 1, 2 and z, with $\theta = \theta_{\mathrm{cr}}$, as shown in Figure 1. If the responses to these uncorrelated individual components of ground motion are denoted by r_1, γr_2 and r_z, respectively, the combined response is given by the SRSS rule

$$r_{\max} = \{r_1^2 + (\gamma r_2)^2 + r_z^2\}^{1/2}, \qquad r_{\min} = \{(\gamma r_1)^2 + r_2^2 + r_z^2\}^{1/2} \tag{12}$$

To determine r_1 and r_2, we specialize Equation (1) for a single horizontal component of ground motion by substituting $\gamma = 0$, delete the response to the vertical component, and substitute for $\sin(\theta_{\mathrm{cr}})$ and $\cos(\theta_{\mathrm{cr}})$ determined from Equation (9). The result is similar to the equations for principal stresses found in textbooks in mechanics:

$$r_{1,2} = \left\{ \frac{r_x^2 + r_y^2}{2} \pm \sqrt{\left(\frac{r_x^2 - r_y^2}{2}\right)^2 + r_{xy}^2} \right\}^{1/2} \tag{13}$$

Finally, by substituting Equation (13) into Equation (12), we obtain the critical response r_{cr}:

$$r_{\mathrm{cr}} = r_{\max} = \left\{ (1+\gamma^2)\left(\frac{r_x^2 + r_y^2}{2}\right) + (1-\gamma^2)\sqrt{\left(\frac{r_x^2 - r_y^2}{2}\right)^2 + r_{xy}^2} + r_z^2 \right\}^{1/2}. \tag{14}$$

The explicit formula given by Equation (14) is convenient for design purposes, especially code applications, because it avoids computation of the two critical angles, as required in previous works [6–8], and provides a rational basis to determine the critical response from r_x, r_y, and r_{xy}. The first two represent responses to an individual component of ground motion applied along the x- and

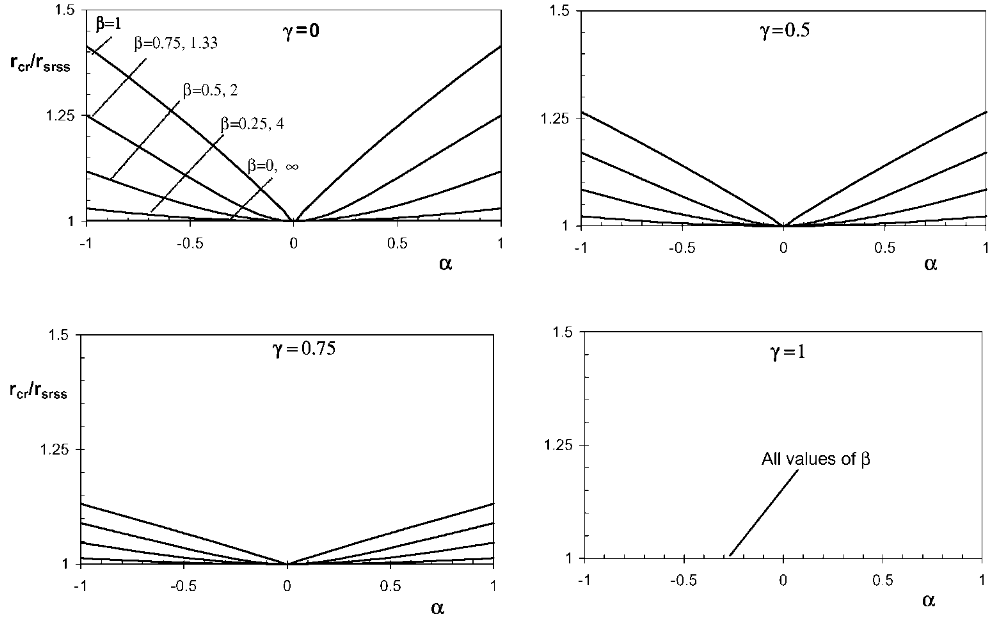

Figure 2. Ratio of critical and SRSS values of response as a function of γ for selected values of α and β.

the y-axis of the structure, respectively, responses that are commonly evaluated by the response spectrum method; r_{xy} is a measure of correlation between responses r_x and r_y. In addition, Equation (14) is not computationally demanding, requiring calculation of r_{ki}^{st}, A_{ki}, and ρ_{ij}, which are readily available if the conventional CQC modal combination rule is implemented in the dynamic analysis software to calculate r_x and r_y.

2.7. Bounds for the critical response

The ratio of the critical response due to horizontal ground motion, Equation (14) with $r_z = 0$, and the response from SRSS analysis (Equations (5) and (6)), is given by

$$\frac{r_{\text{cr}}}{r_{\text{srss}}} = \left\{ \frac{(1+\gamma^2)((1+\beta^2)/2) + (1-\gamma^2)\sqrt{((1-\beta^2)/2)^2 + (\alpha\beta)^2}}{1+(\gamma\beta)^2 \text{ or } \gamma^2+\beta^2} \right\}^{1/2} \tag{15}$$

wherein the denominator has two alternatives expressions: the first is valid if $r(\theta = 0°) \geqslant r(\theta = 90°)$, implying that $r_x \geqslant r_y$ or $\beta \leqslant 1$; the second applies if $r(\theta = 0°) \leqslant r(\theta = 90°)$, implying that $r_x \leqslant r_y$ or $\beta \geqslant 1$. Note that the ratio $r_{\text{cr}}/r_{\text{srss}}$ depends on dimensionless parameters α, β, and γ. It can be shown that $r_{\text{cr}}/r_{\text{srss}}$ is identical for β values that are reciprocal to each other.

Figure 2 plots Equation (16) as a function of correlation coefficient α for several values of β and four values of γ: as the spectrum intensity ratio γ increases, we would expect r_{srss} and r_{cr} to increase, however, $r_{\text{cr}}/r_{\text{srss}}$ decreases. For $\gamma = 1$, $r_{\text{cr}}/r_{\text{srss}} = 1$, independent of α and β, implying that the SRSS analysis is correct only if both horizontal components of ground motion have the same intensity. For fixed values of $\gamma < 1$ and β, the response ratio $r_{\text{cr}}/r_{\text{srss}}$ is largest at $\alpha = \pm 1$, i.e. when

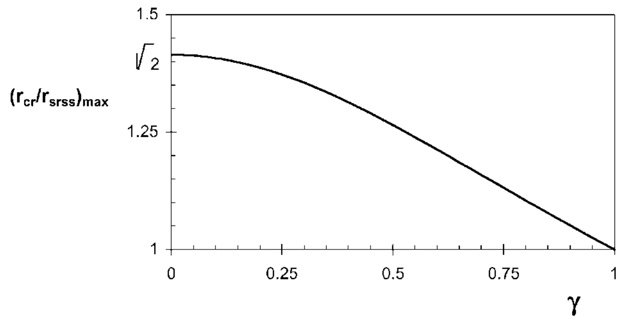

Figure 3. Upperbound for the ratio of critical and SRSS values of response as a function of the spectrum intensity ratio γ.

responses r_x and r_y are perfectly correlated. Among all values of β, r_{cr}/r_{srss} is largest for $\beta = 1$, i.e. $r_x = r_y$. For $\alpha = 0$, $r_{cr}/r_{srss} = 1$, independent of β and γ, implying that the SRSS analysis is correct if responses r_x and r_y are uncorrelated. For a fixed value of γ, r_{cr}/r_{srss} is largest when $\alpha = \pm 1$ and $\beta = 1$, simultaneously; the latter condition implies that $\theta_{cr} = 45$ or $135°$.

The critical response is bounded as follows (see Appendix B):

$$r_{srss} \leqslant r_{cr} \leqslant r_{srss}\sqrt{\frac{2}{1+\gamma^2}} \tag{16}$$

Plotted as a function of γ (see Figure 3), the upper bound value, $(r_{cr}/r_{srss})_{max}$, has values $\sqrt{2}$, 1.26, 1.13, and 1.08 for $\gamma = 0$, 0.5, 0.75, and 0.85, respectively, implying that the critical response value does not exceed $\sqrt{2}$ times the resut of the SRSS analysis [14]. For typical values of the spectrum intensity ratio γ, say 0.75, this ratio is 1.13. Equation (16) and the resulting conclusions are valid for any structure and any spectral shape $A(T_n)$.

3. ONE-STOREY SYMMETRICAL-PLAN BUILDINGS

3.1. System and ground motion

Consider an idealized one-storey system with a rigid square slab of mass m supported on four massless columns clamped at the slab and at the base (Figure 4). The height of the columns is 0.4 times the bay length, L. The first natural vibration mode involves uncoupled motion in the x-direction at period T_x, and the second mode describes uncoupled motion in the y-direction at period T_y. The damping ratio ζ is assumed to be 5 per cent in both modes.

The ground motion consists of two horizontal components. The major principal component is defined by the pseudo-acceleration design spectrum [15] shown in Figure 5, for peak ground acceleration, velocity, and displacement equal to 0.5 g, 24 in/s and 18 in, respectively, 84.1 percentile response and 5 per cent damping. The intermediate principal component of ground motion is defined by γ times the spectrum shown in Figure 5.

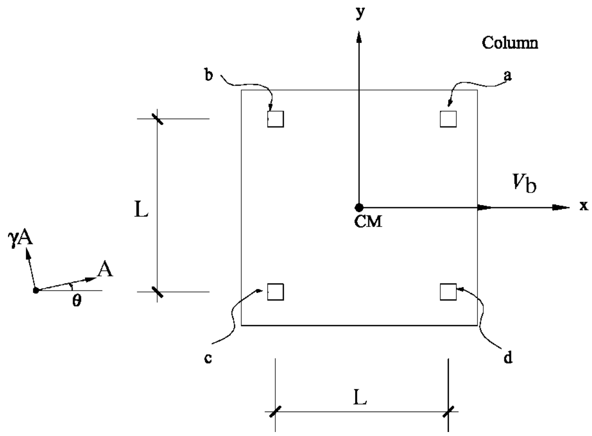

Figure 4. One-storey square building with four columns.

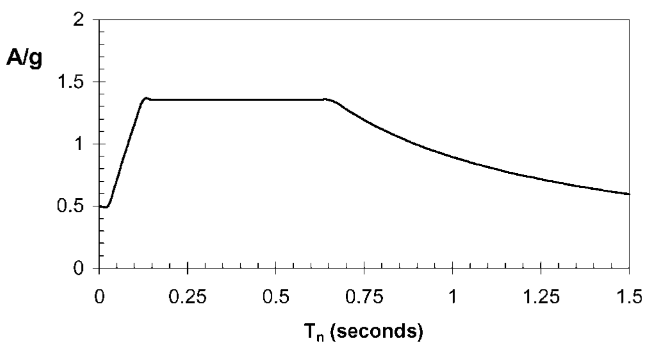

Figure 5. Pseudo-acceleration design spectrum.

3.2. Structural response

By specializing Equation (4) for the system depicted in Figure 4, and recognizing that only the first mode contributes to r_x and only the second mode to r_y, we obtain

$$r_x = |r_{x1}^{\mathrm{st}}| A_{x1}, \quad r_y = |r_{y2}^{\mathrm{st}}| A_{y2}, \quad r_{xy} = \rho_{12} r_{x1}^{\mathrm{st}} r_{y2}^{\mathrm{st}} A_{x1} A_{y2} \tag{17}$$

Substituting Equation (17) into Equations (7) and (11), gives α and β for the system shown in Figure 4:

$$\alpha = \rho_{12} \ \mathrm{sign}\left(\frac{r_{y2}^{\mathrm{st}}}{r_{x1}^{\mathrm{st}}}\right), \qquad \beta = \left|\frac{r_{y2}^{\mathrm{st}}}{r_{x1}^{\mathrm{st}}}\right| \frac{A_{y2}}{A_{x1}} \tag{18}$$

where the notation "sign" means that α is positive if r_{x1}^{st} and r_{y2}^{st} have the same algebraic sign and is negative if the two algebraic signs are different. The response correlation coefficient α is simply the modal correlation coefficient ρ_{12}, but with a positive or negative sign.

Table I. Modal static responses, response correlation coefficient and response ratio.

Response quantity	r_{x1}^{st}	r_{y2}^{st}	α	β
V_b	m	0	Undefined	0
N_a	$+0.1m$	$+0.1m$	$+\rho_{12}$	A_{y2}/A_{x1}
N_b	$-0.1m$	$+0.1m$	$-\rho_{12}$	A_{y2}/A_{x1}
N_c	$-0.1m$	$-0.1m$	$+\rho_{12}$	A_{y2}/A_{x1}
N_d	$+0.1m$	$-0.1m$	$-\rho_{12}$	A_{y2}/A_{x1}

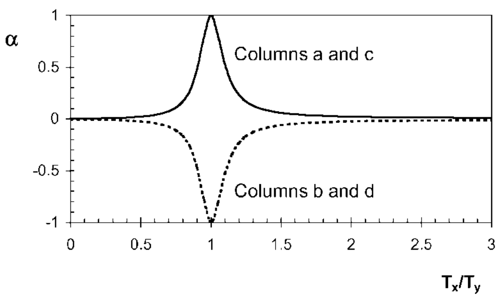

Figure 6. Correlation coefficient α for the axial force in four columns of the one-storey symmetrical building.

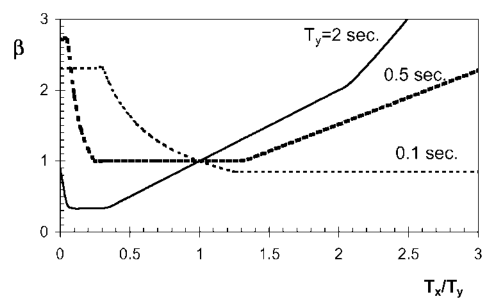

Figure 7. Response ratio β for the axial forces in four columns of the one-storey symmetrical building.

Table I shows the modal static responses, the response correlation coefficient α, and the response ratio β for selected response quantities: base shear V_b in the x-direction and axial forces N_a, N_b, N_c, and N_d in columns a, b, c, and d, respectively (Figure 4). Although not shown in Table I, shear forces and bending moments along the x-direction in the columns, have the same α and β values as for the base shear V_b.

The response correlation coefficient α for the axial forces in columns a, b, c and d is plotted against the period ratio T_x/T_y in Figure 6. This is the well-known modal correlation coefficient found in texbooks [13], but with a positive or negative sign depending upon the response quantity being considered. Figure 6 confirms the earlier result (Equation (8)) that α is bounded by -1 and 1; $\alpha = +1$ or -1 when $T_x = T_y$, i.e., the x- and y-vibration modes have identical vibration periods. For the design spectrum of Figure 5, the response ratio β for the axial force in the four columns (Table I) is plotted against the period ratio T_x/T_y in Figure 7, for three different values of T_y. Observe that $\beta = 1$ if $T_x = T_y$, but may be larger or smaller than 1 depending on the values of T_x/T_y and T_y.

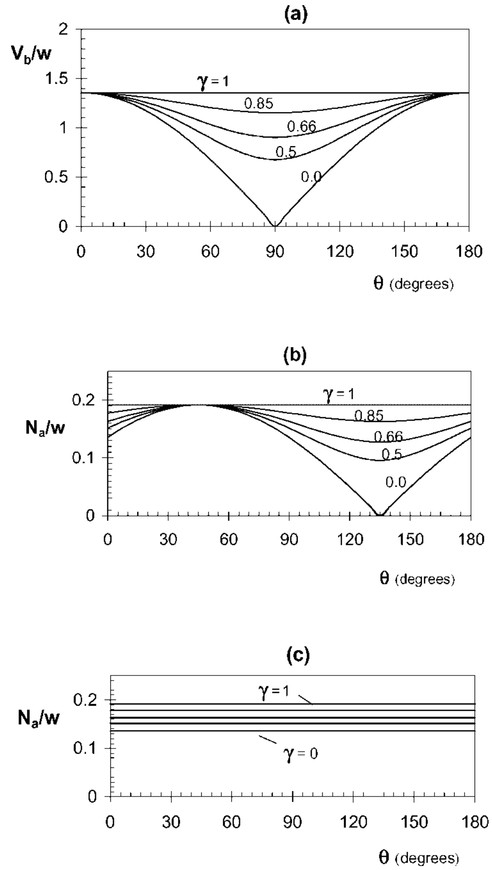

Figure 8. Variation of response with incident angle for several values of γ: (a) base shear in x-direction for systems with $T_x = 0.5$ s, (b) column axial force for systems with $T_x = T_y = 0.5$ s, (c) column axial force for systems with $T_x = T_y/3$, $T_y = 0.5$ s.

3.3. Variation of response with incident angle

The variation of V_b, normalized relative to the structural weight ($w = \mathrm{mg}$) with θ, is presented in Figure 8(a) for systems with $T_x = 0.5$ s and several values of γ. Because $r_y = 0$ and $r_{xy} = 0$ (Equation (17) and Table I), these results are not dependent on T_y. The value of r_x is obtained from Equation (17), Table I, and the design spectrum (Figure 5): $r_x = 1.355w$.

To interpret these results, Equation (1) is specialized whereby r is replaced by V_b/w:

$$\frac{V_b(\theta)}{w} = \frac{A(T_x)}{g}\sqrt{\cos^2\theta + \gamma^2 \sin^2\theta} \tag{19}$$

wherein for this system $A(T_x)/g = 1.355$. Note in Figure 8(a), if both components of ground motion have the same intensity ($\gamma = 1$), the x-base shear is independent of θ; Equation (19) gives $V_b(\theta) = 1.355w$. For a fixed θ, the response increases as γ increases, indicating increasing intensity of the weaker component of ground motion, as intuition would suggest and illustrated by

Equation (1). As we shall see later, these two observations apply to all response quantities, but the following observations are restricted to V_b: for any value of $\gamma < 1$, V_b is largest if $\theta = 0°$ and smallest if $\theta = 90°$.

These observations can be explained easily: $\theta = 0°$ implies that the stronger component of ground motion is applied along the x-direction, which clearly gives the maximum base shear in the x-direction: $V_b(\theta = 0°) = 1.355w$ (from Equation (19)); the weaker component of ground motion does not contribute to the x-base shear. $\theta = 90°$ means that the weaker component of ground motion is applied along the x-direction, which clearly gives the smallest base shear in the x-direction: $V_b(\theta = 90°) = 1.355\gamma w$ (from Equation (19)), with no contribution from the stronger component of ground motion acting in the y-direction. For other values of θ, both components of ground motion contribute to V_b, which takes on intermediate values.

The variation of the axial force N_a in column a, normalized relative to the structural weight with θ, is presented in Figure 8(b) for a system with identical periods $T_x = T_y = 0.5$ s. For this response quantity, $r_x \equiv N_{ax}$, $r_y \equiv N_{ay}$, and $r_{xy} \equiv N_{axy}$ are all non-zero and influenced by both T_x and T_y. For this system, $N_{ax} = N_{ay} = 0.1mA(T_x)$ (from Equation (17) and Table I), and $\alpha = +1$ and $\beta = 1$ (from Equation (18) and Table I).

To interpret the results shown in Figure 8(b), we specialize Equation (1) by replacing r with N_a/w:

$$\frac{N_a(\theta)}{w} = \frac{0.1A(T_x)}{g}\{(1+\gamma^2) + 2(1-\gamma^2)\sin\theta\cos\theta\}^{1/2} \tag{20}$$

Observe in Figure 8(b) that the axial force for any angle θ may be larger or smaller than the axial force for $\theta = 0$ or 90°; because $N_{ax} = N_{ay}$, N_a is the same for these two θ values. The maximum and the minimum values of N_a occur for incident angles when $\theta = 45$ and 135°, respectively, confirming the earlier result for $\alpha = +1$ and $\beta = 1$, as in this example.

The variation of N_a with θ can be explained as follows: consider first a single component of ground motion, i.e. $\gamma = 0$, in which case Equation (20) gives $N_a = 0.1355w$ for $\theta = 0$ and 90°, $N_a = 0.1355w\sqrt{2}$ for $\theta = 45°$, and $N_a = 0$ for $\theta = 135°$. Ground motion in the x-direction ($\theta = 0°$) excites only the x-mode of vibration, and the axial force is $N_a = 0.1355w$. Similarly, ground motion in the y-direction ($\theta = 90°$) excites only the y-mode of vibration, and the axial force is $N_a = 0.1355w$.

Ground motion $u_g(t)$ along a diagonal ($\theta = 45°$) can be resolved using two components: $u_g(t)\cos 45°$ in the x-direction and $u_g(t)\sin 45°$ in the y-direction. These resolved components result in axial force $N_{ax} = (0.1355w)\cos 45°$ and $N_{ay} = (0.1355w)\sin 45°$, respectively. Because the responses are perfectly correlated ($\rho_{12} = 1$), these two peak values occur simultaneously. Thus the combined peak value becomes $N_a = (0.1355w)\cos 45° + (0.1355w)\sin 45° = 0.192w$. A parallel development for $\theta = 135°$ leads to $N_a = (0.1355w)\cos 135° + (0.1355w)\sin 135° = 0$; both results are the same as obtained in Equation (20), shown in Figure 8(b).

Next, we consider two components of ground motion. As the intensity of the second component increases, i.e. γ increases, the response value increases for all values of θ, consistent with intuition. As discussed earlier, because the less intense component of ground motion along $\theta = 135°$ causes no axial force, the response is unaffected by γ if the incident angle is $\theta = 45°$.

The variation of the axial force N_a in column a normalized relative to the structural weight w with θ, is presented in Figure 8(c) for a system with well-separated periods: $T_x = 0.167$ s and $T_y = 0.5$ s. For this system, $N_{ax} = N_{ay} = 0.1355w$ (from Equation (17), Table I, and Figure 5); $\beta = 1$

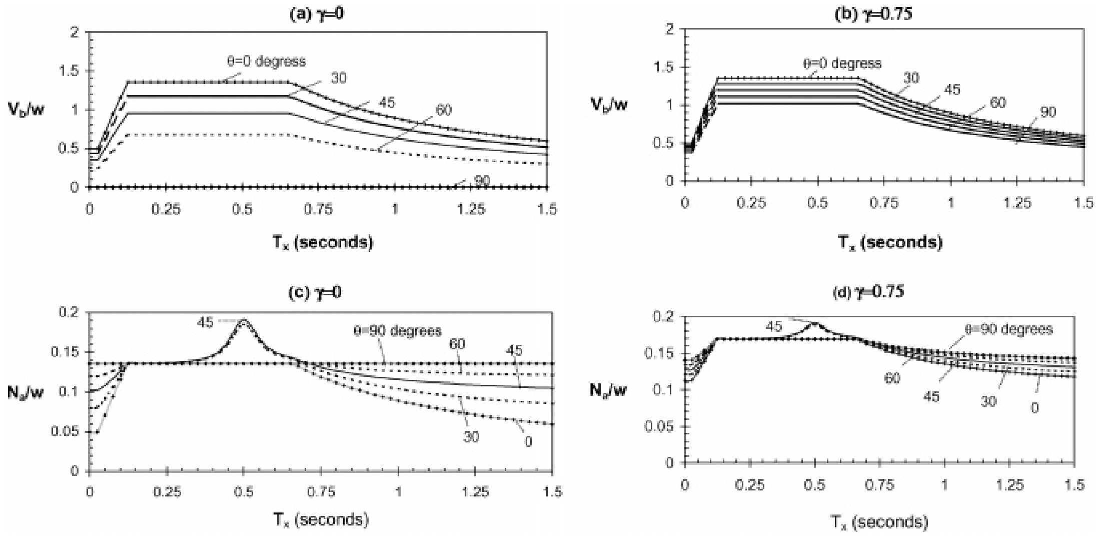

Figure 9. Variation of structural response with T_x for several values of θ and two values of γ. Parts (a) and (b) show results for V_b/w; parts (c) and (d) are for N_a/w.

and $\rho_{12} \cong 0$, hence $N_{axy} \cong 0$ and $\alpha \cong 0$ (from Equation (18), Table I, and Figures 6 and 7). To interpret the results shown in Figure 8(c), we specialize Equation (1) with r replaced by N_a/w:

$$\frac{N_a(\theta)}{w} \cong 0.1355\sqrt{1+\gamma^2} \tag{21}$$

Observe in Figure 8(c) that the axial force is essentially independent of the incident angle θ, which is the case because for a system with well separated periods, $N_{axy} = 0$ and $N_{ax} = N_{ay}$.

3.4. Variation of response with T_x

The peak value of V_b, normalized relative to the weight w, for systems with a fixed $T_y = 0.5$ s is plotted for $\gamma = 0$ and 0.75 against T_x in Figures 9(a) and 9(b). For $\theta = 0°$, the plot is identical to the design spectrum shown in Figure 5; for other values of θ, the plot appears to be multiplied by a θ-dependent factor that is less than one. In Equation (19) this factor is equal to one for $\theta = 0°$ and γ for $\theta = 90°$, and takes on values between one and γ for other values of θ. This simple variation of V_b with T_x is because the x-base shear due to ground motion in the y-direction is zero.

The variation of N_a with T_x shown in Figures 9(c)–(d) is more complicated because it is affected by ground motion in both x- and y-directions. These results permit several observations. If $\theta = 0°$, the variation of N_a/w with T_x is identical to the design spectrum, except for a scale factor that is dependent on the spectrum intensity ratio, as suggested by Equation (20). For other values of θ, the variation of N_a/w with T_x is more complicated and is dependent on the values of θ and γ, reaching a peak when $T_x = T_y$, the largest being when $\theta = 45°$; reasons for these trends were identified in the preceding section.

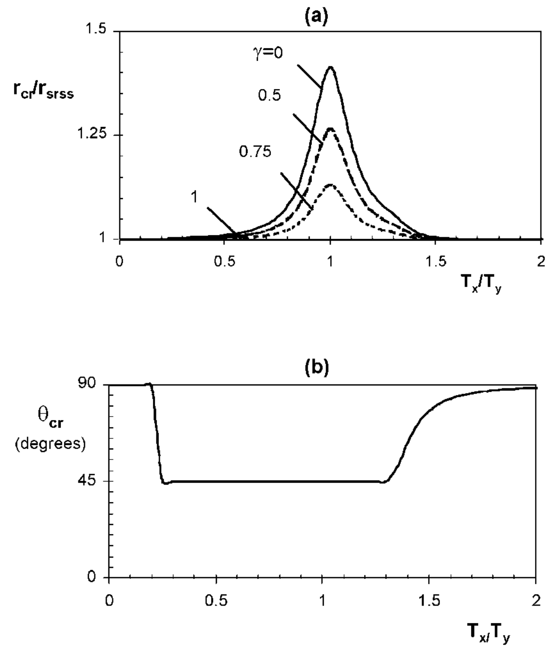

Figure 10. Variation of the ratio of critical and SRSS responses and critical angle with T_x/T_y for the axial force in column "a" of the one-storey symmetrical building; $T_y = 0.5$ s.

For values of θ different than 45°, increasing the value of γ smoothes the differences between the axial force at each angle θ, although significant differences remain when $\gamma = 0.75$ as shown in Figure 9(d). For the limit case of $\gamma = 1$, the responses are independent of θ.

3.5. *Critical response and critical angle*

Next, the ratio r_{cr}/r_{srss} of the critical response and the response from the SRSS analysis (Equation (15)), and the critical angle (Equation (10)) are examined for two response quantities: base shear V_b in the x-direction and axial force N_a in column a. For the x-base shear, this ratio is always equal to one and the critical angle is zero. Such is the case because the x-base shear is not affected by the y-component of ground motion; $r_y = \beta = 0$, as shown in Table I.

The above-mentioned results for the axial force in column a are presented in Figure 10 for a one-storey system with fixed $T_y = 0.5$ s and T_x over a range of values. As the spectrum intensity ratio γ increases, we expect r_{srss} and r_{cr} to increase, however r_{cr}/r_{srss} decreases.

The SRSS analysis gives the correct critical response if the vibration periods, T_x and T_y, are well separated, because the responses r_x and r_y are then essentially uncorrelated. The ratio r_c/r_{srss} is largest for systems with $T_x = T_y$, as this condition implies that responses r_x and r_y are perfectly correlated; the largest value of r_c/r_{srss} is equal to the upperbound in Equation (16) and Figure 3. Thus the discrepancy between r_{srss} and r_{cr} may be significant for systems with closely spaced periods of vibration and smaller values of the spectrum intensity ratio.

The critical angle of incidence depends on the period ratio T_x/T_y, but not on the spectrum intensity ratio. It varies between 45 and 90°, as shown in Figure 10(b). If T_x/T_y is much smaller or much larger than one, θ_{cr} is close to 90°, implying that the response reaches its critical value when the stronger component of horizontal ground motion is applied along the y-axis of the structure. Such is the case because in this range of period ratios, $\beta > 1$ or $r_y > r_x$ for systems with $T_y = 0.5$ s. For systems where T_x/T_y is close to one, the critical angle is 45° for reasons identified in the preceding section.

4. ONE-STOREY UNSYMMETRICAL-PLAN BUILDINGS

4.1. System and ground motion

Consider an idealized one-storey unsymmetrical-plan building with a rigid slab supported by any number of lateral resisting elements oriented along directions x and y (Figure 11). The system has three degrees of freedom: translations of the centre of mass (CM) along x- and y-directions and rotation of the slab about a vertical axis passing through the CM. The eccentricity of the centre of rigidity CR relative to CM is given by distances e_x and e_y (Figure 11); e_x/r and e_y/r are the normalized eccentricities, where r is the radius of gyration of the floor about the vertical axis passing through the CM. A 5 per cent damping ratio is assumed for each of the three vibration modes. Introduced for reference purposes, T_x, T_y and T_θ are the vibration periods of the corresponding symmetrical-plan (or torsionally uncoupled) system with $e_x = e_y = 0$, but the mass and x-, y-, and θ-stiffnesses are identical to the coupled system.

The major and the intermediate principal components of ground motion are defined by $A(T_n)$ and $\gamma A(T_n)$, respectively, applied at incident angle θ (Figure 11), where $A(T_n)$ is the design spectrum of Figure 5. No vertical ground motion is considered.

4.2. Critical response of one-way unsymmetrical systems

Consider first a system with mass and stiffness properties symmetrical about the y-axis ($e_x/r = 0$) but unsymmetrical about the x-axis with $e_y/r = 0.3$. Its uncoupled vibration periods are defined as follows: $T_y = 0.66$ s (the intersection of the constant and hyperbolic branches of the design spectrum); T_x and T_θ are varied, but $T_x/T_\theta = 1$. Ground motion in the x-direction excites the two natural vibration modes (periods T_1 and T_2) that contain coupled x-lateral and torsional motion. Ground motion in the y-direction excites only the mode (period T_3) that describes uncoupled motion in the y-lateral direction. The periods of the three natural vibration modes are plotted against T_x/T_y in Figure 12(a).

We will study the edge displacement of the system in the y-direction ($d_x/r = 1.225$ for a square plan) due to a single component ($\gamma = 0$) of ground motion acting at an arbitrary angle θ, which is characterized by the design spectrum shown in Figure 5. For this response quantity, the correlation coefficient α (Equation (7)), the response ratio β (Equation (11)), the critical angle θ_{cr} (Equation (10)), and the critical response r_{cr} (Equation (14)) are computed and plotted as a function of the uncoupled period ratio T_x/T_y. The response has been normalized relative to the displacement of the corresponding symmetric system, which is given by $A(T_y) \div (2\pi/T_y)^2 = 14.67$ cm. Also plotted are the response values r ($\theta = 0°$) and r ($\theta = 90°$), Equation (15), for the two special cases of ground motion applied in the x-direction ($\theta = 0°$) and in the y-direction ($\theta = 90°$), respectively. The latter response is independent of T_x/T_y because the response to ground motion in the y-direction, the axis of symmetry, is independent of T_x and T_y is fixed. For $\theta = 0°$, the displacement increases as T_x becomes larger.

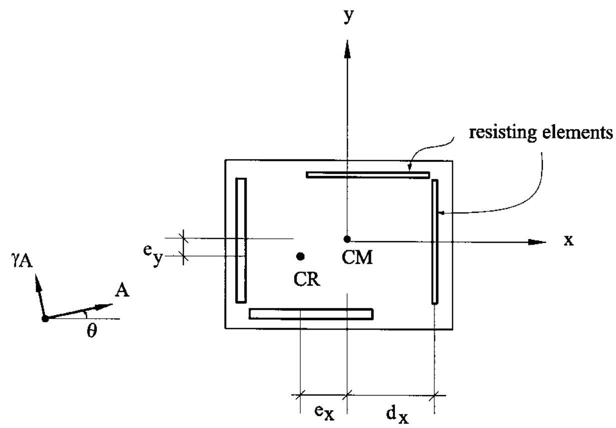

Figure 11. One-storey unsymmetric building.

For the cases when $T_x/T_y \ll 1$ or $\gg 1$, the periods T_1 and T_2 are well separated from T_3 (Figure 12(a)), the correlation coefficient α approaches zero (Figure 12(b)), the critical angle θ_{cr} approaches 90° when $T_x/T_y \ll 1$ and 0° when $T_x/T_y \gg 1$ (Figure 12(c)), and the critical response r_{cr} approaches r $(\theta = 90°)$ for $T_x/T_y \ll 1$ and r $(\theta = 0°)$ when $T_x/T_y \ll 1$ (Figure 12(d)). These results can be explained as follows: when $T_x/T_y \ll 1$, $\beta > 1$ (Figure 12(b)), which implies r_y is much larger than r_x, therefore the response is the largest when the ground motion is applied in the y-direction. The opposite situation occurs when $T_x/T_y \gg 1$, where $\beta < 1$ (Figure 12(b)), implying that r_x is much larger than r_y; therefore the response is largest when the ground motion is applied in the x-direction. For systems with $T_x = T_y$, $\alpha = 0$ (Figure 12(b)); thus based on Figure 2, it would be expected that $r_{cr}/r_{srss} = 1$. This is confirmed by the results shown later in Figure 13(a).

On the contrary, observe that when one of the natural vibration periods, T_1 or T_2, is equal to vibration period T_3 in Figure 12(a), the correlation coefficient α is close to -1 or $+1$, respectively, β tends to 1 (Figure 12(b)), the critical angle is close to 135 or 45°, respectively (Figure 12(c)), and the critical response r_{cr} (Figure 12(d)) has two peaks that exceed the two responses r $(\theta = 0°)$ and r $(\theta = 90°)$. This observation can be explained as follows: When T_1 or T_2 is equal to T_3, the correlation between the modal responses increases, therefore the cross term r_{xy} and the correlation coefficient α increases; $T_1 = T_3$ implies that $T_x/T_y = 0.85$, and $T_2 = T_3$ occurs at $T_x/T_y = 1.15$, which define the two peaks in the critical response values shown in Figure 12(d). In addition, the parameter β is close to 1 in the same period range. Thus, as pointed out previously in Figure 2, the simultaneity of β approaching 1 and α approaching -1 or $+1$ leads to an increase in the critical response with respect to the responses r $(\theta = 0°)$ and r $(\theta = 90°)$, as confirmed by the results shown in Figure 12(d).

4.3. Critical response of unsymmetric systems

The ratio of the critical value of the response to its SRSS value, r_{cr}/r_{srss}, is computed from Equation (15) and plotted against the period ratio T_x/T_y in Figure 13. This figure is organized in three parts to show (a) the effect of the spectrum intensity ratio γ, for $e_x/r = 0$, $e_y/r = 0.3$ and $T_x/T_\theta = 1$; (b) the effect of T_x/T_θ, for $e_x/r = 0$, $e_y/r = 0.3$ and $\gamma = 0$; and (c) the effect of the eccentricity e_x/r, for $T_x/T_\theta = 1$, $e_y/r = 0.3$ and $\gamma = 0$. The system presented in Figure 13(a) is the same one-way, unsymmetrical system subjected to a single ground motion component that was presented

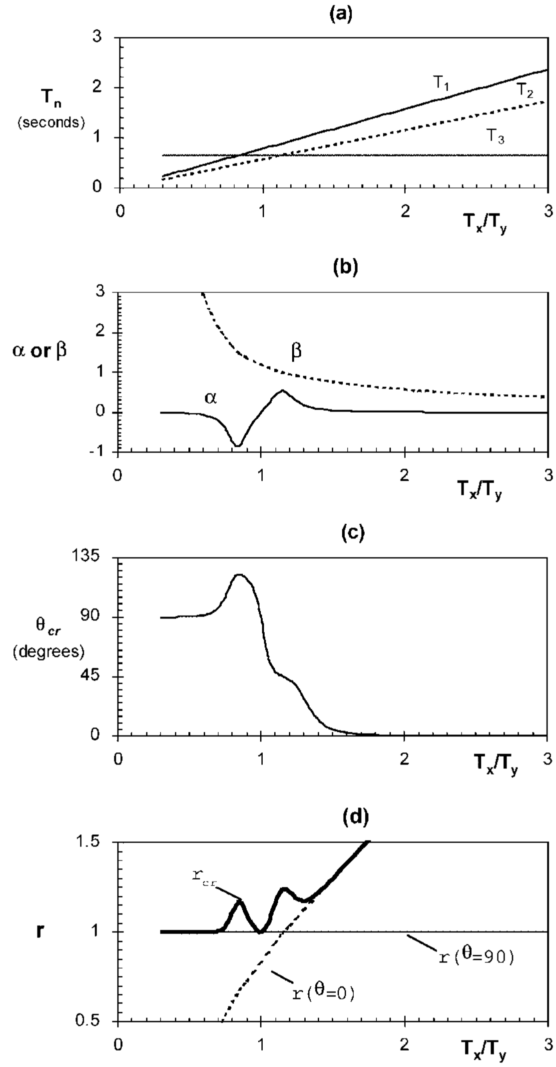

Figure 12. (a) Natural vibration periods, (b) parameters α and β, (c) critical angle, and (d) normalized response for a one-way unsymmetrical building.

previously in Figure 12. The largest value of r_{cr}/r_{srss} is 1.24 when $\gamma = 0$ and $T_x/T_y = 1.15$. When $\gamma = 0.5$ and 0.75, the largest value of r_{cr}/r_{srss} is reduced to 1.15 and 1.08, respectively.

Varying T_x/T_θ (Figure 13(b)) leads to a decrease or increase in r_{cr}/r_{srss}, depending on the period ratio T_x/T_y; r_{cr}/r_{srss} is largest for systems when $T_x/T_\theta = 1$ and $T_x/T_y = 1.15$. Similarly, varying the eccentricity e_x/r (Figure 13(c)) may lead to an increase or a decrease in the values of r_{cr}/r_{srss}, depending on T_x/T_y; r_{cr}/r_{srss} is largest for systems when $e_x/r = 0$ and $T_x/T_y = 1.15$. For all systems considered herein, the largest value of r_{cr}/r_{srss} is 1.24, corresponding to $e_x/r = 0$, $e_y/r = 0.3$, $T_x/T_\theta = 1$, and $T_x/T_y = 1.15$, when the excitation is a single seismic component ($\gamma = 0$). For more realistic values of the spectrum intensity ratio γ, such as 0.75, the largest value of r_{cr}/r_{srss} is 1.08;

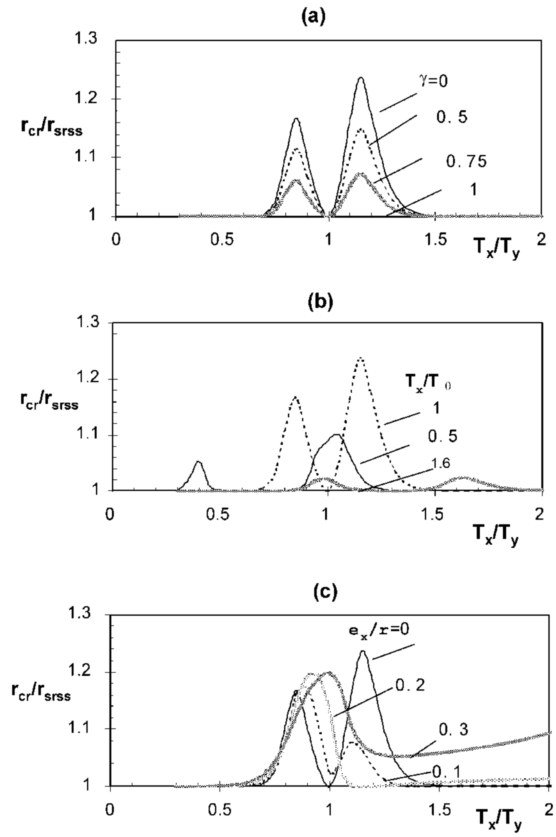

Figure 13. Ratio of critical and SRSS responses: (a) Effect of γ ($e_x/r=0$, $e_y/r=0.3$, $T_x/T_\theta=1$), (b) effect of T_x/T_θ ($e_x/r=0$, $e_y/r=0.3$, $\gamma=0$), (c) effect of e_x/r ($e_y/r=0.3$, $T_x/T_\theta=1$, $\gamma=0$).

these values are below the upperbound values presented in Figure 3. Results not presented here indicate that $r_{\rm cr}/r_{\rm srss}=1.12$ when $\gamma=0.75$ for the displacement of some resisting elements of a building with a rectangular plan and an aspect ratio of 4.

5. CONCLUSIONS

1. An explicit formula has been derived to calculate the critical values of structural response to two principal components of horizontal ground motion acting along any incident angle and the vertical component of ground motion; the critical response is defined as the largest value of response for all possible incident angles. Using this formula, the critical value can be computed from the responses to single components of ground motion applied separately along the three structural axes and the correlation between these responses. This explicit formula is convenient for design purposes, especially code applications, because it avoids computation of the critical incident angle.

2. The ratio between the critical value of response and the SRSS response—corresponding to the principal components of ground acceleration applied along the structural axes—depends on three dimensionless parameters: the spectrum intensity ratio γ between the two principal components of horizontal ground motion characterized by design spectra $A(T_n)$ and $\gamma A(T_n)$; the correlation

coefficient α of responses r_x and r_y due to design spectrum $A(T_n)$ applied in the x- and y-directions, respectively; and $\beta = r_y/r_x$.

3. The ratio r_{cr}/r_{srss} is bounded by 1 and $[2/(1+\gamma^2)]^{1/2}$, for any structure and spectral shape under the assumption that both horizontal spectra have the same shape. Thus the largest value of this ratio is $\sqrt{2}$,1.26, 1.13, and 1.08 for $\gamma = 0$, 0.5, 0.75 and 0.85, respectively, implying that the critical response is 1.13 times the SRSS response for typical values of the spectrum intensity ratio, say 0.75; and never exceeds $\sqrt{2}$ times the SRSS response. The SRSS analysis gives the correct critical response only if $\gamma = 1$, i.e. the two principal components of horizontal ground motion have the same intensity, or if responses r_x and r_y are uncorrelated.

4. The correlation coefficient α depends on the structural properties, but is always bounded between -1 and 1 for any structure and spectral shape. For example, $\alpha = 1$ or -1 for some responses of one-storey symmetrical buildings with identical vibration periods along the axes of symmetry. Or, α is close to $+1$ or -1 for some responses of one-storey unsymmetrical buildings if the vibration period of the mode with the largest contribution in the response to ground motion in the x-direction coincides with the vibration period of the mode with largest contribution in the response to ground motion in the y-direction.

5. For a fixed value of γ, the ratio r_{cr}/r_{srss} is largest if $\beta = 1$ and $\alpha = \pm 1$. The parametric variations presented for one-storey buildings indicate that this condition can be satisfied by axial forces in columns of symmetrical buildings or can be approximated by lateral displacements in resisting elements of unsymmetrical buildings.

6. The incident angle of the ground motion for which a structural response quantity is largest, may define any direction in the horizontal plane, depending upon the system properties, the location of the system natural periods on the spectrum being considered, and the shape of the spectrum; however, r_{cr}/r_{srss} is largest when the incident angle is either 45 or 135°.

APPENDIX A: BOUNDS FOR THE RESPONSE CORRELATION COEFFICIENT α

The cross term r_{xy} and the response correlation coefficient α are defined by Equations (3) and (7), respectively. First we recognize that any set of peak modal responses to seismic motion in a given direction, satisfy the following inequality:

$$\sum_i \sum_j \rho_{ij} r_i r_j \geqslant 0 \tag{A1}$$

From Equation (2), specialized for $k = x$ and y, we can write

$$r_x^2 = \sum_i \sum_j \rho_{ij} r_{xi} r_{xj}, \qquad r_y^2 = \sum_i \sum_j \rho_{ij} r_{yi} r_{yj} \tag{A2}$$

Defining $R_{xi} = r_{xi}/r_x$ and $R_{yi} = r_{yi}/r_y$ and using Equation (A2), we note that

$$\sum_i \sum_j \rho_{ij} R_{xi} R_{xj} = \sum_i \sum_j \rho_{ij} R_{yi} R_{yj} = 1 \tag{A3}$$

Let u_i and v_i be two modal responses defined as

$$u_i = R_{xi} + R_{yi}, \qquad v_i = R_{xi} - R_{yi} \tag{A4}$$

Then, according to Equation (A1) we can write

$$\sum_i \sum_j \rho_{ij} u_i u_j \geqslant 0 \quad \text{and} \quad \sum_i \sum_j \rho_{ij} v_i v_j \geqslant 0 \tag{A5}$$

By substituting Equations (A4) into Equations (A5), and using Equations (A3), (3) and (7), it can be shown that

$$\sum_i \sum_j \rho_{ij} u_i u_j = 2 + 2\frac{\sum_i \sum_j \rho_{ij} r_{xi} r_{yj}}{r_x r_y} = 2 + 2\alpha \tag{A6}$$

$$\sum_i \sum_j \rho_{ij} v_i v_j = 2 - 2\frac{\sum_i \sum_j \rho_{ij} r_{xi} r_{yj}}{r_x r_y} = 2 - 2\alpha \tag{A7}$$

From Equations (A5), (A6) and (A7), we can write

$$2 + 2\alpha \geqslant 0, \qquad 2 - 2\alpha \geqslant 0 \tag{A8}$$

Finally, from Equations (A8) we conclude that

$$-1 \leqslant \alpha \leqslant 1 \tag{A9}$$

Hence the correlation coefficient is bounded between -1 and $+1$. This result is valid for any structure and any spectral shape, under the assumption that both horizontal spectra have the same shape.

APPENDIX B: BOUNDS FOR THE CRITICAL RESPONSE

Consider structural response to two principal components of horizontal ground motion acting along any incident angle θ, relative to the structural axes. The critical response r_{cr}, defined as the maximum response considering all possible incident angles θ, is given by Equation (14). Also, the SRSS response r_{srss} was defined as the larger of the two responses for $\theta = 0$ and $90°$. Obviously,

$$r_{cr} \geqslant r_{srss} \tag{B1}$$

From Equation (14) we note that $r_{cr} = r_{srss}$ only if $r_{xy} = 0$ (i.e. the correlation coefficient $\alpha = 0$). Now, from Equation (1), we can write

$$r(\theta, \gamma = 1) = (r_x^2 + r_y^2)^{1/2} \tag{B2}$$

Note that the right-hand term is independent of θ and hence equal to the critical response. For any other value of γ, the critical response will be smaller. Thus,

$$r_{cr}^2 \leqslant r_x^2 + r_y^2 \tag{B3}$$

Furthermore, from Equation (14) it can be noted that $r_{cr}^2 = r_x^2 + r_y^2$ only when $r_{xy} = r_x r_y$ (i.e. $|\alpha| = 1$).

Let's assume that r_{srss} is defined by the response $r(\theta = 0°) = (r_x^2 + \gamma^2 r_y^2)^{1/2}$ (Equation (5)). Then, Equation (B3) can be written as

$$r_{cr}^2 \leqslant (r_x^2 + \gamma^2 r_y^2)\frac{(r_x^2 + r_y^2)}{(r_x^2 + \gamma^2 r_y^2)} \tag{B4}$$

or

$$r_{cr}^2 \leqslant r_{srss}\frac{(r_x^2 + r_y^2)}{(r_x^2 + \gamma^2 r_y^2)} \tag{B5}$$

which, by using the definitions of α and β (Equations (7) and (11)), becomes

$$r_{\rm cr}^2 \leqslant r_{\rm srss}^2 \frac{(1+\beta^2)}{(1+\beta^2\gamma^2)} \tag{B6}$$

The function $(1+\beta^2)/(1+\beta^2\gamma^2)$ in Equation (B6) is an increasing function of β with a maximum at $\beta = 1$. Hence, Equation (B6) can be rewritten as

$$r_{\rm cr}^2 \leqslant r_{\rm srss}^2 \frac{2}{(1+\gamma^2)} \tag{B7}$$

If we had assumed that $r_{\rm srss}$ was defined by the response $r(\theta = 90^\circ) = (\gamma^2 r_x^2 + r_y^2)^{1/2}$ (Equation (5)), following a similar procedure it is easy to show that the same Equation (B7) is obtained.

Finally, using Equations (B1) and (B7), the lower and upper bounds of the critical response are

$$r_{\rm srss} \leqslant r_{\rm cr} \leqslant r_{\rm srss}\sqrt{\frac{2}{1+\gamma^2}} \tag{B8}$$

where the lower bound is reached if the correlation coefficient $\alpha = 0$ and the upperbound if $|\alpha| = \beta = 1$.

It should be noted that these results are valid for any structure and any spectral shape, under the assumption that both horizontal spectra have the same shape.

REFERENCES

1. Newmark NM. Seismic design criteria for structures and facilities, Trans-Alaska pipeline system. *Proceedings of the U.S. National Conference on Earthquake Engineering*. Earthquake Engineering Institute, 1975; 94–103.
2. Rosenblueth E, Contreras H. Approximate design for multicomponent earthquakes. *Journal of Engineering Mechanics Division ASCE* 1977; **103**: 895–911.
3. Cheng FY, Ger JF. The effect of multicomponent seismic excitation and direction on response behavior of 3-D structures. *Proceedings of the 4th U.S. National Conference on Earthquake Engineering*, Palm Springs, 1990; **2**: 5–14.
4. Clough RW, Penzien J. *Dynamics of Structures*. New York: McGraw-Hill, 1993.
5. Wilson EL, Suharwardy I, Habibullah A. A clarification of the orthogonal effects in a three-dimensional seismic analysis. *Earthquake Spectra* 1995; **11**: 659–666.
6. Smeby W, Der Kiureghian AD. Modal combination rules for multicomponent earthquake excitation. *Journal of Earthquake Engineering and Structural Dynamics* 1983; **13**: 1–12.
7. López OA, Torres R. The critical angle of seismic incidence and the maximum structural response. *Journal of Earthquake Engineering and Structural Dynamics*. 1997; **26**: 881–894.
8. Menun C, Der Kiureghian AD. A replacement for the 30%, 40%, and the SRSS rules for multicomponent seismic analysis. *Earthquake Spectra* 1998; **14**(1): 153–163.
9. MacRae GA, Mattheis J. Three-dimensional steel building response to near-fault motions. *Journal of Structural Engineering ASCE* 2000; **126**(1): 117–126.
10. Penzien J, Watabe M. Characteristics of 3-dimensional earthquake ground motion. *Journal of Earthquake Engineering and Structural Dynamics* 1975; **3**: 365–374.
11. Loh C-H. Analysis of the spatial variation of seismic waves and ground movements from smart-1 array data. *Journal of Earthquake Engineering and Structural Dynamics* 1985; **13**: 561–581.
12. Wilson EL, Der Kiureghian A, Bayo E.P. A replacement of the SRSS method in seismic analysis. *Journal of Earthquake Engineering and Structural Dynamics* 1981; **9**: 187–194.
13. Chopra AK. *Dynamics of Structures*. New York: Prentice Hall, 1995.
14. Torres R. "Influencia de la direccionalidad del movimiento sismico sobre la respuesta dinamica de estructuras", Trabajo de Grado para Magister Scientarium, IMME, Fac. de Ingenieria, Universidad Central de Venezuela, 1996.
15. Newmark NM, Hall WJ. *Vibration of Structures Induced by Ground Motion. Shock and Vibration Handbook*, (2nd edn). New York: McGraw Hill, 1976.