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INELASTIC RESPONSE OF ONE-STOREY ASYMMETRIC-PLAN SYSTEMS SUBJECTED TO BI-DIRECTIONAL EARTHQUAKE MOTIONS

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SUMMARY

The inelastic response of one-storey systems with one axis of asymmetry subjected to bi-directional base motion is studied in this paper. The effect of the system parameters on response is also evaluated: uncoupled torsional-to-lateral frequency ratio, stiffness eccentricity, and yield strength of the lateral resisting elements. The ensemble of earthquake records used consists of 15 two-component strong ground motions. The response to uni-directional excitation is considered first to examine the influence of the system parameters and to serve as a basis to examine the results of the bi-directional case, which are presented in terms of average spectra for bi- over uni-directional lateral-deformation ratios. It is shown that the effect of inelastic behaviour is, on the average, noteworthy for stiff structures, in turn, the same structures are the most affected by the action of bi-directional ground motions. The effect of the relative intensity of the two orthogonal ground motion components is also studied. Copyright © 1999 John Wiley & Sons, Ltd.

KEY WORDS: torsion; earthquake; inelastic response; torsional coupling; bi-directional motion

INTRODUCTION

The coupled lateral-torsional response of the most simple asymmetric-plan system is an intrinsically complex problem due to the number of parameters involved. Further, things complicate when inelastic response is considered, subject that has attracted the attention of researchers in the last decade.[1–6] Indeed, inconsistencies of the conclusions of various studies available in the literature have been attributed to limitations of the models used.[3,5,6] One of the matters under discussion is the behaviour of the elements that provide resistance in the direction transverse to the asymmetric direction, the latter being the direction of earthquake input motion in uni-directional excitation studies. In fact, if the transverse elements remain elastic because their resistance is sufficiently large, the effect of the transverse component of ground motion is null as far as the torsional response of the system is concerned. However, if such elements do have limited strength, the presence of transverse earthquake loading may represent a reduction of their available capacity to provide torsional stiffness, hence the torsional effects may eventually increase.

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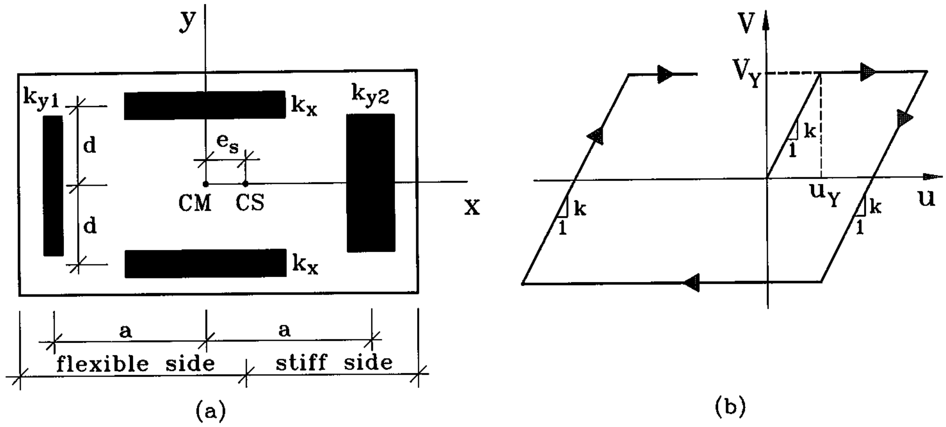

Figure 1. System considered and lateral load–deformation relationship

The general purpose of this investigation is to study the effect of simultaneous bi-directional ground motion on single-storey asymmetric inelastic systems. To enable comparison with other studies, the same parameters controlling the response of asymmetric inelastic systems used by Goel and Chopra[3] are considered here; they also presented a comprehensive review of the available literature on the subject.

SYSTEMS CONSIDERED

The system considered is an idealized one-storey building consisting of a rigid deck supported on frames or walls that provide lateral resistance in both directions of the plan, as shown in Figure 1(a). The resisting planes in the y-direction are symmetrically located about the Centre of Mass (CM) but have different stiffnesses, causing an eccentricity e_s between the Centre of Stiffness or rigidity (CS) and the CM where the storey mass is lumped; the resisting planes in the x-direction have equal stiffness and are symmetric about the x-axis.

Only two resisting elements are considered in each direction since it has been shown that such a simple system provides a satisfactory estimate of the response of another with a larger number of elements, provided the system parameters are kept the same;[1] this conclusion, however, was obtained for the case of one-component base motion. All the resisting planes are assumed to be elastoplastic (Figure 1(b)), having the same yield deformation in their planes; their out of plane and torsional stiffnesses and strengths are neglected. The x- and y-direction planes are unconnected, i.e., no bi-axial plasticity interaction takes place at the element level.

Denoting by k_x the elastic lateral stiffness of each resisting plane in the x-direction, and k_{yi} the corresponding stiffnesses of the elements oriented along the y-direction, the lateral stiffnesses of the structure in the x and y directions are $K_x = 2k_x$ and $K_y = (k_{y1} + k_{y2})$, and the torsional stiffness with respect to the center of mass is $K_\theta = 2d^2k_x + a^2(k_{y1} + k_{y2})$. The co-ordinates of the centre of stiffness are $e_{sy} = 0$ and $e_{sx} = e_s = a(k_{y1} - k_{y2})/K_y$, where e_s is called the stiffness eccentricity of the system. In turn, if it is assumed that the element strengths are proportional to their stiffnesses, the location of the resultant of the yield forces of the resisting elements, or Plastic Centre (CP), has co-ordinates $e_{py} = 0$ and $e_{px} = e_s$.

The relative torsional to lateral stiffness of the system is defined by the ratio $\Omega_\theta = \omega_{\theta s}/\omega_y$, where $\omega_{\theta s} = \sqrt{K_{\theta s}/mr^2}$ and $\omega_y = \sqrt{K_y/m}$ correspond to the natural frequencies of an associated symmetric ($e_s = 0$) elastic system with the same mass and stiffnesses K_y and $K_{\theta s}$ as in the coupled system, where $K_{\theta s} = K_\theta - e_s^2 K_y$ is the torsional stiffness of the structure about the CS, and r is the radius of gyration of the deck about the CM. The values of the parameter Ω_θ selected for this study are 0·5, 0·8, 1, 1·25, and 2.

Because in most real buildings ω_x and ω_y are similar ($\omega_x = \sqrt{K_x/m}$), the ratio of the uncoupled lateral vibration frequencies ω_x/ω_y was taken equal to 1, thus, defining $\omega = \omega_x = \omega_y$, the uncoupled translation period becomes $T = 2\pi/\omega$. The values of T used in this study are: 0·1, 0·2, 0·3, 0·4, 0·5, 0·6, 0·75, 0·8, 0·9, 1, 1·5, 2, 4, 6, 8, 9, 20, and 50 sec; the very long periods were included as a mean to check the physical response limits for infinitely flexible structures. Another parameter necessary to define the properties of the system is γ_x, the relative contribution of the resisting elements in the x-direction to the total torsional stiffness with respect to the center of stiffness, which in this study was taken equal to 0.5

The strength of the resisting elements is defined through the dimensionless reduction factor c, with reference to the uni-directional response of the corresponding symmetric-plan elastic system, as

$$u_{Yl} = c u_{ml} \tag{1}$$

where u_{ml} is the maximum elastic lateral displacement of the symmetric system subjected to component l (i.e. u_{ml} is the spectral displacement for ground motion component l at period T), and u_{Yl} is the yield deformation used for all the resisting elements in the analysis of the asymmetric inelastic system subjected to the bi-directional motion with component l acting in the y-direction. The advantage of this scheme is that scaling (or normalizing) the ground motions becomes unnecessary, and average responses to a number of earthquakes records can be directly computed. The values of c used in this study are 0·25, 0·5, and 1.

According to the above, the parameters necessary to characterize the systems considered are: T, Ω_θ, γ_x, and the normalized stiffness eccentricity e_s/r, wherefrom the stiffnesses and the locations of the resisting elements become determined; the corresponding expressions for d/r, a/r, k_x, k_{y1} and k_{y2} are available.[3] It must be pointed out, however, that the system parameters may not be freely chosen since they have some limiting values. For example, assuming that only the weight of the slab contributes to the mass of the system, and assuming a uniform distribution of mass over the plan, for a square deck the maximum possible value of Ω_θ is 1.73, which corresponds to the case when the resisting planes reach the edge of the slab. For a circular slab the maximum feasible Ω_θ is 2. For rectangular slabs, the limiting value of Ω_θ is less than 1.73, value that decreases as the aspect ratio of the deck departs from 1 and as e_s/r increases form zero. In a real structure not only the deck has weight (mass) but also the vertical elements; thus, a structure with reinforced concrete core walls (additional mass near the center of mass) will have a radius of gyration smaller than that of framed-tube building (additional mass near the perimeter). In practice, however, it is customary to assume that mass is uniformly distributed. The mentioned restriction for the value of Ω_θ must be kept in mind since the range of values used in this study may go beyond the limits applicable to some specific cases.

EARTHQUAKE RECORDS USED AND EQUATIONS OF MOTION

Fifteen two-component ground motion records were used as base motion. Information about the ensemble of records is given in Table I, where it can be seen that they cover a variety of situations

Table I. Data of ground motion records used

Site and date	Site geology	Magnitude (Ms) and approx. epicentral distance (km)	Component	Maximum accel. (cm/sec^2)	Maximum velocity (cm/sec)	Maximum disp. (cm)	Record duration used (sec)
Quintay, Chile (3/3/95)	Rock (Soil Type I)	7·8 17	LONG TRAN	0·237 0·26	12·52 19·33	2·77 3·58	50
Zapallar, Chile (3/3/85)	Rock (Soil Type I)	7·8 85	N90E N00E	0·305 0·27	13·47 11·21	1·67 1·11	42
Melipilla, Chile (3/3/85)	Dense sand (Soil Type II)	7·8 76	N90E N00E	0·529 0·687	40·32 34·25	5·88 12·11	42
Llolleo, Chile (3/3/85)	Dense sand (Soil Type II)	7·8 46	N10E S80E	0·713 0·446	40·29 23·29	10·5 4·25	50
Viña del Mar, Chile (3/3/85)	Sand (Soil Type III)	7·8 38	S20W N70W	0·363 0·238	30·74 25·51	5·42 4·12	50
Llay Lay, Chile (3/3/85)	Gravel and soft lime (Soil Type III)	7·8 94	N80W S10W	0·475 0·331	36·66 36·59	6·38 8·42	42
Kushiro, Japan (23/4/62)	Volcanic ash and stiff sand over sanstone	7·0 100	N90E N00E	0·478 0·244	20·01 13·61	5·22 3·24	30
Hachinohe, Japan (16/5/68)	Deep cohesionles soil	7·9 80	N90E N00E	0·207 0·269	34·95 35·43	10·38 9·68	60
Aomori, Japan (16/5/68)	Soft sandy soil	7·9 230	N00E N90E	0·257 0·196	39·12 31·59	19·97 17·82	50
Mexico SCT, Mexico (19/9/85)	Soft clay	8·1 385	EW NS	0·171 0·105	60·51 38·54	12·36 20·07	65
El Centro, USA (18/5/40)	Stiff clay over deep shale	6·3 100	S00E S90W	0·349 0·214	33·45 36·93	21·16 19·19	30
Castaic, U.S.A (17/1/94)	Sedimentary rock	6·8 41	N90E N00E	0·568 0·515	51·51 52·56	9·18 15·33	20
Sylmar, U.S.A. (17/1/94)	Alluvium	6·8 16	N00E N90E	0·844 0·605	128·9 76·94	32·55 15·97	20
Newhall, U.S.A. (17/1/94)	Alluvium	6·8 20	N00E N90E	0·591 0·583	94·72 74·84	28·84 16·76	18
Corralitos, U.S.A. (17/10/89)	Landslide deposits (rock)	7·1 7	N00E N90E	0·603 0·479	55·2 46·13	12·03 20·7	15

regarding tectonic environment, site conditions, epicentral distance, and intensity and duration of motion. The common factors are that, first, at all sites at least one component exceeds a peak ground acceleration of 0·25g or a peak ground velocity of 50 cm/sec, and second, structural and/or soil damage occurred in almost all these sites when the corresponding earthquakes struck. It is therefore believed that the selected motions are sufficiently severe, and cover a wide range of conditions, so as to be relevant for seismic design. The soil types indicated for Chilean records in Table I correspond to the categories these sites classify in according to the Chilean seismic design code; a detailed description of the soil types defined in the code is available.[7]

The two horizontal orthogonal components $\ddot{u}_{g1}$ and $\ddot{u}_{g2}$ were used in the bi-directional analyses, the component with larger peak ground acceleration was applied in the y-direction. All 30 records were independently used in the uni-directional response analyses. Because of the definition of the factor c, no scaling of the records was necessary to account for their different intensities, except for the study of the effect of the relative intensity of the x- and y-direction motions, as it will be explained later on.

Within the range of elastic behaviour, the equations of motion of the system of Figure 1(a) subjected to bi-directional ground acceleration are:

$$\begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_y \\ r^2\ddot{u}_\theta \end{Bmatrix} + \frac{2\xi}{\omega_2+\omega_3}\begin{bmatrix} \omega_1(\omega_2+\omega_3) & 0 & 0 \\ 0 & \omega_1^2+\omega_2\omega_3 & e_s\omega_1^2 \\ 0 & e_s\omega_1^2 & (\omega_\theta^2+\omega_2\omega_3)r^2 \end{bmatrix}\begin{Bmatrix} \dot{u}_x \\ \dot{u}_y \\ \dot{u}_\theta \end{Bmatrix}$$

$$+\,\omega_y^2\begin{bmatrix} (\omega_x/\omega_y)^2 & 0 & 0 \\ 0 & 1 & e_s \\ 0 & e_s & \Omega_\theta^2 r^2 + e_s^2 \end{bmatrix}\begin{Bmatrix} u_x \\ u_y \\ u_\theta \end{Bmatrix} = -\begin{Bmatrix} \ddot{u}_{gx} \\ \ddot{u}_{gy} \\ 0 \end{Bmatrix} \tag{2}$$

where ω_1, ω_2, and ω_3 are the natural frequencies of the coupled system, and ω_θ is the uncoupled torsional frequency, i.e., according to the definitions in the previous section: $\omega_1 = \omega_x = \omega$ and $r^2\omega_\theta^2 = K_\theta/m = d^2\omega_x^2 + a^2\omega_y^2$. The classical damping matrix was determined by superposition of the modal damping matrices assuming the same damping coefficient ξ, equal to 5 per cent of critical in this case, for all three modes. When one or more resisting elements yield Equation (2) is not longer valid, and the third term of the first member must be replaced by $\{F\}/m$, where $\{F\}$ is the vector of the resultant restoring forces and torque at that instant.

INELASTIC RESPONSE RESULTS

The response of asymmetric-plan systems subjected to uni-directional ground motion acting in the y-direction is examined first with the objective of identifying the influence of the system parameters combined with inelastic behaviour. For this purpose, response spectra were computed for the 30 ground motions considered in this study. Since the maxima of the basic coordinates u_y and u_θ cannot be combined because they do not occur at the same time, the results were presented in terms of u^+, the maximum lateral deformation of the stiff side element in the y-direction, and u^-, the maximum lateral deformation of the flexible side element in the same direction. These response variables are convenient because they directly represent the deformation demands on the structural elements.

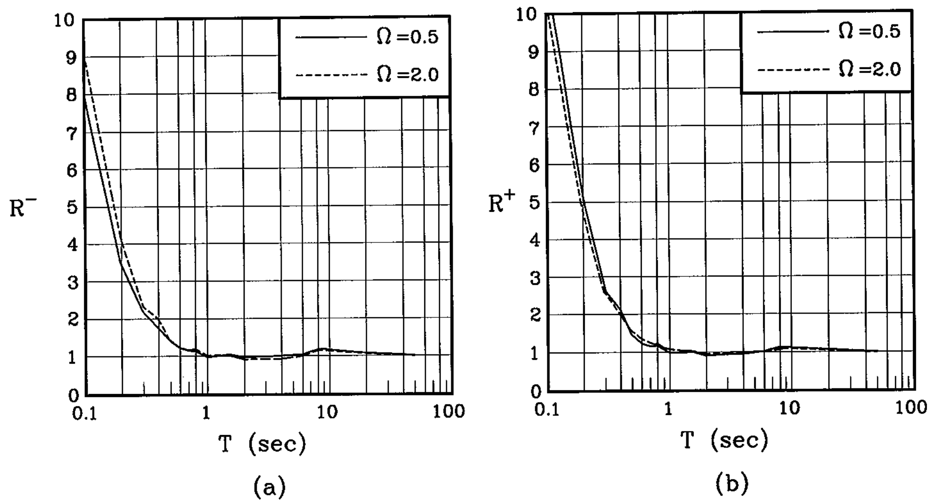

Figure 2. Average spectra for the ratio of maximum inelastic to elastic lateral deformation of the: (a) flexible-side element; and (b) stiff-side element. Systems with $e_s/r = 0{\cdot}2$ and $c = 0{\cdot}5$ subjected to uni-directional ground motion

The influence of Ω_θ, the uncoupled torsional-to-lateral frequency ratio, is studied first. The responses are presented in the form of average spectra of the ratios of peak inelastic to elastic lateral deformations:

$$R^+ = \frac{1}{n}\sum_i (u^+_{in}/u^+_{el})_i \tag{3a}$$

$$R^- = \frac{1}{n}\sum_i (u^-_{in}/u^-_{el})_i \tag{3b}$$

where i denotes each of the ground motion records, $n = 30$, and the elastic and inelastic systems are identical except for the limiting yield strength of the latter defined by parameter c (Equation (1)). Figure 2 shows the spectra for R^+ and R^- for the highest and lowest values of Ω_θ considered in this study, and for fixed values of the yield reduction factor and the stiffness eccentricity, $c = 0{\cdot}5$ and $e_s/r = 0{\cdot}2$, respectively. It is apparent from these figures that the maximum lateral deformations of both the stiff- and flexible-side element of the plan are affected very little by Ω_θ. From the results shown in Figure 2, it is also concluded that the effect of yielding strongly depends on the period of vibration of the system: as the period decreases from 1 sec, the lateral deformation ratios rapidly increase, reaching average inelastic to elastic response ratios between 8 and 9 for the flexible-side element, and over 10 for the element at the stiff side of the plan, whereas, in the intermediate and long period range ($T > 1$), lateral deformations are not affected by yielding so that inelastic and elastic systems experience, on the average, essentially the same lateral deformation.

The influence of the yield strength on the inelastic response of these systems is illustrated in Figure 3(a) in terms of factor c (Equation (1)). For systems with $T < 2$ the deformations increase as the yield factor decreases. For periods longer than 2 sec the lateral deformations do not necessarily increase when the yield level is reduced, and the average ratio changes only around 10% when c varies from 1 to 0·25. From the previous observations, it is concluded that the response of intermediate to long period asymmetric systems is primarily controlled by the ground displacement, as it occurs with Single-Degree-Of-Freedom (SDOF) systems.

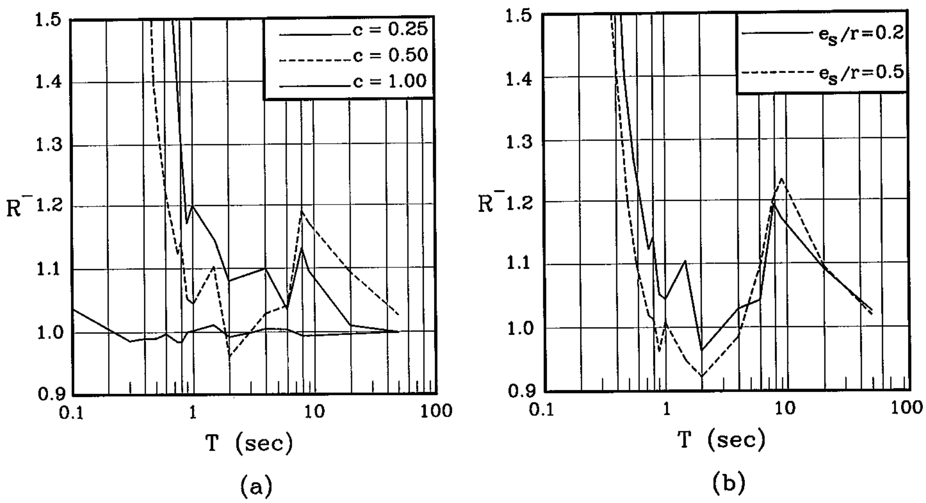

Figure 3. Average spectra for the ratio of inelastic to elastic lateral deformation of the flexible-side element, systems with $\Omega_\theta = 1$ subjected to uni-directional ground motion: (a) effect of yield factor for fixed eccentricity $e_s/r = 0{\cdot}2$; and (b) effect of stiffness eccentricity for a fixed yield factor $c = 0{\cdot}5$

The observations made with regard to Figures 2 and 3(a) are consistent with the findings of Goel and Chopra[3] and Correnza *et al.*:[5] for medium- and long-period structures the elements in the x-direction of the plan remain essentially elastic, thus, the structure behaves as it were torsionally rigid and responds in translation in the y-direction like a SDOF system. In turn, for a SDOF system it can be shown that on the average—for a number of records—for $T \geqslant 1$, the maximum inelastic displacement response u_{in} is approximately equal to the maximum response of the associated elastic system u_{el}. Indeed, $u_{\text{in}} = \mu u_Y = \mu u_{\text{el}}/R_\mu$, where μ is the ductility, and $R_\mu = u_{\text{el}}/u_Y$ is the response modification factor, which on the average is always greater than or equal to μ for $T \geqslant 1$, regardless of the soil conditions.[9] It is also worth to note in Figure 3(a) that although R^- is approximately 1 for $c = 1$, it is not necessarily equal to 1; the reason is that c defines the yield strength on the basis of the response of the symmetric elastic system, while the asymmetric system experiences torsion and slight inelastic response despite c being equal to 1.

Figure 3(b) illustrates how the stiffness eccentricity influences the response. At first sight it may surprise that for periods less than 5 sec the average response ratio decreases when the stiffness eccentricity increases. This is explained, however, by the fact that the effect of increasing e_s/r is larger for elastic systems compared to inelastic systems, thus the inelastic to elastic response ratios decrease when e_s/r increase. For very long-period systems, $T > 5$ in this case, the response ratio is not sensitive to e_s/r, finding consistent with the fact that in the limit $(T \to \infty)$ the lateral displacement tends to the peak ground displacement regardless of the value of e_s/r.

The effect of the action of a double-component ground motion is investigated by comparing the responses of the inelastic asymmetric-plan systems subjected to bi-directional and uni-directional input motions; the results are presented in terms of Q, the average spectra of the ratio of the peak deformations of the former to the latter cases:

$$Q = \frac{1}{n'} \sum_j (u_{bi}/u_{uni})_j \tag{4}$$

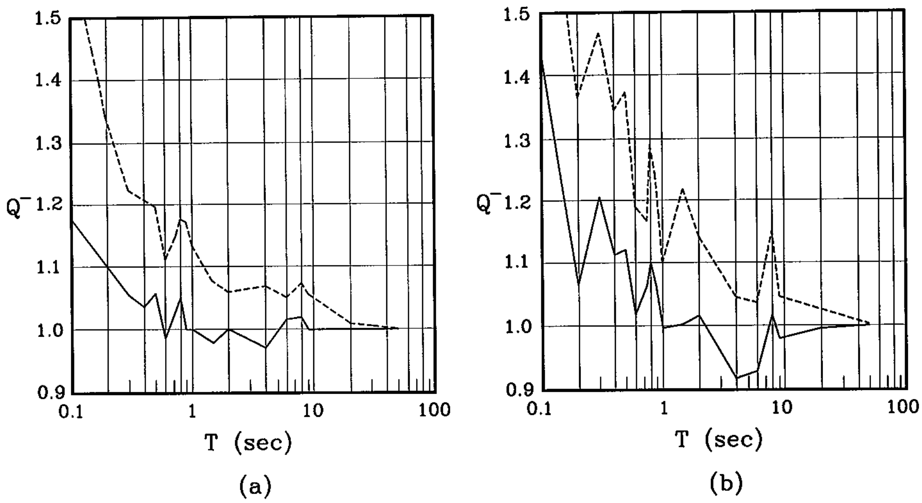

Figure 4. Mean and mean-plus-one-standard-deviation spectra for the ratio of bi-directional to uni-directional lateral deformation of the flexible-side element, systems with $\Omega_\theta = 1$ and $c = 0{\cdot}5$: (a) $e_s/r = 0{\cdot}2$; (b) $e_s/r = 0{\cdot}5$

where $n = 15$, and Q and the deformations u may wear a plus or minus superscript to denote the stiff- or flexible-side y-direction element, respectively. In both the bi- and uni-directional cases the record component with the largest peak ground acceleration was applied in the y-direction.

Figure 4 shows the mean and the mean-plus-one-standard-deviation spectra for Q^-, i.e. for the flexible-side element, for $c = 0{\cdot}5$, $\Omega_\theta = 1$, and $e_s/r = 0{\cdot}2$ and 0·5. The following observations can be made from this figure: (a) on the average, systems with periods larger than 1 are essentially unaffected by the presence of the second ground motion component, however, if a larger degree of conservatism is required, the effect shall be accounted for by considering the mean-plus-one-standard-deviation spectrum; (b) short-period systems are affected by the bi-directional lateral loading, the average response-ratio increases as T reduces from 1 to 0·1; and (c) the response amplification increases as the stiffness eccentricity increases, resulting in about 40 per cent average amplification for $T = 0{\cdot}1$ and $e_s/r = 0{\cdot}5$, and 20 per cent for $e_s/r = 0{\cdot}2$. These results are in agreement with the findings of Correnza et al.;[5] as they have indicated, accurate assessment of the response of the flexible-side element of short-period systems can be achieved only by means of bi-directional analyses.

The stiff-side element is slightly sensitive to the bi-directional ground motion, as inferred from the Q^+ spectra in Figures 5(a) and 5(b). It is worth noting from these figures that increasing e_s/r from 0·2 to 0·5 reduces the average response ratio at the stiff side of the plan for systems with $T \leqslant 2$, while the contrary occurs at the flexible side (Figure 4). It is therefore concluded that the bi-directional ground motion affects principally the flexible-side element of stiff structures, effect that further increases the already larger deformations such structures experience due to inelastic behaviour for uni-directional motion (Figure 2).

It is worth to note that the dispersion of computed responses (measured by the standard deviation of Q or by its coefficient of variation) is relatively small compared with typical dispersion of elastic response spectra for single-degree-of-freedom systems for a number of earthquake records. It can be seen in Figure 4(a) that COV(Q) varies from approximately 0·22 for

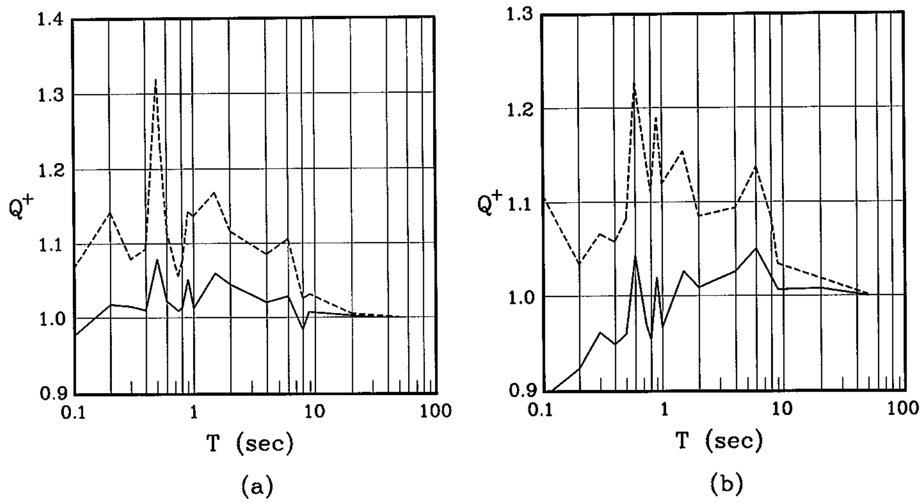

Figure 5. Mean and mean-plus-one-standard-deviation spectra for the ratio of bi-directional to uni-directional lateral deformation of the stiff-side element, systems with $\Omega_\theta = 1$ and $c = 0{\cdot}5$: (a) $e_s/r = 0{\cdot}2$; (b) $e_s/r = 0{\cdot}5$

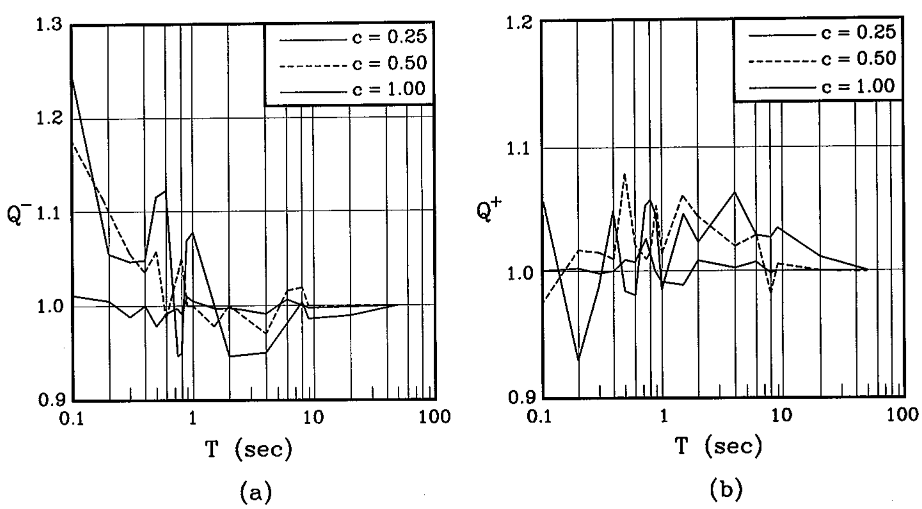

Figure 6. Average spectra for the ratio of bi-directional to uni-directional lateral deformation for systems with $e_s/r = 0{\cdot}2$, $\Omega_\theta = 1$ and various yield levels: $c = 0{\cdot}25$, $0{\cdot}5$ and $1{\cdot}0$: (a) flexible-side element; (b) stiff-side element

$T = 0{\cdot}2$ sec to 0.05 for $T = 10$ sec; in Figure 4(b) COV(Q) varies from 0·28 to 0·06, respectively, for the mentioned periods. COV values of the same order of magnitude can be inferred from Figure 5. The small COV of the bi-directional to uni-directional response ratio ($U_{\rm bi}/U_{\rm uni}$) is due to the fact that the random variables $U_{\rm bi}$ and $U_{\rm uni}$ are strongly correlated, i.e. their covariance is large and positive. The physical significance of such correlation is that the values of $u_{\rm bi}$ and $u_{\rm uni}$ for a given system tend to be both large (intense excitation and large torsional response) or both small (weak excitation and small torsional response) with respect to their respective means. Therefore, their ratio (Q) has small dispersion, i.e. tends to be clustered.

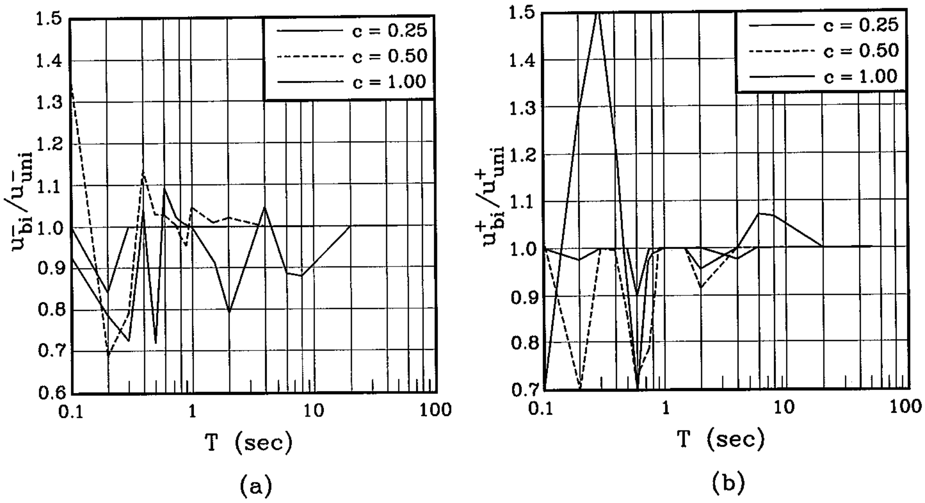

Fig. 7. Spectra for the ratio of responses to bi-directional and uni-directional Sylmar ground motion for systems with $e_s/r = 0{\cdot}5$, $\Omega_\theta = 1$, and various yield levels: $c = 0{\cdot}25$, $0{\cdot}5$ and 1: (a) flexible-side element; (b) stiff-side element

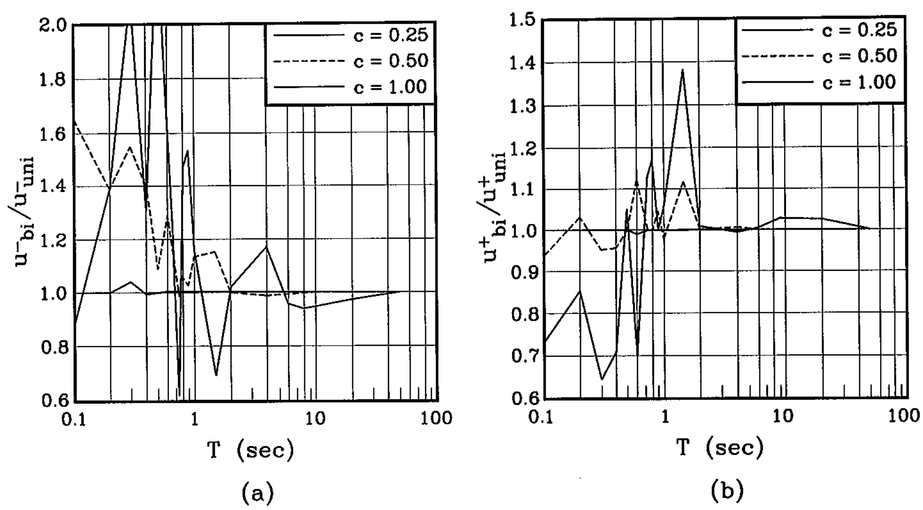

Figure 8. Spectra for the ratio of responses to bi-directional and uni-directional Llo–Lleo ground motion for systems with $e_s/r = 0{\cdot}5$, $\Omega_\theta = 1$, and various yield levels: $c = 0{\cdot}25$, $0{\cdot}5$ and 1: (a) flexible-side element; (b) stiff-side element

The effect of the yield factor c is illustrated in Figure 6. It is apparent that Q^+ is not much affected by c, and no clear trends are apparent either. For low-period systems however, Q^- increases when c reduces from 1, but there is no significant difference between the cases $c = 0{\cdot}25$ and $0{\cdot}5$.

The effect of c was further investigated by examining the u_{bi}/u_{uni} response ratios for individual records. Figure 7 shows the spectra for the Sylmar motion. It can be seen that for the flexible-side

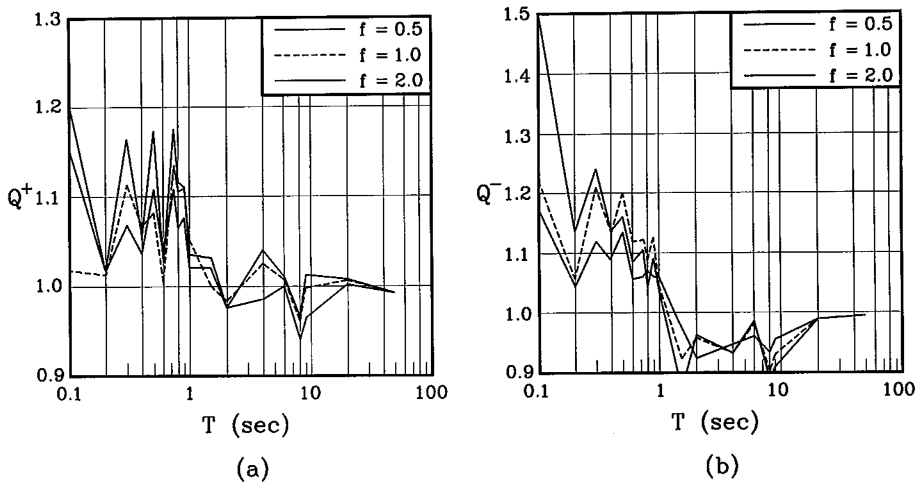

Figure 9. Average spectra for the ratio of bi-directional to uni-directional lateral deformation for systems with $e_s/r = 0{\cdot}2$, $\Omega_\theta = 1$, and $c = 0{\cdot}25$: (a) stiff-side element; (b) flexible-side element

element, $c = 0{\cdot}25$, i.e., the lowest strength case, results in the smallest lateral displacement ratios for almost all periods (Figure 7(a)), while practically the contrary occurs for the Llo–Lleo motion (Figure 8(a)). For the stiff-side element, however, the lowest yield factor c gives the largest response ratios for the Sylmar motion in the 0·15–0·5 period range (Figure 7(b)), while the opposite occurs for Llo–Lleo (Figure 8(b)). The previous examples explain why no clear trends were found regarding the effect of c in Figure 6; they are also useful to illustrate, first, the complex nature of the problem being addressed, and second, that responses to particular motions may feature significant differences, hence the need to consider a number of records to arrive to general conclusions.

The effect of the relative intensity of the two ground motion components was studied by varying the intensity of the x-direction component while the intensity of the y-direction component was kept constant. As before, the component with the largest peak ground acceleration was applied in the y-direction. Let $A_1 = \ddot{u}_{g1}^{max}$ and $A_2 = \ddot{u}_{g2}^{max}$ be the peak accelerations of the actually recorded components at one site, with $A_1 > A_2$, then $\ddot{u}_{gy}(t) = \ddot{u}_{g1}(t)$. The second component was normalized first to have a peak equal to the first component, then it was scaled by a factor f with values 0.5, 1 or 2. Thus, the ground acceleration histories applied in the x-direction correspond to

$$\ddot{u}_{gx}(t) = f\frac{A_1}{A_2}\ddot{u}_{g2}(t) \tag{5}$$

Clearly, when $f = 1$, the system is subjected to a bi-directional ground motion such that both components have the same peak acceleration, which in turn is equal to the original y-component peak.

The results are also presented in terms of Q (equation (4)) for the stiff- and flexible-side elements in the y-direction. Note that the reference uni-directional case remains the same regardless of the value of f used in the various bi-directional cases. Figure 9 shows the average spectra for the three values of f; it is apparent from these figures that the effect of the intensity of the x-direction excitation is not significant, except at the flexible side of the plan of very short-period systems (Figure 9(b)). For the stiff-side element (Figure 9(a)) the deformation generally increases as f increases, whereas no clear trends are found for the flexible-side element (Figure 9(b)) unless the period is very short. Response ratios u_{bi}/u_{uni} for individual records were also analysed but no clear trends were revealed.

The question may arise whether there could be differences in the conclusions if the records were grouped according to the seismic region they belong. For this purpose the two most numerous groups were considered: The six two-component Chilean records, and the five pairs of records in California, U.S.A. The comparison of responses of these groups did not reveal clear differences attributable to the tectonic environment.

CONCLUSIONS

The main observations presented in the previous section are summarized below:

1. The investigation of the response of inelastic systems to uni-directional ground motion led to the following conclusions: (a) the maximum lateral deformations of the elements parallel to the axis of asymmetry are not affected by a variation of the uncoupled torsional-to-lateral frequency ratio Ω_θ; (b) the effect of yielding is significant for short-period systems, resulting on average lateral deformations of the inelastic systems many times larger than those of the corresponding elastic systems, in turn, the deformations substantially increase as the yield strength decreases; and (c) in the intermediate and long-period ranges ($T > 1$ sec) lateral deformations are not much affected by yielding, so that inelastic and elastic systems experience, on the average, essentially the same lateral deformation, regardless of the yield strength.
2. The effect of the bi-directional ground motion, inferred from average spectra for the ratio of peak deformations for bi- and uni-directional excitation, is significant for the flexible-side element of the plan in short-period systems. The effect increases as the period decreases from 1, the response ratio being up to 1·2 and 1·4 for $T = 0{\cdot}1$ for a yield strength associated to a response reduction factor of 2 and plan eccentricities of 20 and 50 per cent of the plan radius of gyration, respectively. The stiff-side element is slightly sensitive to bi-directional ground motion.
3. The effect of the relative intensity of the two ground motion components, also inferred from average spectra for the ratio of responses to bi- and uni-directional excitation, is not significant, except at the flexible side of the plan of short-period systems. No clear trends are found when the response ratios for particular records are considered for various values of the factor used to scale one of the ground motion components to modify their relative intensity.
4. From the previous observations, it is concluded that the effect of inelastic behaviour in torsional systems subjected to uni-directional ground motion is significant for short-period systems ($T < 1$). Furthermore, if bi-directional excitation is taken into consideration, the average deformations at the flexible side of the plan of such systems increase. The deformations increase as the eccentricity and the intensity of the second component of ground motion increase, and as the yield strength and the period decrease.
5. Comparison of responses for ground motion records grouped according to their source—Chile and California in particular—does not reveal clear differences attributable to the tectonic environment. With regard to this subject, a detailed study using a larger number of records is advisable.

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