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# ASSESSMENT ON PILE EFFECTIVE LENGTHS AND THEIR EFFECTS ON DESIGN—I. ASSESSMENT

**Y. Chen**

Civil Engineering Program, Penn State University, Harrisburg, Middletown, PA 17057-4898, U.S.A.

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**Abstract**—Various methods for determining depths to fixity as required in an equivalent cantilever pile model, including the current practical method (LRFD method) and the proposed method with a comprehensive numerical procedure, are described and discussed. Suitability and applicability of the methods are carefully assessed with applications to steel H piles. Copyright © 1996 Elsevier Science Ltd.

## NOTATION

| | |
|---|---|
| $A, B$ | constants describing $k_h$ |
| $A_j, B_j$ | soil constants associated with $k_{hj}$ |
| $A(i), B(i)$ | soil constants associated with $k_{hi}$ (matrix notation) |
| $DIFF_1$ | variable defined as LRFD $L_f - L_{fm}$ |
| $DIFF_2$ | variable defined as LRFD $L_f - L_{fb}$ |
| $E_c$ | soil modulus for clays |
| $E_p$ | modulus of elasticity of pile |
| Error | the convergence allowance for $L_0$ |
| $H$ | total soil depth (Fig. 3) |
| $H_A$ | abutment height |
| $h_i, h(i)$ | soil layer depth (Fig. 3) |
| $ht(i)$ | terminating soil layer depth |
| $I_p$ | moment of inertia of pile |
| $I_{py}$ | moment of inertia of pile about $y$ (weak) axis |
| $i, j$ | index number |
| Ierr | control code for errors |
| Iquit | control code for quitting |
| $K$ | the soil layer number depending upon $L_0$ value |
| $k$ | a site-dependent coefficient (=0.25, usually) |
| $k_{ave}$ | weighted $k_h$ value |
| $k_e$ | effective horizontal soil stiffness |
| $k_h$ | horizontal stiffness corresponding to $j$th soil layer |
| $k_z$ | vertical subgrade modulus |
| $L_a$ | actual total pile length |
| $L_c$ | pile length at which the pile behaves flexibly |
| $L_e$ | embedded pile length |
| $L_f$ | depth to fixity (general) |
| $L_0$ | the active pile length in bending ($\approx 0.5L_c$) |
| $L_u$ | unbraced pile length (general) |
| $L_c(1)$ | $L_c$ considering predrilled hole |
| $L_c(2)$ | $L_c$ ignoring predrilled hole |
| $L_b, L_{fb}$ | depth to fixity for buckling |
| $L_m, L_{fm}$ | depth to fixity for bending moment |
| $L_s, L_{fs}$ | depth to fixity for horizontal stiffness |
| $L_u(1)$ | unbraced pile length considering predrilled hole |
| $L_u(2)$ | unbraced pile length ignoring predrilled hole |
| $N$ | the total number of soil layers |
| $N_0$ | the terminating layer number depending on $L_0$ value |
| $N'_{70}$ | standard blow count for energy ratio of 0.70 |
| $N_{ave}$ | average standard blow count for energy ratio of 0.70 |
| $n_h$ | rate of increase of soil modulus with depth for sands |
| $P$ | axial load |
| $Q_{m,max}$ | maximum $Q_m(z)$ as found from eqns (20)–(22) |
| $q_u$ | undrained compressive strength |
| $R$ | a length defined by eqn (7) |
| $S_u$ | undrained shear strength of clays |
| $T$ | a length defined by eqn (8) |
| $TL_b$ | total effective pile length for buckling |
| $TL_m$ | total effective pile length for bending |
| $TL_s$ | total effective pile length for horizontal stiffness |
| $u$ | a mathematical parameter defined as $\beta_c L_u$ |
| $uc$ | unit conversion factor |
| $v$ | a mathematical parameter defined as $\alpha L_u$ |
| $Wk_h$ | weighted $k_h$ |
| $x$ | length ratio defined as $L_u/L_c$ |
| $x(1)$ | $L'_u(1)/L_c(1)$ |
| $x(2)$ | $L'_u(2)/L_c(2)$ |
| $Y_b$ | length ratio defined as $L_{fb}/L_c$ |
| $Y_m$ | length ratio defined as $L_{fm}/L_c$ |
| $Y_s$ | length ratio defined as $L_{fs}/L_c$ |
| $y, z$ | coordinates (Fig. 2) |
| $\alpha$ | a mathematical parameter defined by eqn (17) |
| $\xi$ | a mathematical parameter given by eqn (23) |
| $\beta_c$ | a mathematical parameter defined by eqn (23) or eqn (25) |
| $\Delta H$ | the layer thickness $[=h(i) - h(i-1)]$ |

## INTRODUCTION

Steel piles with I shape (i.e. H piles) are commonly employed in civil engineering construction, particularly for bridge structures. For example, steel H piles are often used to support integral abutments because of their structural efficiency and cost competitiveness. In analysis, as shown in Fig. 1, four possible pile models may be used: (1) three-dimensional bridge–soil–pile model, (2) equivalent soil springs model, (3) equivalent base spring model, and (4) equivalent cantilever model. Descriptions of these models follows.

The *full three-dimensional model*, usually working together with the finite element (FE) method, is the most sophisticated model and generally gives better results. But this method requires substantial modeling effort that would be quite difficult for design engineers. Also, the total cost involved in the

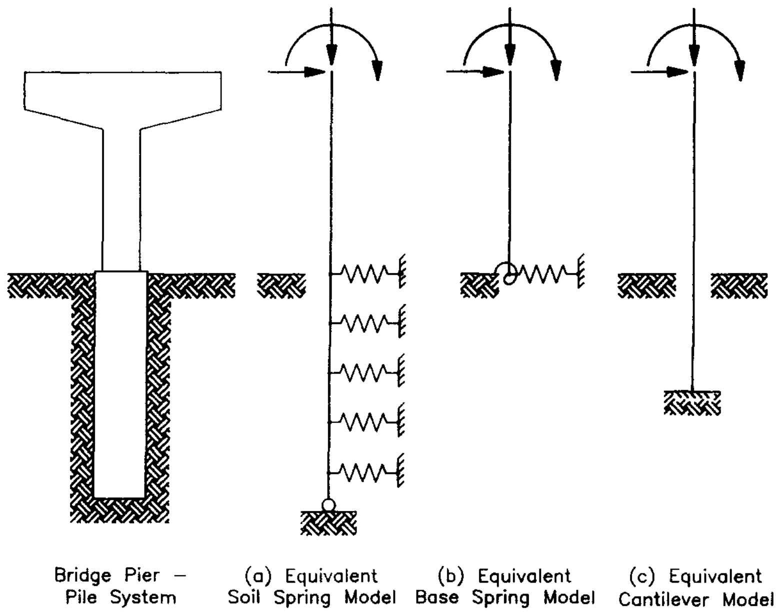


Fig. 1. Soil-pile models.

three-dimensional FE model could be prohibitive and thus becomes impractical. The *equivalent soil springs model* involves the inclusion of piles into the superstructure model and the use of axial load–lateral displacement curves to represent the soil. The accuracy of this model primarily depends upon the spacing between the nodes used to attach the soil springs to the pile. The closer the spacing is, the better the accuracy would be. So, the total cost of computation associated with the model might be very high too. The *equivalent base spring model* assumes elastic soil behavior and a set of at least six independent springs (three sliding plus three rocking) acting at the ground surface. This model can be quite satisfactory provided that the cross coupling terms, often ignored for footings, are included in the stiffness matrix. However, calculating these terms can be a major effort as it is frequently done by a substructuring technique.

The *equivalent cantilever model* is the simplest approach, and is currently adopted in practice. The cantilever section is the same as that of the actual pile being modeled as a beam–column, but its embedded length is adjusted to give either the same stiffness at

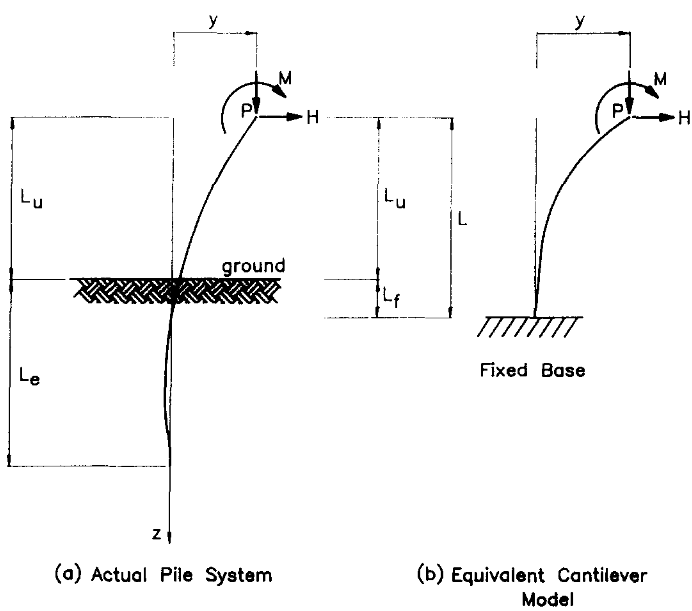


Fig. 2. Equivalent cantilever pile model.

Table 1. Representative $N'_{70}$ values for saturated clays [4]

| Clay type | $N'_{70}$ |
|---|---|
| Soft | 3–5 |
| Medium | 6–9 |
| Stiff | 10–16 |
| Very stiff | 17–30 |

the ground level, the same buckling strength, or the same maximum bending moment as in the actual soil–pile system. The adjusted embedded pile length is referred as *depth to fixity* ($L_f$). The equivalent cantilever model can produce good results if proper $L_f$ values are furnished.

The $L_f$ values as needed in the equivalent cantilever model can be determined from a detailed substructure model or from considerations of the relative stiffnesses of the pile and soil. Information related to $L_f$ is sparsely scattered in the literature and limited. A literature survey also reveals that the suitability and applicability of the existing formulas need to be further clarified in some cases or established in other cases. Moreover, engineers may have misused or misinterpreted the available $L_f$ equations.

This paper first describes and discusses various methods for computing $L_f$ values, including the proposed method with a comprehensive numerical procedure (Part I). Then, the effects of $L_f$ values and lateral movement on the strength an design of steel H piles are discussed in detail (Part II). The objectives of this study are to clarify and establish the suitability and applicability of the various $L_f$ equations, and to assist engineers to analyze and design piles.

## DEPTH TO FIXITY FOR PILES

### *Governing equation*

The depth to fixity for *prismatic* piles, $L_f$, can be derived using the beam-on-elastic-foundation theory, namely the following governing differential equation [1], Fig. 2:

$$E_p I_p \frac{d^4 y}{dz^4} + P\frac{d^2 y}{dz^2} + k_z y = 0, \tag{1}$$

where $E_p$ is the modulus of elasticity of the pile, $I_p$ the moment of inertia of the pile, $P$ the axial load, $y$ and $z$ the coordinates shown on Fig. 2, and $k_z$ is the vertical subgrade modulus.

To minimize the stresses at the top of the piles attached to an integral abutment, it is necessary to place the steel piles such that they are subject to weak-axis bending regardless of bridge skew, namely to orient the weak ($y$) axis of the H piles perpendicular to the longitudinal axis of the bridge. $I_p$ is therefore replaced by $I_{py}$ in the following equations, unless noted otherwise.

Table 2. Representative $E_e$ values for saturated clays

| Clay type | $N_{ave}$ | $q_u$[1] (ksf) | $S_u$[2] (ksf) | $S_u$ (tsf) | $E_e$[3] (tsf) |
|---|---|---|---|---|---|
| Soft | 4 | 1 | 0.5 | 0.25 | 16.75 |
| Medium | 7.5 | 1.875 | 0.94 | 0.47 | 31.4 |
| Stiff | 13 | 3.25 | 1.625 | 0.81 | 54.4 |
| Very stiff | 23.5 | 5.875 | 2.94 | 1.47 | 98.5 |

[1]By eqn (5); [2]by eqn (4); [3]by eqn (3).

Table 3. Representative $n_h$ values for sands [2]

| Sand type | Saturation condition | $n_h$ (tsf ft$^{-1}$) |
|---|---|---|
| Loose | Moist/dry | 30 |
| | Submerged | 15 |
| Medium | Moist/dry | 80 |
| | Submerged | 40 |
| Dense | Moist/dry | 200 |
| | Submerged | 100 |

### *Simplified method (the LRFD method)*

AASHTO LRFD [2] adopts the simplified formulas proposed by Davison and Robinson in 1965 [3] to determine the depth to fixity ($L_f$). For piles in *clays*, $L_f$, measured from the ground, is calculated from

$$L_f = 1.4\left[\frac{E_p I_{py}}{E_e}\right]^{0.25}, \tag{2}$$

where $E_p$ is in ton per square foot (tsf), $I_{py}$ in ft$^4$, and $E_e$ is the soil modulus for clays in tsf.

$E_e$ can be found from eqns (3)–(5) [2, 4]

$$E_e = 67S_u \tag{3}$$

$$S_u = \frac{q_u}{2} \tag{4}$$

$$q_u = kN_{ave}, \tag{5}$$

where $S_u$ is the undrained shear strength of clays (tsf), $q_u$ the undrained compressive strength, $k$ a site-dependent coefficient (=0.25, usually), and $N_{ave}$ is the average standard blow count for energy ratio of 0.70 (i.e. $N'_{70}$). Representative $N'_{70}$ and $E_e$ values are shown in Tables 1 and 2, respectively.

For piles in *sands*, $L_f$, also measured from the ground, is computed by

$$L_f = 1.8\left[\frac{E_p I_{py}}{n_h}\right]^{0.20}, \tag{6}$$

Table 4. Representative $k_h$ relations [6]

| Soil type | $k_h$ (k) |
|---|---|
| Loose sand | $8z$ |
| Medium sand | $27z$ |
| Dense sand | $72z$ |
| Soft clay | $24 + 5.8z \leq 72$ |
| Medium clay | $107 + 23.4z \leq 326$ |
| Stiff clay | $190 + 41z \leq 580$ |
| Very stiff clay | $750 + 610z \leq 2200$ |

where $n_h$ is the rate of increase of soil modulus with depth for sands (tsf ft$^{-1}$), Table 3.

Equations (2) and (6) were based on beam-on-elastic-foundation theory, and are intended for *partially* embedded piles. The coefficients of 1.4 in eqn (2) and 1.8 in eqn (6) were suggested for simplification and compromise such that each equation is applicable to both *bending* and *buckling* behaviors. The error involved in eqn (2) (for piles in clays) is 8% or less provided that the unbraced pile length ($L_u$) is not less than $2R$ [$R$, by eqn (7)], and is under 4% in eqn (6) (for piles in sands) if $L_u$ is not less than $T$ [$T$, by eqn (8)] [3].

$$R = \left[\frac{E_p I_p}{E_s}\right]^{0.25} \tag{7}$$

$$T = \left[\frac{E_p I_p}{n_h}\right]^{0.20}. \tag{8}$$

Equations (2) and (6) are also included in the FHWA report [5] which deals with the seismic design of highway bridges. During the installation of piles for integral abutment, it is a good practice to provide at least 10 ft of predrilled hole filled with soft granular material to alleviate the piling stress [6, 7]. However, in terms of lateral support the predrilled hole portion might be questionable. Therefore, as a conservative approach, $L_u$ may be taken as the pile length above the ground, plus the predrilled hole depth.

Furthermore, LRFD eqns (2) and (6) do not distinguish between fixed-headed piles (FHP) and pinned-headed piles (PHP), and are applicable to a *single* soil layer only. For multiple soil layers, one has to come up with an equivalent soil layer (sand or clay) when exercising these equations.

*Theoretical method (analytical method)*

In addition to bending and buckling, Greimann *et al.* [6] considered the *horizontal stiffness* mode. They also considered two typical support conditions at the pile head: fixed, and pinned.

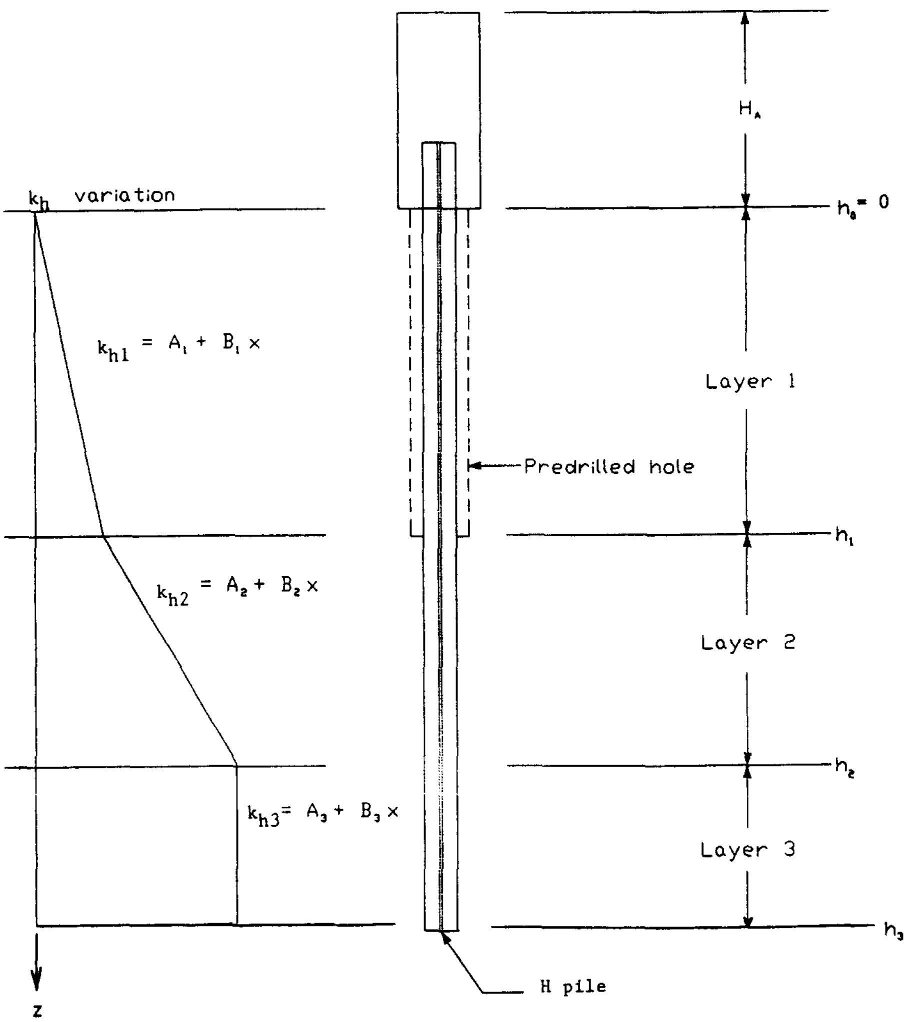


Fig. 3. Multi-layered soil (three layers being shown).

Let us define $x = L_u/L_c$, $Y_s = L_{fs}/L_c$, $Y_m = L_{fm}/L_c$, and $Y_b = L_{fb}/L_c$, where $L_{fs}$, $L_{fm}$ and $L_{fb}$ are the depth to fixity for horizontal stiffness, bending and buckling, respectively, and $L_c$ is the pile length at which the pile behaves flexibly defined as [8]

$$L_c = 4\left[\frac{E_p I_{py}}{k_e}\right]^{0.25}, \tag{9}$$

where $k_e$ is the effective horizontal soil stiffness determined by

$$k_e = \frac{3}{L_0^3}\int_0^{L_0} k_h (L_0 - z)^2 \, dz, \tag{10}$$

where $L_0$ is the active pile length in bending approximate to one half of $L_c$, and $k_h$ is the horizontal soil stiffness.

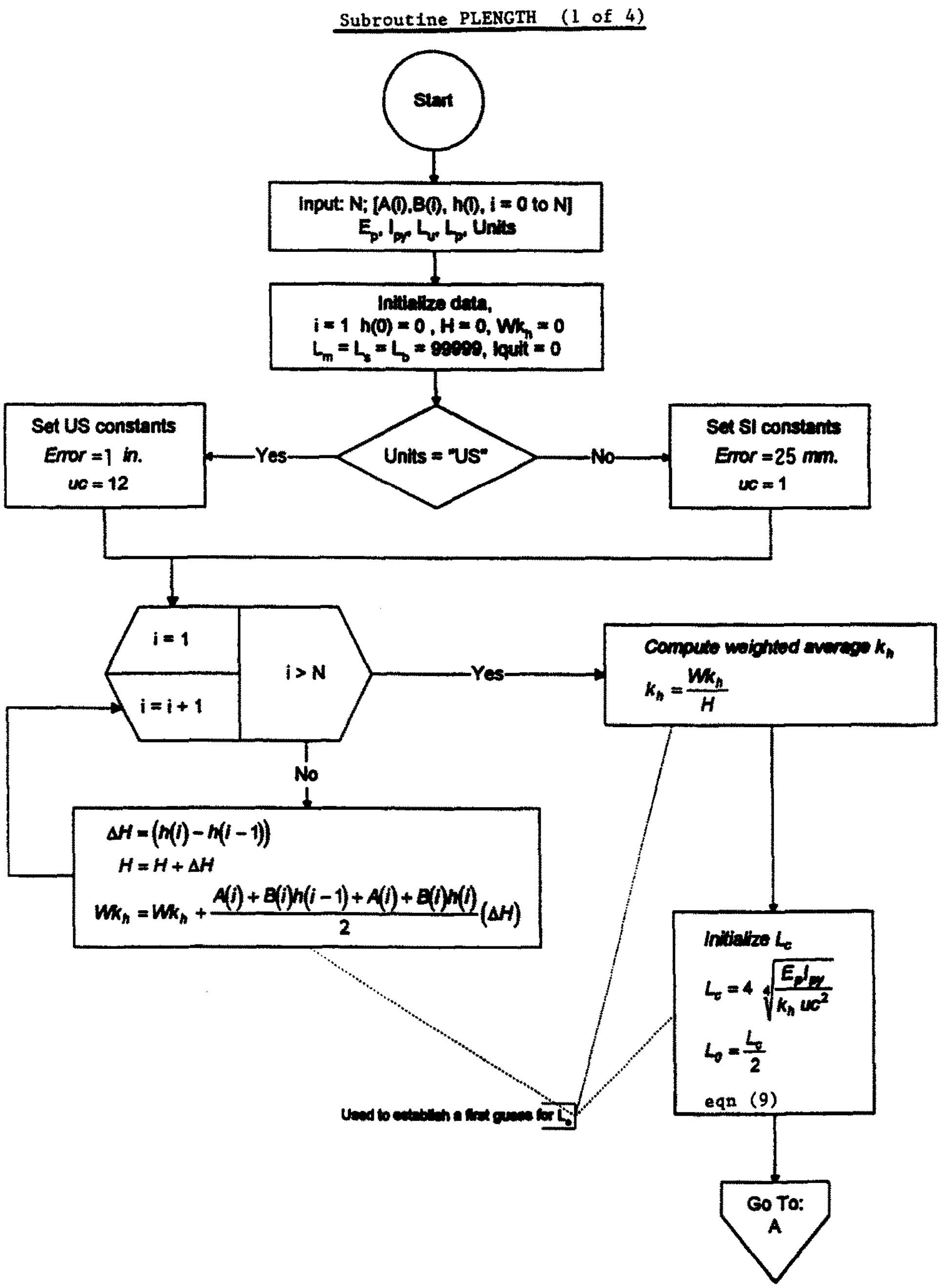


Fig. 4. Proposed numerical procedure "PLENGTH" for computing total effective pile lengths—calculating weighted average $k_h$ value and initializing $L_c$ value.

$k_h$ is constant for sands, but varies linearly with the soil depth ($z$), namely

$$k_h = A + Bz, \tag{11}$$

where $A$ and $B$ are constants dependent on the soil.

Representative $k_h$ values are summarized in Table 4 [6]. Exact formulas for $Y$s (normalized $L_f$s) are presented in eqns (12)–(14) (FHP), and eqns (18), (19) and (24) (PHP), as follows.

*Fixed-headed piles* (*FHP*)

For horizontal stiffness,

$$Y_s = \left[\frac{256x^4 + 362x^3 + 192x^2 + 67.9x + 12}{256x + 90.5}\right]^{1/3} - x. \tag{12}$$

For bending moment,

$$Y_m = \left[\frac{128x^4 + 181x^3 + 96x^2 + 33.9x + 6}{128x^2 + 90.5x + 16}\right]^{1/2} - x. \tag{13}$$

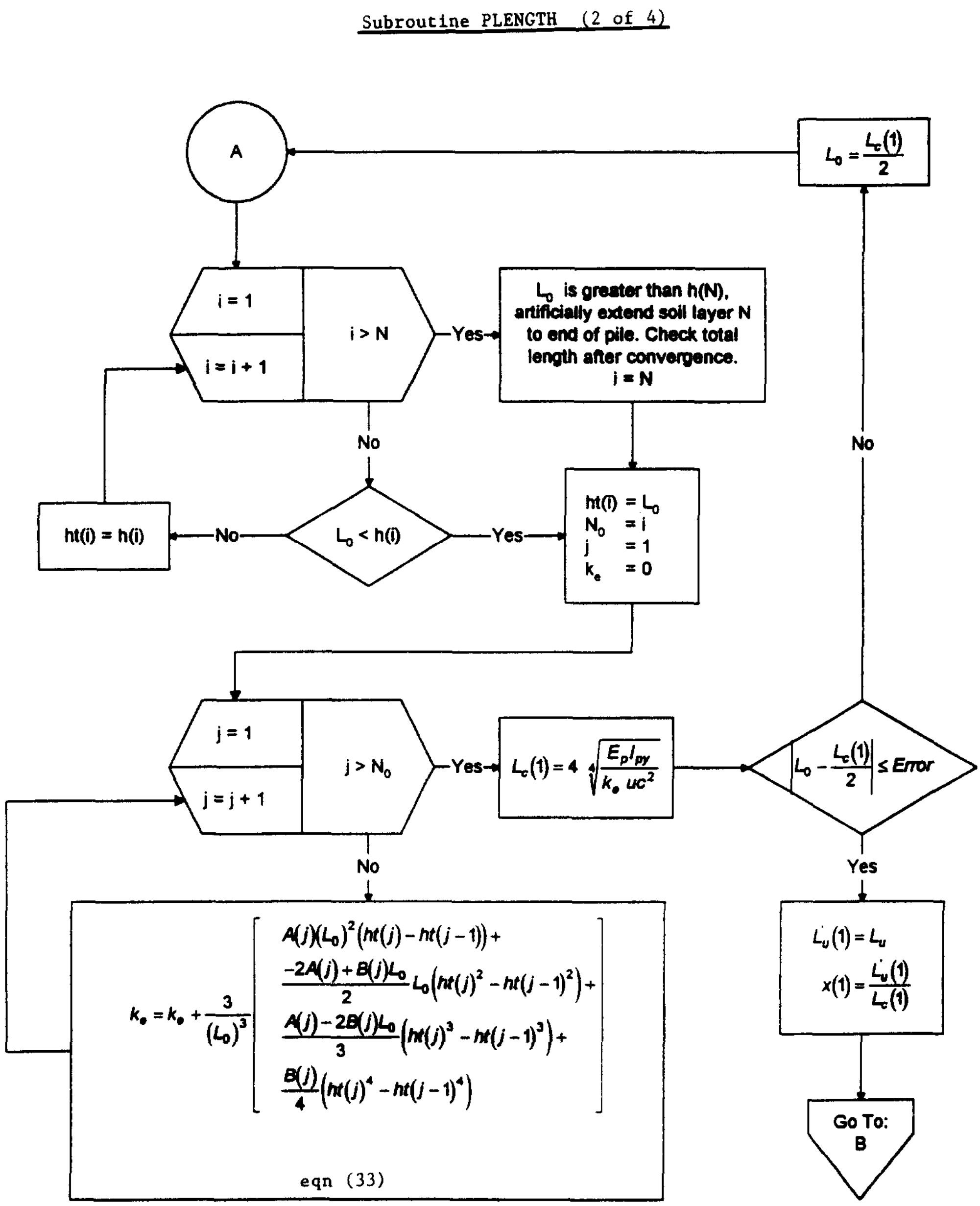


Fig. 5. Proposed numerical procedure "PLENGTH" for computing total effective pile lengths—calculating $k_e$ and $L_c$ by considering predrilled hole depth.

For buckling,

$$Y_b = \left[\frac{2\pi}{\beta_e L_u} - 1\right]x, \tag{14}$$

where $\beta_e$ is defined as

$$\beta_e = \sqrt{\frac{P}{E_p I_p}}, \tag{15}$$

where $P$ is the axial load of pile.

Mathematically, $\beta_e$ is found from

$$u\sin(u)\left\{1 - \left(\frac{u}{v}\right)^2 - \frac{1}{v}\left[1 + \left(\frac{u}{v}\right)^2\right]\sqrt{2-(u/v)^2}\right\}$$

$$+\cos(u)\left\{2 - \left(\frac{u}{v}\right)^4 + v\left(\frac{u}{v}\right)^2\sqrt{2-(u/v)^2}\right\} - 2 = 0, \tag{16}$$

Subroutine PLENGTH (3 of 4)

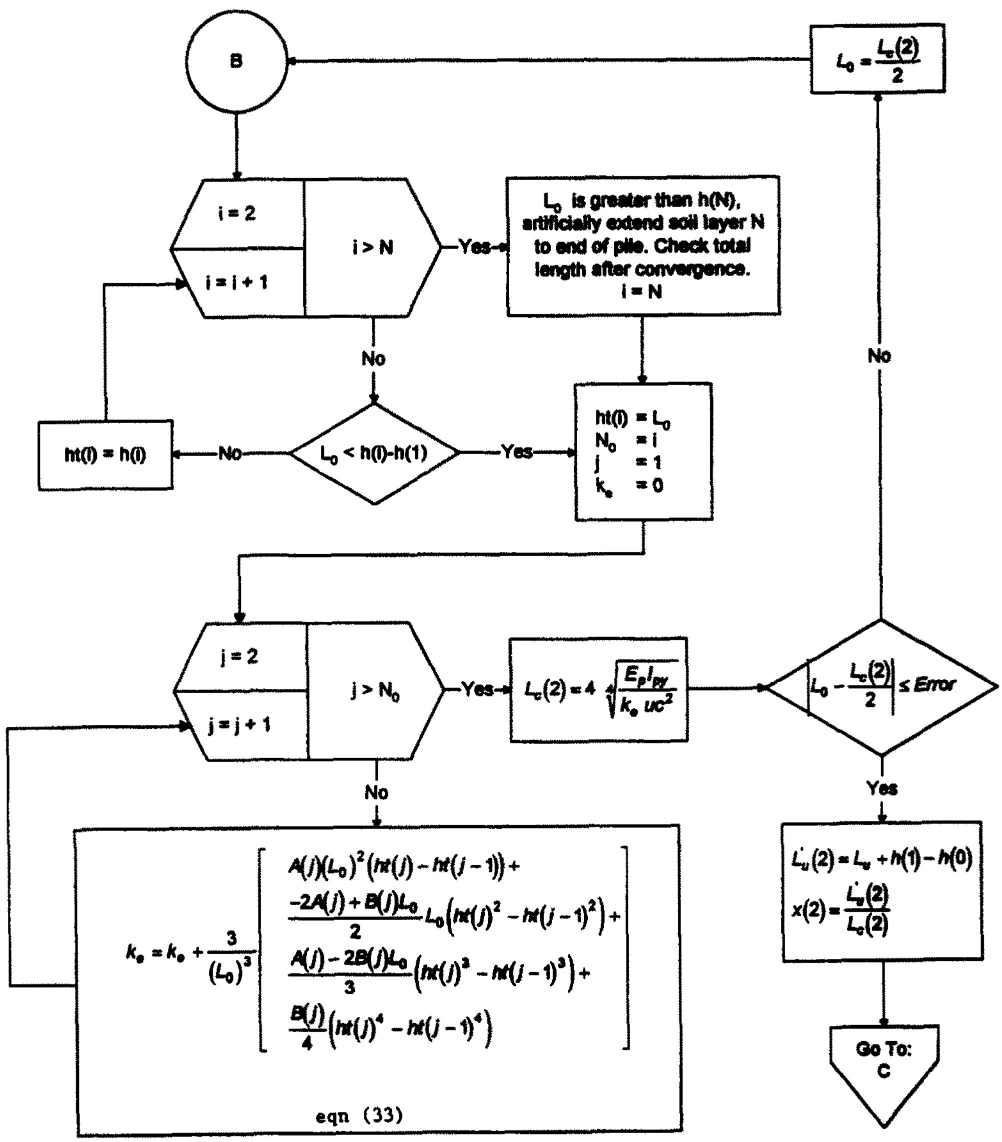


Fig. 6. Proposed numerical procedure "PLENGTH" for computing total effective pile lengths—calculating $k_e$ and $L_c$ by ignoring predrilled hole depth.

where $u = \beta_e L_u$, and $v = \alpha L_u$ with $\alpha$ defined by

$$\alpha = \sqrt{\frac{k_h}{E_p I_{py}}}. \tag{17}$$

*Pinned-headed piles (PHP)*

For horizontal stiffness,

$$Y_s = [x^3 + 4.2x^2 + 0.375x]^{1/3} - x. \tag{18}$$

For bending moment,

$$Y_m = \sqrt{\frac{1}{5.33 Q_{m,max}}[15.1x^3 + 16x^2 + 5.6x + 1]} - x, \tag{19}$$

where $Q_{m,max}$ is the maximum value found from eqns (20)–(22)

$$Q_m(z) = k_h L_c K_{mh}(z) - 2.8x K_{mm}(z) \tag{20}$$

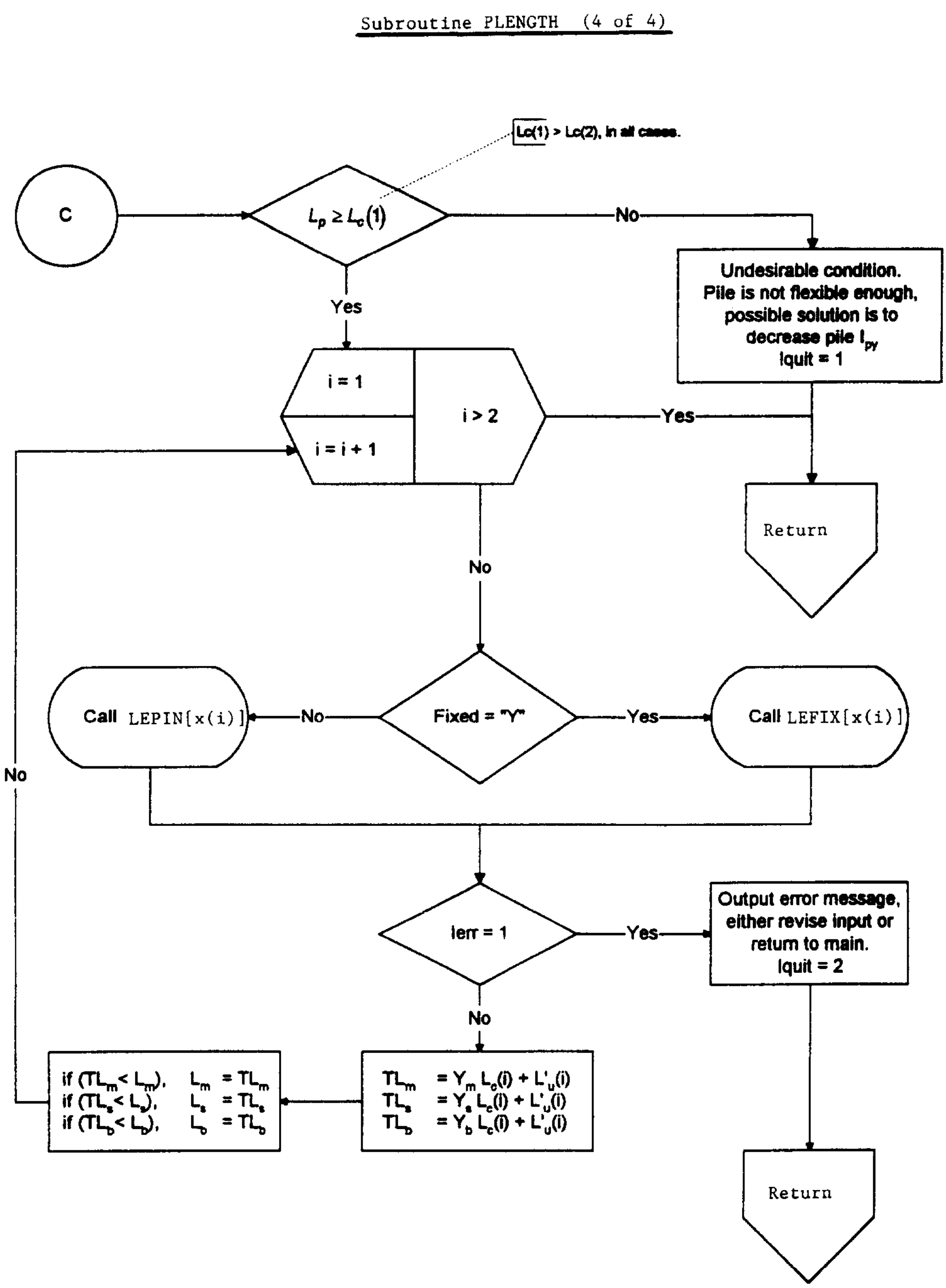


Fig. 7. Proposed numerical procedure "PLENGTH" for computing total effective pile lengths—determining the critical total effective pile lengths ($TL_m$, $TL_s$, $TL_b$).

$$K_{mh}(z) = \frac{\sin\xi\,\sin(\xi z/L_a)\sinh[\xi(1 - z/L_a)] - \sin\xi\,\sinh(\xi z/L_a)\sin[\xi(1 - z/L_a)]}{\sinh^2\xi - \sin^2\xi} \quad (21)$$

$$K_{mm}(z) = \frac{\sinh\xi\{\sinh[\xi(1 - z/L_a)]\cos(\xi z/L_a) + \cosh[\xi(1 - z/L_a)]\sin(\xi z/L_a)\} + \sin\xi\{\sinh(\xi z/L_a)\cos[\xi(1 - z/L_a)] + \cosh(\xi z/L_a)\sin[\xi(1 - z/L_a)]\}}{\sinh^2\xi - \sin^2\xi}, \quad (22)$$

where $L_a$ is the actual total pile length, and $\xi$ is given by

$$\xi = \frac{4L_a}{\sqrt{2L_c}}. \quad (23)$$

For buckling,

$$Y_b = \left[\frac{\pi}{0.70\beta_c L_c} - 1\right]x, \quad (24)$$

Function LEFIX

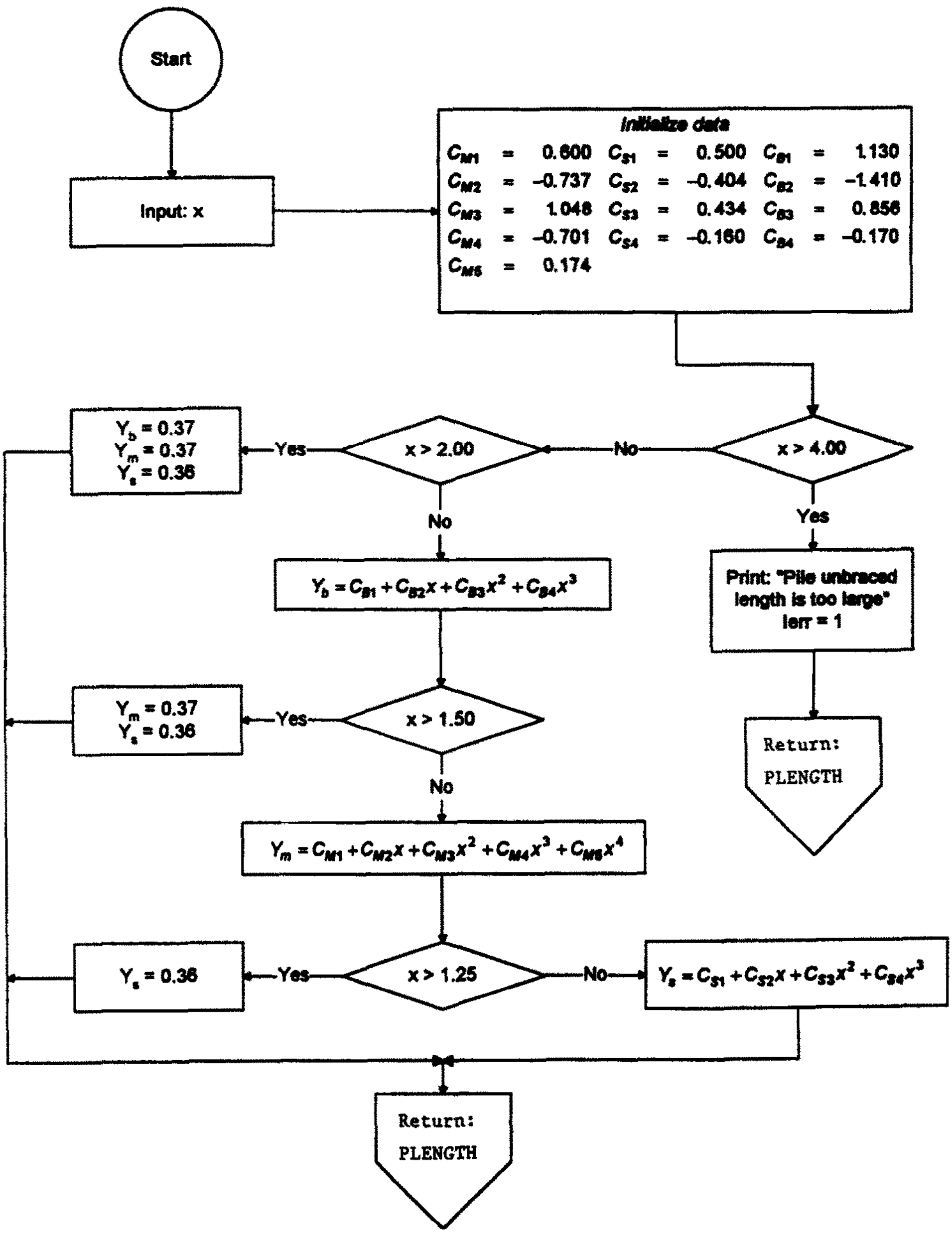


Fig. 8. Proposed numerical procedure "LEFIX" for calculating the fixity depths for fixed-headed piles.

where $\beta_e$ is solved from

$$\frac{\tan(u)}{u} = \frac{1 - \left(\frac{u}{v}\right)^2 - \frac{1}{v}\left(\frac{u}{v}\right)^2 \sqrt{2 - (u/v)^2}}{1 + \left(\frac{u}{v}\right)^2 - \left(\frac{u}{v}\right)^4 + v\left(\frac{u}{v}\right)^2 \sqrt{2 - (u/v)^2}} \tag{25}$$

*Approximate/numerical method (the proposed method)*

Solving the above analytical equations, especially eqns (14), (19) and (24), is a very difficult task. Therefore, the approximate formulas, which are intended for use in practice, were derived as follows.

*Fixed-headed piles (FHP).* For horizontal stiffness,

$$Y_s = L_c(0.500 - 0.404x + 0.434x^2 - 0.160x^3),\ 0 \leqslant x \leqslant 1.25, \tag{26a}$$

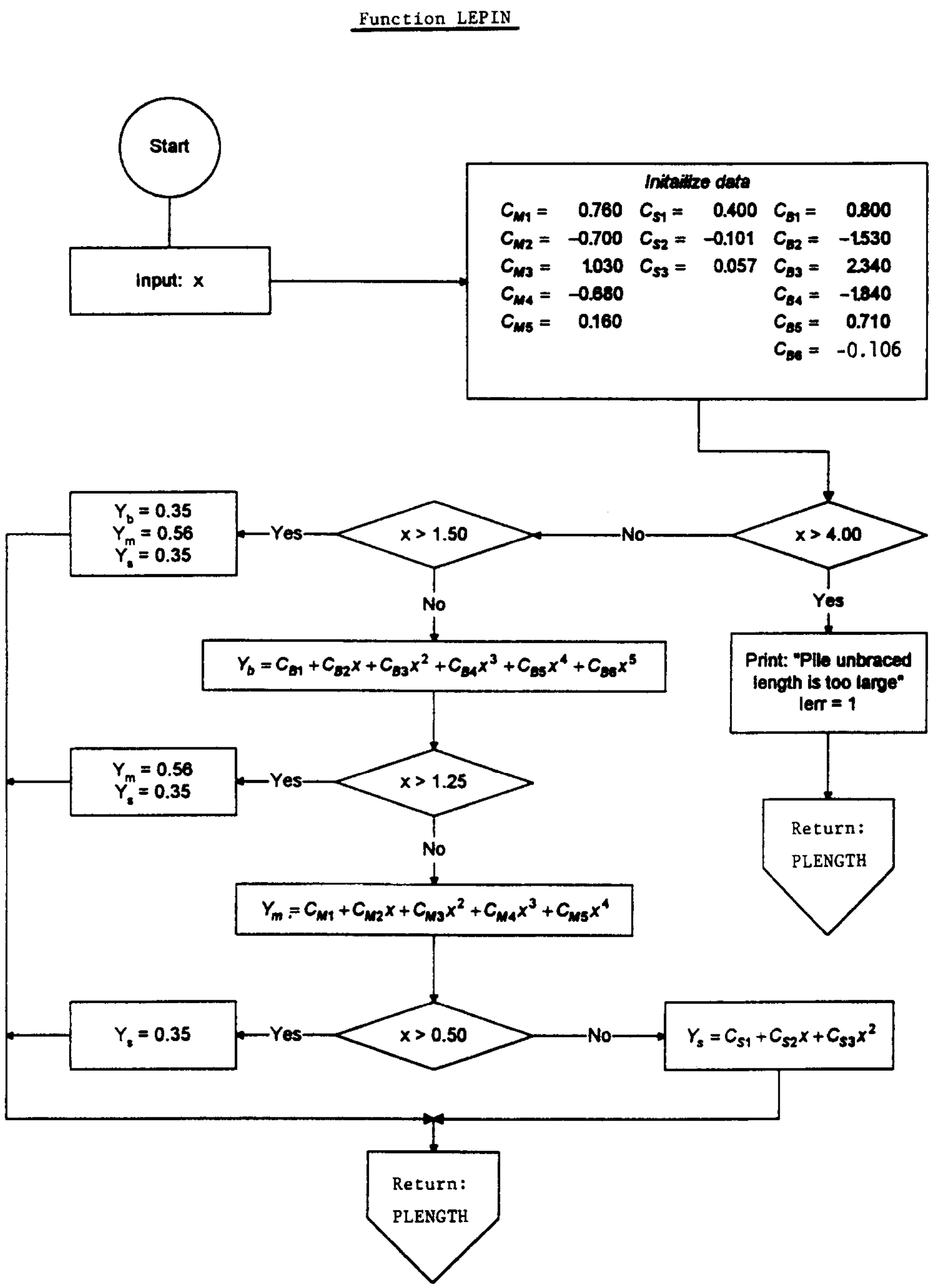


Fig. 9. Proposed numerical procedure "LEPIN" for calculating the fixity depths for pinned-headed piles.

$$Y_s = 0.36L_c, \quad 1.25 \leqslant x \leqslant 4.0. \quad (26b)$$

Here, it should be noted that 4.0 is the practical maximum value for $x$.

For bending moment,

$$Y_m = L_c(0.600 - 0.737x + 1.048x^2 - 0.701x^3 + 0.174x^4), \quad 0 \leqslant x \leqslant 1.5, \quad (27a)$$

$$Y_m = 0.37L_c, 1.5 < x < 4.0. \quad (27b)$$

For buckling,

$$Y_b = L_c(1.13 - 1.41x + 0.856x^2 - 0.17x^3), \quad 0 \leqslant x \leqslant 2.0, \quad (28a)$$

$$Y_b = 0.37L_c, \quad 2.0 < x \leqslant 4.0. \quad (28b)$$

*Pinned-headed piles* (*PHP*). For horizontal stiffness,

$$Y_s = L_c(0.400 - 0.101x + 0.057x^2), \quad 0 \leqslant x \leqslant 0.50, \quad (29a)$$

$$Y_s = 0.35L_c, \quad 0.50 < x \leqslant 4.0. \quad (29b)$$

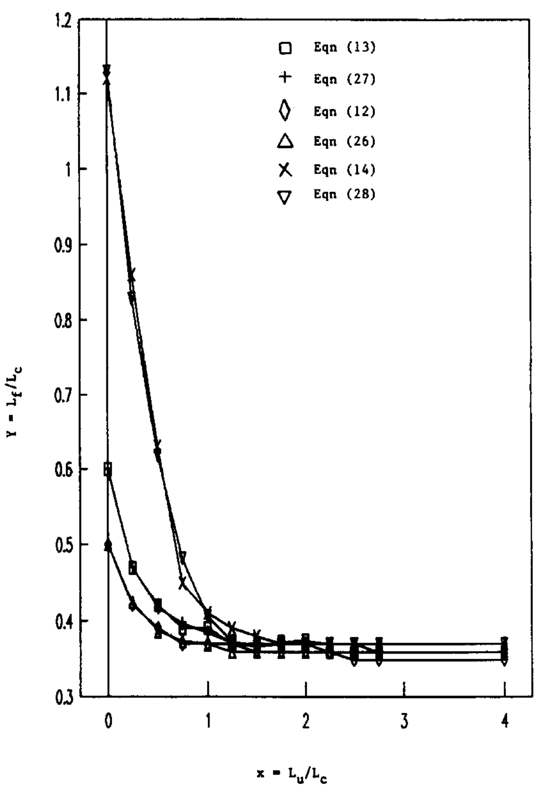


Fig. 10. Comparisons between eqns (26)–(28) and eqns (12)–(14) (fixed-headed piles).

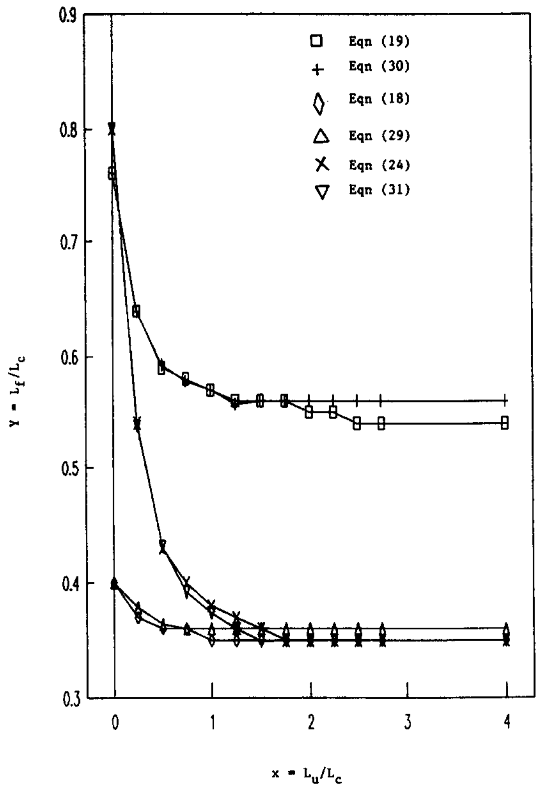


Fig. 11. Comparisons between eqns (29)–(31) and eqns (18)–(19) and (24) (pinned-headed piles).

For bending moment,

$$Y_m = L_c(0.760 - 0.700x + 1.030x^2 - 0.680x^3 + 0.160x^4), \quad 0 \leqslant x \leqslant 1.25, \quad (30a)$$

$$Y_m = 0.56L_c, \quad 1.25 < x \leqslant 4.0. \quad (30b)$$

For buckling,

$$Y_b = L_c(0.80 - 1.53x + 2.34x^2 - 1.84x^3 + 0.71x^4 - 0.106x^5), \quad 0 \leqslant x \leqslant 1.5, \quad (31a)$$

$$Y_b = 0.35L_c, \quad 1.5 < x \leqslant 4.0. \quad (31b)$$

*Computation of effective horizontal soil stiffness* ($k_e$)

For single soil layer, one can conclude from eqns (10) and (11) that

$$k_e = A + \frac{BL_0}{4}. \quad (32)$$

For multi-layered soil as shown in Fig. 3 ($H_A$ = abutment height, $h_0$–$h_N$ = $z$ coordinates defining the layers with $h_0 = 0$), numerical integration over

Table 5. $L_f$ values determined by eqn (2) for H piles in clays

| HP | $I_{pt}$ (in$^4$) | LRFD $L_f$ (ft) Soft clay | Medium clay | Stiff clay | Very stiff clay |
|---|---|---|---|---|---|
| 14 × 117 | 443 | 10.1 | 8.6 | 7.5 | 6.5 |
| 14 × 102 | 380 | 9.7 | 8.3 | 7.2 | 6.2 |
| 14 × 89 | 326 | 9.3 | 8.0 | 6.9 | 6.0 |
| 14 × 73 | 261 | 8.8 | 7.5 | 6.6 | 5.7 |
| 13 × 100 | 294 | 9.1 | 7.8 | 6.8 | 5.8 |
| 13 × 87 | 250 | 8.7 | 7.4 | 6.5 | 5.6 |
| 13 × 73 | 207 | 8.3 | 7.1 | 6.2 | 5.3 |
| 13 × 60 | 165 | 7.9 | 6.7 | 5.9 | 5.0 |
| 12 × 84 | 213 | 8.4 | 7.2 | 6.2 | 5.4 |
| 12 × 74 | 186 | 8.1 | 6.9 | 6.0 | 5.2 |
| 12 × 63 | 153 | 7.7 | 6.6 | 5.7 | 5.0 |
| 12 × 53 | 127 | 7.4 | 6.3 | 5.5 | 4.7 |
| 10 × 57 | 101 | 6.9 | 5.9 | 5.2 | 4.5 |
| 10 × 42 | 71.7 | 6.4 | 5.5 | 4.8 | 4.1 |
| 8 × 36 | 40.3 | 5.5 | 4.7 | 4.1 | 3.5 |

Table 6. $L_f$ values determined by eqn (6) for H piles in sands

| HP | $I_{pt}$ (in$^4$) | LRFD $L_f$ (ft) Loose sand moist/dry | Loose sand submerged | Medium sand moist/dry | Medium sand submerged | Dense sand moist/dry | Dense sand submerged |
|---|---|---|---|---|---|---|---|
| 14 × 117 | 443 | 7.8 | 8.9 | 6.4 | 7.3 | 5.3 | 6.1 |
| 14 × 102 | 380 | 7.5 | 8.6 | 6.2 | 7.1 | 5.1 | 5.9 |
| 14 × 89 | 326 | 7.3 | 8.4 | 6.0 | 6.9 | 5.0 | 5.7 |
| 14 × 73 | 261 | 7.0 | 8.0 | 5.7 | 6.6 | 4.8 | 5.5 |
| 13 × 100 | 294 | 7.1 | 8.2 | 5.9 | 6.7 | 4.9 | 5.6 |
| 13 × 87 | 250 | 6.9 | 7.9 | 5.7 | 6.5 | 4.7 | 5.4 |
| 13 × 73 | 207 | 6.7 | 7.7 | 5.5 | 6.3 | 4.6 | 5.2 |
| 13 × 60 | 165 | 6.4 | 7.3 | 5.2 | 6.0 | 4.4 | 5.0 |
| 12 × 84 | 213 | 6.7 | 7.7 | 5.5 | 6.3 | 4.6 | 5.3 |
| 12 × 74 | 186 | 6.5 | 7.5 | 5.4 | 6.2 | 4.5 | 5.1 |
| 12 × 63 | 153 | 6.3 | 7.2 | 5.2 | 5.9 | 4.3 | 4.9 |
| 12 × 53 | 127 | 6.0 | 6.9 | 5.0 | 5.7 | 4.1 | 4.7 |
| 10 × 57 | 101 | 5.8 | 6.6 | 4.7 | 5.4 | 3.9 | 4.5 |
| 10 × 42 | 71.7 | 5.4 | 6.2 | 4.4 | 5.1 | 3.7 | 4.2 |
| 8 × 36 | 40.3 | 4.8 | 5.5 | 3.9 | 4.5 | 3.3 | 3.8 |

Table 7. $L_f$ values for fixed-headed piles in moist/dry loose sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_t$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 7.8 | 29.6 | 29.6 | 22.1 | 13.2 | 12.1 | 1.70 | 2.85 |
| 14 × 102 | 7.5 | 28.8 | 28.7 | 21.1 | 12.7 | 11.6 | 1.69 | 2.81 |
| 14 × 89 | 7.3 | 27.8 | 27.9 | 20.3 | 12.3 | 11.3 | 1.69 | 2.78 |
| 14 × 73 | 7.0 | 26.6 | 26.7 | 19.0 | 11.7 | 10.7 | 1.67 | 2.72 |
| 13 × 100 | 7.1 | 27.2 | 27.3 | 19.7 | 12.0 | 11.0 | 1.68 | 2.75 |
| 13 × 87 | 6.9 | 26.4 | 26.4 | 18.8 | 11.5 | 10.6 | 1.67 | 2.71 |
| 13 × 73 | 6.7 | 25.4 | 25.5 | 17.8 | 11.0 | 10.1 | 1.66 | 2.67 |
| 13 × 60 | 6.4 | 24.4 | 24.3 | 16.6 | 10.5 | 9.6 | 1.64 | 2.61 |
| 12 × 84 | 6.7 | 25.6 | 25.6 | 17.9 | 11.1 | 10.2 | 1.66 | 2.67 |
| 12 × 74 | 6.5 | 25.0 | 24.9 | 17.2 | 10.8 | 9.9 | 1.65 | 2.64 |
| 12 × 63 | 6.3 | 24.0 | 23.9 | 16.2 | 10.3 | 9.5 | 1.64 | 2.59 |
| 12 × 53 | 6.0 | 23.0 | 23.1 | 15.4 | 9.9 | 9.1 | 1.63 | 2.55 |
| 10 × 57 | 5.8 | 22.0 | 22.1 | 14.4 | 9.3 | 8.6 | 1.62 | 2.49 |
| 10 × 42 | 5.4 | 20.6 | 20.6 | 12.9 | 8.6 | 8.0 | 1.60 | 2.40 |
| 8 × 36 | 4.8 | 18.4 | 18.3 | 10.8 | 7.5 | 7.0 | 1.57 | 2.24 |

[1]Equation (6); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Moist/dry loose sand $n_h = 30$ (tsf ft$^{-1}$); $k_h = 8z$ (ksf).

Table 8. $L_f$ values for fixed-headed piles in submerged loose sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 8.9 | 29.6 | 29.6 | 22.1 | 13.2 | 12.1 | 1.48 | 2.48 |
| 14 × 102 | 8.6 | 28.8 | 28.7 | 21.1 | 12.7 | 11.6 | 1.47 | 2.44 |
| 14 × 89 | 8.4 | 27.8 | 27.9 | 20.3 | 12.3 | 11.3 | 1.47 | 2.42 |
| 14 × 73 | 8.0 | 26.6 | 26.7 | 19.0 | 11.7 | 10.7 | 1.46 | 2.37 |
| 13 × 100 | 8.2 | 27.2 | 27.3 | 19.7 | 12.0 | 11.0 | 1.46 | 2.40 |
| 13 × 87 | 7.9 | 26.4 | 26.4 | 18.8 | 11.5 | 10.6 | 1.45 | 2.36 |
| 13 × 73 | 7.7 | 25.4 | 25.5 | 17.8 | 11.0 | 10.1 | 1.44 | 2.32 |
| 13 × 60 | 7.3 | 24.4 | 24.3 | 16.6 | 10.5 | 9.6 | 1.43 | 2.27 |
| 12 × 84 | 7.7 | 25.6 | 25.6 | 17.9 | 11.1 | 10.2 | 1.44 | 2.33 |
| 12 × 74 | 7.5 | 25.0 | 24.9 | 17.2 | 10.8 | 9.9 | 1.44 | 2.29 |
| 12 × 63 | 7.2 | 24.0 | 23.9 | 16.2 | 10.3 | 9.5 | 1.43 | 2.25 |
| 12 × 53 | 6.9 | 23.0 | 23.1 | 15.4 | 9.9 | 9.1 | 1.42 | 2.22 |
| 10 × 57 | 6.6 | 22.0 | 22.1 | 14.4 | 9.3 | 8.6 | 1.41 | 2.17 |
| 10 × 42 | 6.2 | 20.6 | 20.6 | 12.9 | 8.6 | 8.0 | 1.39 | 2.09 |
| 8 × 36 | 5.5 | 18.4 | 18.3 | 10.8 | 7.5 | 7.0 | 1.37 | 1.95 |

[1]Equation (6); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Moist/dry loose sand $n_h = 15$ (tsf ft$^{-1}$); $k_h = 8z$ (ksf).

Table 9. $L_f$ values for fixed-headed piles in moist/dry medium sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 6.4 | 78.30 | 23.2 | 15.5 | 9.9 | 9.2 | 1.56 | 2.44 |
| 14 × 102 | 6.2 | 76.28 | 22.5 | 14.8 | 9.6 | 8.8 | 1.55 | 2.39 |
| 14 × 89 | 6.0 | 73.58 | 21.9 | 14.2 | 9.2 | 8.5 | 1.54 | 2.36 |
| 14 × 73 | 5.7 | 70.88 | 20.9 | 13.2 | 8.8 | 8.1 | 1.53 | 2.30 |
| 13 × 100 | 5.9 | 72.23 | 21.4 | 13.7 | 9.0 | 8.3 | 1.53 | 2.33 |
| 12 × 87 | 5.7 | 70.20 | 20.7 | 13.0 | 8.7 | 8.0 | 1.53 | 2.29 |
| 13 × 73 | 5.5 | 67.50 | 19.9 | 12.3 | 8.3 | 7.7 | 1.52 | 2.25 |
| 13 × 60 | 5.2 | 64.13 | 19.1 | 11.5 | 7.9 | 7.3 | 1.51 | 2.19 |
| 12 × 84 | 5.5 | 67.50 | 20.1 | 12.4 | 8.4 | 7.8 | 1.52 | 2.26 |
| 12 × 74 | 5.4 | 66.15 | 19.5 | 11.9 | 8.1 | 7.5 | 1.51 | 2.22 |
| 12 × 63 | 5.2 | 63.45 | 18.8 | 11.2 | 7.8 | 7.2 | 1.50 | 2.17 |
| 12 × 53 | 5.0 | 60.75 | 18.1 | 10.6 | 7.4 | 6.9 | 1.50 | 2.13 |
| 10 × 57 | 4.7 | 58.05 | 17.3 | 9.8 | 7.1 | 6.6 | 1.49 | 2.07 |
| 10 × 42 | 4.4 | 54.68 | 16.1 | 8.8 | 6.5 | 6.1 | 1.47 | 1.98 |
| 8 × 36 | 3.9 | 48.60 | 14.4 | 7.3 | 5.7 | 5.4 | 1.45 | 1.84 |

[1]Equation (6); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Moist/dry loose sand $n_h = 80$ (tsf ft$^{-1}$); $k_h = 27z$ (ksf).

Table 10. $L_f$ values for fixed-headed piles in submerged medium sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 7.3 | 78.30 | 23.2 | 15.5 | 9.9 | 9.2 | 1.36 | 2.12 |
| 14 × 102 | 7.1 | 76.28 | 22.5 | 14.8 | 9.6 | 8.8 | 1.35 | 2.08 |
| 14 × 89 | 6.9 | 73.58 | 21.9 | 14.2 | 9.2 | 8.5 | 1.34 | 2.06 |
| 14 × 73 | 6.6 | 70.88 | 20.9 | 13.2 | 8.8 | 8.1 | 1.33 | 2.00 |
| 13 × 100 | 6.7 | 72.23 | 21.4 | 13.7 | 9.0 | 8.3 | 1.34 | 2.03 |
| 12 × 87 | 6.5 | 70.20 | 20.7 | 13.0 | 8.7 | 8.0 | 1.33 | 1.99 |
| 13 × 73 | 6.3 | 67.50 | 19.9 | 12.3 | 8.3 | 7.7 | 1.32 | 1.96 |
| 13 × 60 | 6.0 | 64.13 | 19.1 | 11.5 | 7.9 | 7.3 | 1.31 | 1.91 |
| 12 × 84 | 6.3 | 67.50 | 20.1 | 12.4 | 8.4 | 7.8 | 1.32 | 1.97 |
| 12 × 74 | 6.2 | 66.15 | 19.5 | 11.9 | 8.1 | 7.5 | 1.32 | 1.93 |
| 12 × 63 | 5.9 | 63.45 | 18.8 | 11.2 | 7.8 | 7.2 | 1.31 | 1.89 |
| 12 × 53 | 5.7 | 60.75 | 18.1 | 10.6 | 7.4 | 6.9 | 1.30 | 1.85 |
| 10 × 57 | 5.4 | 58.05 | 17.3 | 9.8 | 7.1 | 6.6 | 1.30 | 1.80 |
| 10 × 42 | 5.1 | 54.68 | 16.1 | 8.8 | 6.5 | 6.1 | 1.28 | 1.72 |
| 8 × 36 | 4.5 | 48.60 | 14.4 | 7.3 | 5.7 | 5.4 | 1.27 | 1.61 |

[1]Equation (6); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Submerged medium sand $n_h = 40$ (tsf ft$^{-1}$); $k_h = 27z$ (ksf).

Table 11. $L_f$ values for fixed-headed piles in moist/dry dense sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 5.3 | 171.0 | 19.1 | 11.5 | 7.9 | 7.4 | 1.49 | 2.17 |
| 14 × 102 | 5.1 | 165.6 | 18.5 | 11.0 | 7.6 | 7.1 | 1.48 | 2.13 |
| 14 × 89 | 5.0 | 162.0 | 17.9 | 10.4 | 7.4 | 6.9 | 1.47 | 2.09 |
| 14 × 73 | 4.8 | 154.8 | 17.2 | 9.7 | 7.0 | 6.5 | 1.47 | 2.03 |
| 13 × 100 | 4.9 | 158.4 | 17.6 | 10.1 | 7.2 | 6.7 | 1.47 | 2.06 |
| 13 × 87 | 4.7 | 153.0 | 17.0 | 9.6 | 6.9 | 6.5 | 1.47 | 2.03 |
| 13 × 73 | 4.6 | 147.6 | 16.4 | 9.0 | 6.6 | 6.2 | 1.46 | 1.98 |
| 13 × 60 | 4.4 | 140.4 | 15.7 | 8.4 | 6.3 | 5.9 | 1.45 | 1.93 |
| 12 × 84 | 4.6 | 147.6 | 16.5 | 9.1 | 6.7 | 6.3 | 1.46 | 1.99 |
| 12 × 74 | 4.5 | 144.0 | 16.1 | 8.7 | 6.5 | 6.1 | 1.46 | 1.95 |
| 12 × 63 | 4.3 | 138.6 | 15.4 | 8.2 | 6.2 | 5.8 | 1.45 | 1.91 |
| 12 × 53 | 4.1 | 133.2 | 14.9 | 7.7 | 6.0 | 5.6 | 1.44 | 1.86 |
| 10 × 57 | 3.9 | 127.8 | 14.2 | 7.1 | 5.7 | 5.3 | 1.43 | 1.81 |
| 10 × 42 | 3.7 | 118.8 | 13.3 | 6.4 | 5.3 | 5.0 | 1.43 | 1.73 |
| 8 × 36 | 3.3 | 106.2 | 11.8 | 5.3 | 4.6 | 4.4 | 1.41 | 1.61 |

[1]Equation (6); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Moist/dry dense sand $n_h = 200$ (tsf ft$^{-1}$); $k_h = 72z$ (ksf).

Table 12. $L_f$ values for fixed-headed piles in submerged dense sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 6.1 | 171.0 | 19.1 | 11.5 | 7.9 | 7.4 | 1.30 | 1.89 |
| 14 × 102 | 5.9 | 165.6 | 18.5 | 11.0 | 7.6 | 7.1 | 1.29 | 1.86 |
| 14 × 89 | 5.7 | 162.0 | 17.9 | 10.4 | 7.4 | 6.9 | 1.28 | 1.82 |
| 14 × 73 | 5.5 | 154.8 | 17.2 | 9.7 | 7.0 | 6.5 | 1.28 | 1.77 |
| 13 × 100 | 5.6 | 158.4 | 17.6 | 10.1 | 7.2 | 6.7 | 1.28 | 1.80 |
| 13 × 87 | 5.4 | 153.0 | 17.0 | 9.6 | 6.9 | 6.5 | 1.28 | 1.76 |
| 13 × 73 | 5.2 | 147.6 | 16.4 | 9.0 | 6.6 | 6.2 | 1.27 | 1.72 |
| 13 × 60 | 5.0 | 140.4 | 15.7 | 8.4 | 6.3 | 5.9 | 1.26 | 1.68 |
| 12 × 84 | 5.3 | 147.6 | 16.5 | 9.1 | 6.7 | 6.3 | 1.27 | 1.73 |
| 12 × 74 | 5.1 | 144.0 | 16.1 | 8.7 | 6.5 | 6.1 | 1.27 | 1.70 |
| 12 × 63 | 4.9 | 138.6 | 15.4 | 8.2 | 6.2 | 5.8 | 1.26 | 1.66 |
| 12 × 53 | 4.7 | 133.2 | 14.9 | 7.7 | 6.0 | 5.6 | 1.26 | 1.62 |
| 10 × 57 | 4.5 | 127.8 | 14.2 | 7.1 | 5.7 | 5.3 | 1.25 | 1.57 |
| 10 × 42 | 4.2 | 118.8 | 13.3 | 6.4 | 5.3 | 5.0 | 1.24 | 1.51 |
| 8 × 36 | 3.8 | 106.2 | 11.8 | 5.3 | 4.6 | 4.4 | 1.23 | 1.40 |

[1]Equation (6); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Submerged dense sand $n_h = 100$ (tsf ft$^{-1}$); $k_h = 72z$ (ksf).

Table 13. $L_f$ values for fixed-headed piles in soft clays

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 10.1 | 43.58 | 26.9 | 12.6 | 10.6 | 10.0 | 1.06 | 1.26 |
| 14 × 102 | 9.7 | 42.85 | 26.0 | 12.0 | 10.2 | 9.7 | 1.06 | 1.24 |
| 14 × 89 | 9.3 | 42.27 | 25.1 | 11.3 | 9.8 | 9.3 | 1.06 | 1.21 |
| 14 × 73 | 8.8 | 41.40 | 23.9 | 10.4 | 9.3 | 8.9 | 1.06 | 1.18 |
| 13 × 100 | 9.1 | 41.84 | 24.5 | 10.9 | 9.6 | 9.1 | 1.06 | 1.20 |
| 13 × 87 | 8.7 | 41.11 | 23.7 | 10.3 | 9.2 | 8.8 | 1.06 | 1.18 |
| 13 × 73 | 8.3 | 40.39 | 22.7 | 9.6 | 8.8 | 8.4 | 1.06 | 1.15 |
| 13 × 60 | 7.9 | 39.52 | 21.5 | 8.9 | 8.3 | 8.0 | 1.06 | 1.13 |
| 12 × 84 | 8.4 | 40.53 | 22.8 | 9.7 | 8.8 | 8.5 | 1.06 | 1.16 |
| 12 × 74 | 8.1 | 39.95 | 22.1 | 9.2 | 8.6 | 8.2 | 1.06 | 1.14 |
| 12 × 63 | 7.7 | 39.37 | 21.2 | 8.6 | 8.1 | 7.8 | 1.56 | 1.12 |
| 12 × 53 | 7.4 | 38.64 | 20.3 | 8.1 | 7.8 | 7.5 | 1.05 | 1.10 |
| 10 × 57 | 6.9 | 37.92 | 19.3 | 7.5 | 7.3 | 7.1 | 1.05 | 1.08 |
| 10 × 42 | 6.4 | 36.91 | 17.8 | 6.7 | 6.7 | 6.5 | 1.05 | 1.06 |
| 8 × 36 | 5.5 | 35.31 | 15.6 | 5.7 | 5.7 | 5.6 | 1.04 | 1.04 |

[1]Equation (2); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 21$ (ft); $E_p = 29{,}000$ (ksi).
Soft clay $E_s = 16.75$ (tsf ft$^{-1}$); $k_h = 24 + 5.8z$ (ksf).

Table 14. $L_f$ values for fixed-headed piles in medium clays

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 8.6 | 163.16 | 19.3 | 7.6 | 7.3 | 7.1 | 0.85 | 0.88 |
| 14 × 102 | 8.3 | 161.41 | 18.7 | 7.2 | 7.1 | 6.8 | 0.85 | 0.87 |
| 14 × 89 | 8.0 | 159.65 | 18.0 | 6.8 | 6.8 | 6.6 | 0.85 | 0.86 |
| 14 × 73 | 7.5 | 157.31 | 17.1 | 6.4 | 6.4 | 62. | 0.85 | 0.85 |
| 13 × 100 | 7.8 | 158.48 | 17.6 | 6.6 | 6.6 | 6.4 | 0.85 | 0.86 |
| 13 × 87 | 7.4 | 156.73 | 16.9 | 6.3 | 6.3 | 6.1 | 0.85 | 0.85 |
| 13 × 73 | 7.1 | 154.39 | 16.2 | 6.0 | 6.0 | 5.8 | 0.84 | 0.85 |
| 13 × 60 | 6.7 | 152.05 | 15.4 | 5.7 | 5.7 | 5.5 | 0.84 | 0.84 |
| 12 × 84 | 7.2 | 154.39 | 16.3 | 6.1 | 6.0 | 5.9 | 0.85 | 0.85 |
| 12 × 74 | 6.9 | 153.22 | 15.8 | 5.8 | 5.8 | 5.7 | 0.84 | 0.84 |
| 12 × 63 | 6.6 | 150.88 | 15.1 | 5.6 | 5.6 | 5.4 | 0.84 | 0.84 |
| 12 × 53 | 6.3 | 149.70 | 14.5 | 5.3 | 5.3 | 5.2 | 0.84 | 0.84 |
| 10 × 57 | 5.9 | 146.78 | 13.7 | 5.0 | 5.1 | 4.9 | 0.85 | 0.85 |
| 10 × 42 | 5.5 | 143.86 | 12.7 | 4.7 | 4.7 | 4.6 | 0.86 | 0.86 |
| 8 × 36 | 4.7 | 139.18 | 11.1 | 4.1 | 4.1 | 4.0 | 0.87 | 0.88 |

[1]Equation (2); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 21$ (ft); $E_p = 29{,}000$ (ksi).
Medium clay $E_s = 31.4$ (tsf ft$^{-1}$); $k_h = 107 + 23.4z$ (ksf).

Table 15. $L_f$ values for fixed-headed piles in stiff clays

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 7.5 | 277.13 | 16.9 | 6.3 | 6.3 | 6.1 | 0.84 | 0.85 |
| 14 × 102 | 7.2 | 274.05 | 16.4 | 6.1 | 6.1 | 5.9 | 0.84 | 0.84 |
| 14 × 89 | 6.9 | 270.98 | 15.8 | 5.8 | 5.8 | 5.7 | 0.84 | 0.84 |
| 14 × 73 | 6.6 | 266.88 | 15.0 | 5.5 | 5.5 | 5.4 | 0.84 | 0.84 |
| 13 × 100 | 6.8 | 268.93 | 15.4 | 5.7 | 5.7 | 5.5 | 0.84 | 0.84 |
| 13 × 87 | 6.5 | 265.85 | 14.8 | 5.4 | 5.4 | 5.3 | 0.84 | 0.84 |
| 13 × 73 | 6.2 | 262.78 | 14.2 | 5.2 | 5.2 | 5.1 | 0.84 | 0.84 |
| 13 × 60 | 5.9 | 258.68 | 13.5 | 5.0 | 5.0 | 4.8 | 0.85 | 0.85 |
| 12 × 84 | 6.2 | 262.78 | 14.3 | 5.2 | 5.2 | 5.1 | 0.84 | 0.84 |
| 12 × 74 | 6.0 | 260.73 | 13.8 | 5.1 | 5.1 | 5.0 | 0.85 | 0.84 |
| 12 × 63 | 5.7 | 257.65 | 13.2 | 4.9 | 4.9 | 4.8 | 0.85 | 0.85 |
| 12 × 53 | 5.5 | 255.60 | 12.7 | 4.7 | 4.7 | 4.6 | 0.85 | 0.86 |
| 10 × 57 | 5.2 | 251.50 | 12.0 | 4.5 | 4.4 | 4.3 | 0.86 | 0.86 |
| 10 × 42 | 4.8 | 246.48 | 11.1 | 4.2 | 4.1 | 4.0 | 0.86 | 0.87 |
| 8 × 36 | 4.1 | 239.20 | 9.7 | 3.6 | 3.6 | 3.5 | 0.87 | 0.87 |

[1]Equation (2); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 21$ (ft); $E_p = 29{,}000$ (ksi).
Stiff clay $E_s = 54.4$ (tsf ft$^{-1}$); $k_h = 190 + 41z$ (ksf).

Table 16. $L_f$ values for fixed-headed piles in very stiff clays

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 6.5 | 1588.75 | 10.9 | 4.1 | 4.1 | 3.9 | 0.63 | 0.64 |
| 14 × 102 | 6.2 | 1558.25 | 10.6 | 4.0 | 3.9 | 3.8 | 0.63 | 0.64 |
| 14 × 89 | 6.0 | 1527.75 | 10.2 | 3.8 | 3.8 | 3.7 | 0.63 | 0.63 |
| 14 × 73 | 5.7 | 1497.25 | 9.7 | 3.6 | 3.6 | 3.5 | 0.64 | 0.64 |
| 13 × 100 | 5.8 | 1512.50 | 10.0 | 3.7 | 3.7 | 3.6 | 0.63 | 0.63 |
| 13 × 87 | 5.6 | 1482.00 | 9.7 | 3.6 | 3.6 | 3.5 | 0.64 | 0.64 |
| 13 × 73 | 5.3 | 1451.50 | 9.3 | 3.4 | 3.4 | 3.3 | 0.64 | 0.64 |
| 13 × 60 | 5.0 | 1421.00 | 8.8 | 3.3 | 3.3 | 3.2 | 0.64 | 0.64 |
| 12 × 84 | 5.4 | 1451.50 | 9.3 | 3.5 | 3.5 | 3.4 | 0.64 | 0.64 |
| 12 × 74 | 5.2 | 1436.25 | 9.0 | 3.3 | 3.3 | 3.3 | 0.64 | 0.64 |
| 12 × 63 | 5.0 | 1405.75 | 8.7 | 3.2 | 3.2 | 3.1 | 0.65 | 0.65 |
| 12 × 53 | 4.7 | 1375.25 | 8.3 | 3.1 | 3.1 | 3.0 | 0.65 | 0.65 |
| 10 × 57 | 4.5 | 1360.00 | 7.9 | 2.9 | 2.9 | 2.8 | 0.65 | 0.65 |
| 10 × 42 | 4.1 | 1314.25 | 7.3 | 2.7 | 2.7 | 2.6 | 0.66 | 0.66 |
| 8 × 36 | 3.5 | 1238.00 | 6.4 | 2.4 | 2.4 | 2.3 | 0.67 | 0.67 |

[1]Equation (2); [2]eqn (28); [3]eqn (27); [4]eqn (26).
Fixed-headed piles $L_u = 21$ (ft); $E_p = 29{,}000$ (ksi).
Very stiff clay $E_s = 98.4$ (tsf ft$^{-1}$); $k_h = 750 - 610z$ (ksf).

Table 17. $L_f$ values for pinned-headed piles in moist/dry loose sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_c$ (ksf) | $L_c$ (ft) | Chen $L_{fh}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 7.8 | 29.6 | 29.6 | 14.5 | 18.3 | 11.0 | 2.36 | 1.87 |
| 14 × 102 | 7.5 | 28.8 | 28.7 | 13.9 | 17.7 | 10.7 | 2.35 | 1.84 |
| 14 × 89 | 7.3 | 27.8 | 27.8 | 13.3 | 17.1 | 10.3 | 2.34 | 1.83 |
| 14 × 73 | 7.0 | 26.6 | 26.7 | 12.6 | 16.3 | 9.9 | 2.33 | 1.80 |
| 13 × 100 | 7.1 | 27.4 | 27.3 | 13.0 | 16.7 | 10.1 | 2.33 | 1.81 |
| 13 × 87 | 6.9 | 26.4 | 26.4 | 12.4 | 16.1 | 9.8 | 2.33 | 1.80 |
| 13 × 73 | 6.7 | 25.4 | 25.5 | 11.8 | 15.4 | 9.4 | 2.32 | 1.77 |
| 13 × 60 | 6.4 | 24.4 | 24.3 | 11.1 | 14.7 | 8.9 | 2.30 | 1.75 |
| 12 × 84 | 6.7 | 25.6 | 25.6 | 11.9 | 15.5 | 9.4 | 2.32 | 1.78 |
| 12 × 74 | 6.5 | 25.0 | 24.9 | 11.5 | 15.1 | 9.2 | 2.31 | 1.76 |
| 12 × 63 | 6.3 | 24.0 | 23.9 | 10.9 | 14.4 | 8.8 | 2.30 | 1.74 |
| 12 × 53 | 6.0 | 23.0 | 23.1 | 10.4 | 13.9 | 8.5 | 2.30 | 1.72 |
| 10 × 57 | 5.8 | 22.0 | 22.1 | 9.8 | 13.2 | 8.1 | 2.28 | 1.70 |
| 10 × 42 | 5.4 | 20.6 | 20.6 | 8.9 | 12.2 | 7.5 | 2.27 | 1.66 |
| 8 × 36 | 4.8 | 18.4 | 18.3 | 7.7 | 10.8 | 6.4 | 2.25 | 1.61 |

[1]Equation (6); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Moist/dry loose sand $n_h = 30$ (tsf ft$^{-1}$); $k_h = 8z$ (ksf).

Table 18. $L_f$ values for pinned-headed piles in submerged loose sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_c$ (ksf) | $L_c$ (ft) | Chen $L_{fh}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 8.9 | 29.6 | 29.6 | 14.5 | 18.3 | 11.0 | 2.05 | 1.62 |
| 14 × 102 | 8.6 | 28.8 | 28.7 | 13.9 | 17.7 | 10.7 | 2.04 | 1.61 |
| 14 × 89 | 8.4 | 27.8 | 27.9 | 13.3 | 17.1 | 10.3 | 2.04 | 1.59 |
| 14 × 73 | 8.0 | 26.6 | 26.7 | 12.6 | 16.3 | 9.9 | 2.03 | 1.57 |
| 13 × 100 | 8.2 | 27.4 | 27.3 | 13.0 | 16.7 | 10.1 | 2.03 | 1.58 |
| 13 × 87 | 7.9 | 26.4 | 26.4 | 12.4 | 16.1 | 9.8 | 2.03 | 1.56 |
| 13 × 73 | 7.7 | 25.4 | 25.5 | 11.8 | 15.4 | 9.4 | 2.02 | 1.54 |
| 13 × 60 | 7.3 | 24.4 | 24.3 | 11.1 | 14.7 | 8.9 | 2.01 | 1.52 |
| 12 × 84 | 7.7 | 25.6 | 25.6 | 11.9 | 15.5 | 9.4 | 2.02 | 1.55 |
| 12 × 74 | 7.5 | 25.0 | 24.9 | 11.5 | 15.1 | 9.2 | 2.01 | 1.53 |
| 12 × 63 | 7.2 | 24.0 | 23.9 | 10.9 | 14.4 | 8.8 | 2.00 | 1.51 |
| 12 × 53 | 6.9 | 23.0 | 23.1 | 10.4 | 13.9 | 8.5 | 2.00 | 1.50 |
| 10 × 57 | 6.6 | 22.0 | 22.1 | 9.8 | 13.2 | 8.1 | 1.99 | 1.48 |
| 10 × 42 | 6.2 | 20.6 | 20.6 | 8.9 | 12.2 | 7.5 | 1.97 | 1.44 |
| 8 × 36 | 5.5 | 18.4 | 18.3 | 7.7 | 10.8 | 6.4 | 1.96 | 1.40 |

[1]Equation (6); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Submerged loose sand $n_h = 15$ (tsf ft$^{-1}$); $k_h = 8z$ (ksf).

Table 19. $L_f$ values for pinned-headed piles in moist/dry medium sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_c$ (ksf) | $L_c$ (ft) | Chen $L_{fh}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 6.4 | 78.30 | 23.2 | 10.5 | 14.0 | 8.5 | 2.19 | 1.64 |
| 14 × 102 | 6.2 | 76.28 | 22.5 | 10.1 | 13.5 | 8.2 | 2.18 | 1.63 |
| 14 × 89 | 6.0 | 73.58 | 21.9 | 9.7 | 13.1 | 8.0 | 2.18 | 1.61 |
| 14 × 73 | 5.7 | 70.88 | 20.9 | 9.1 | 12.4 | 7.6 | 2.16 | 1.59 |
| 13 × 100 | 5.9 | 72.23 | 21.4 | 9.4 | 12.8 | 7.8 | 2.17 | 1.60 |
| 13 × 87 | 5.7 | 70.20 | 20.7 | 9.0 | 12.3 | 7.5 | 2.16 | 1.58 |
| 13 × 73 | 5.5 | 67.50 | 19.9 | 8.6 | 11.8 | 7.0 | 2.16 | 1.57 |
| 13 × 60 | 5.2 | 64.13 | 19.9 | 8.1 | 11.3 | 6.7 | 2.15 | 1.55 |
| 12 × 84 | 5.5 | 67.50 | 20.1 | 8.7 | 11.9 | 7.3 | 2.16 | 1.57 |
| 12 × 73 | 5.4 | 66.15 | 19.5 | 8.3 | 11.5 | 6.8 | 2.15 | 1.56 |
| 12 × 63 | 5.2 | 63.45 | 18.8 | 7.9 | 11.1 | 6.6 | 2.15 | 1.54 |
| 12 × 53 | 5.0 | 60.75 | 18.1 | 7.6 | 10.7 | 6.3 | 2.15 | 1.53 |
| 10 × 57 | 4.7 | 58.05 | 17.3 | 7.2 | 10.1 | 6.1 | 2.14 | 1.51 |
| 10 × 42 | 4.4 | 54.68 | 16.1 | 6.6 | 9.4 | 5.6 | 2.12 | 1.48 |
| 8 × 36 | 3.9 | 48.60 | 14.4 | 5.7 | 8.3 | 5.0 | 2.11 | 1.45 |

[1]Equation (6); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Moist/dry medium sand $n_h = 80$ (tsf ft$^{-1}$); $k_h = 27z$ (ksf).

Table 20. $L_f$ values for pinned-headed piles in submerged medium sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 7.3 | 78.30 | 23.2 | 10.5 | 14.0 | 8.5 | 1.91 | 1.43 |
| 14 × 102 | 7.1 | 76.28 | 22.5 | 10.1 | 13.5 | 8.2 | 1.90 | 1.42 |
| 14 × 89 | 6.9 | 73.58 | 21.9 | 9.7 | 13.1 | 8.0 | 1.90 | 1.40 |
| 14 × 73 | 6.6 | 70.88 | 20.9 | 9.1 | 12.4 | 7.6 | 1.88 | 1.38 |
| 13 × 100 | 6.7 | 72.23 | 21.4 | 9.4 | 12.8 | 7.8 | 1.89 | 1.39 |
| 13 × 87 | 6.5 | 70.20 | 20.7 | 9.0 | 12.3 | 7.5 | 1.88 | 1.38 |
| 13 × 73 | 6.3 | 67.50 | 19.9 | 8.6 | 11.8 | 7.0 | 1.88 | 1.36 |
| 13 × 60 | 6.0 | 64.13 | 19.9 | 8.1 | 11.3 | 6.7 | 1.87 | 1.35 |
| 12 × 84 | 6.3 | 67.50 | 20.1 | 8.7 | 11.9 | 7.3 | 1.88 | 1.37 |
| 12 × 73 | 6.2 | 66.15 | 19.5 | 8.3 | 11.5 | 6.8 | 1.87 | 1.36 |
| 12 × 63 | 5.9 | 63.45 | 18.8 | 7.9 | 11.1 | 6.6 | 1.87 | 1.34 |
| 12 × 53 | 5.7 | 60.75 | 18.1 | 7.6 | 10.7 | 6.3 | 1.87 | 1.33 |
| 10 × 57 | 5.4 | 58.05 | 17.3 | 7.2 | 10.1 | 6.1 | 1.86 | 1.32 |
| 10 × 42 | 5.1 | 54.68 | 16.1 | 6.6 | 9.4 | 5.6 | 1.85 | 1.29 |
| 8 × 36 | 4.5 | 48.60 | 14.4 | 5.7 | 8.3 | 5.0 | 1.84 | 1.26 |

[1]Equation (6); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Submerged medium sand $n_h = 40$ (tsf ft$^{-1}$); $k_h = 27z$ (ksf).

Table 21. $L_f$ values for pinned-headed piles in moist/dry dense sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 5.3 | 171.0 | 19.1 | 8.1 | 11.3 | 6.7 | 2.13 | 1.53 |
| 14 × 102 | 5.1 | 167.4 | 18.5 | 7.8 | 10.9 | 6.5 | 2.12 | 1.51 |
| 14 × 89 | 5.0 | 162.0 | 17.9 | 7.5 | 10.5 | 6.3 | 2.11 | 1.50 |
| 14 × 73 | 4.8 | 154.8 | 17.2 | 7.1 | 10.1 | 6.0 | 2.11 | 1.49 |
| 13 × 100 | 4.9 | 158.4 | 17.6 | 7.3 | 10.3 | 6.2 | 2.11 | 1.50 |
| 13 × 87 | 4.7 | 153.0 | 17.0 | 7.0 | 10.0 | 6.0 | 2.11 | 1.48 |
| 13 × 73 | 4.6 | 147.6 | 16.4 | 6.7 | 9.6 | 5.7 | 2.10 | 1.47 |
| 13 × 60 | 4.4 | 140.4 | 15.7 | 6.3 | 9.1 | 5.5 | 2.10 | 1.46 |
| 12 × 84 | 4.6 | 147.6 | 16.5 | 6.8 | 9.7 | 5.8 | 2.10 | 1.48 |
| 12 × 74 | 4.5 | 144.0 | 16.1 | 6.5 | 9.4 | 5.6 | 2.10 | 1.46 |
| 12 × 63 | 4.3 | 138.6 | 15.4 | 6.2 | 9.0 | 5.4 | 2.10 | 1.45 |
| 12 × 53 | 4.1 | 133.2 | 14.9 | 6.0 | 8.7 | 5.2 | 2.09 | 1.44 |
| 10 × 57 | 3.9 | 127.8 | 14.2 | 5.6 | 8.2 | 5.0 | 2.09 | 1.42 |
| 10 × 42 | 3.7 | 118.8 | 13.3 | 5.2 | 7.7 | 4.6 | 2.08 | 1.41 |
| 8 × 36 | 3.3 | 106.2 | 11.8 | 4.5 | 6.8 | 4.1 | 2.07 | 1.38 |

[1]Equation (6); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Moist/dry dense sand $n_h = 200$ (tsf ft$^{-1}$); $k_h = 72z$ (ksf).

Table 22. $L_f$ values for pinned-headed piles in submerged dense sands

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 6.1 | 171.0 | 19.1 | 8.1 | 11.3 | 6.7 | 1.85 | 1.33 |
| 14 × 102 | 5.9 | 167.4 | 18.5 | 7.8 | 10.9 | 6.5 | 1.84 | 1.32 |
| 14 × 89 | 5.7 | 162.0 | 17.9 | 7.5 | 10.5 | 6.3 | 1.84 | 1.31 |
| 14 × 73 | 5.5 | 154.8 | 17.2 | 7.1 | 10.1 | 6.0 | 1.83 | 1.29 |
| 13 × 100 | 5.6 | 158.4 | 17.6 | 7.3 | 10.3 | 6.2 | 1.84 | 1.30 |
| 13 × 87 | 5.4 | 153.0 | 17.0 | 7.0 | 10.0 | 6.0 | 1.83 | 1.29 |
| 13 × 73 | 5.2 | 147.6 | 16.4 | 6.7 | 9.6 | 5.7 | 1.83 | 1.28 |
| 13 × 60 | 5.0 | 140.4 | 15.7 | 6.3 | 9.1 | 5.5 | 1.83 | 1.27 |
| 12 × 84 | 5.3 | 147.6 | 16.5 | 6.8 | 9.7 | 5.8 | 1.83 | 1.28 |
| 12 × 74 | 5.1 | 144.0 | 16.1 | 6.5 | 9.4 | 5.6 | 1.83 | 1.27 |
| 12 × 63 | 4.9 | 138.6 | 15.4 | 6.2 | 9.0 | 5.4 | 1.82 | 1.26 |
| 12 × 53 | 4.7 | 133.2 | 14.9 | 6.0 | 8.7 | 5.2 | 1.82 | 1.25 |
| 10 × 57 | 4.5 | 127.8 | 14.2 | 5.6 | 8.2 | 5.0 | 1.82 | 1.24 |
| 10 × 42 | 4.2 | 118.8 | 13.3 | 5.2 | 7.7 | 4.6 | 1.81 | 1.23 |
| 8 × 36 | 3.8 | 106.2 | 11.8 | 4.5 | 6.8 | 4.1 | 1.80 | 1.20 |

[1]Equation (6); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 10$ (ft); $E_p = 29{,}000$ (ksi).
Submerged dense sand $n_h = 100$ (tsf ft$^{-1}$); $k_h = 72z$ (ksf).

Table 23. $L_f$ values for pinned-headed piles in soft clays

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 10.1 | 43.58 | 26.9 | 104. | 15.5 | 9.4 | 1.54 | 1.04 |
| 14 × 102 | 9.7 | 42.85 | 26.0 | 10.0 | 15.0 | 9.1 | 1.55 | 1.04 |
| 14 × 89 | 9.3 | 42.27 | 25.1 | 9.6 | 14.5 | 8.8 | 1.55 | 1.03 |
| 14 × 73 | 8.8 | 41.40 | 23.9 | 9.1 | 13.7 | 8.4 | 1.56 | 1.03 |
| 13 × 100 | 9.1 | 41.84 | 24.5 | 9.4 | 14.1 | 8.6 | 1.55 | 1.03 |
| 13 × 87 | 8.7 | 41.11 | 23.7 | 9.0 | 13.6 | 8.3 | 1.56 | 1.03 |
| 13 × 73 | 8.3 | 40.39 | 22.7 | 8.5 | 13.0 | 7.9 | 1.56 | 1.02 |
| 13 × 60 | 7.9 | 39.52 | 21.5 | 8.0 | 12.3 | 7.5 | 1.57 | 1.02 |
| 12 × 84 | 8.4 | 40.53 | 22.8 | 8.6 | 13.1 | 8.0 | 1.56 | 1.03 |
| 12 × 74 | 8.1 | 39.95 | 22.1 | 8.3 | 12.7 | 7.7 | 1.56 | 1.02 |
| 12 × 63 | 7.7 | 39.37 | 21.2 | 7.8 | 12.1 | 7.4 | 1.56 | 1.02 |
| 12 × 53 | 7.4 | 38.79 | 20.3 | 7.4 | 11.5 | 7.1 | 1.57 | 1.01 |
| 10 × 57 | 6.9 | 37.92 | 19.3 | 7.0 | 10.9 | 6.7 | 1.57 | 1.00 |
| 10 × 42 | 6.4 | 36.91 | 17.8 | 6.3 | 10.0 | 6.2 | 1.56 | 0.99 |
| 8 × 36 | 5.5 | 35.31 | 15.6 | 5.3 | 8.7 | 5.5 | 1.58 | 0.95 |

[1]Equation (2); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 21$ (ft); $E_p = 29{,}000$ (ksi).
Soft clay $E_s = 16.75$ (tsf ft$^{-1}$); $k_h = 24 + 5.8z$ (ksf).

Table 24. $L_f$ values for pinned-headed piles in medium clays

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 8.6 | 163.16 | 19.3 | 7.0 | 11.0 | 6.8 | 1.27 | 0.82 |
| 14 × 102 | 8.3 | 161.41 | 18.7 | 6.7 | 10.5 | 6.5 | 1.27 | 0.81 |
| 14 × 89 | 8.0 | 159.65 | 18.0 | 6.4 | 10.1 | 6.3 | 1.27 | 0.81 |
| 14 × 73 | 7.5 | 157.31 | 17.1 | 6.0 | 9.5 | 6.0 | 1.26 | 0.79 |
| 13 × 100 | 7.8 | 158.48 | 17.6 | 6.2 | 9.9 | 6.2 | 1.27 | 0.80 |
| 13 × 87 | 7.4 | 156.73 | 16.9 | 5.9 | 9.4 | 5.9 | 1.27 | 0.79 |
| 13 × 73 | 7.1 | 154.39 | 16.2 | 5.6 | 9.1 | 5.7 | 1.28 | 0.78 |
| 13 × 60 | 6.7 | 152.05 | 15.4 | 5.2 | 8.6 | 5.4 | 1.28 | 0.77 |
| 12 × 84 | 7.2 | 154.97 | 16.3 | 5.6 | 9.1 | 5.7 | 1.28 | 0.78 |
| 12 × 74 | 6.9 | 153.22 | 15.8 | 5.4 | 8.9 | 5.5 | 1.28 | 0.78 |
| 12 × 63 | 6.6 | 150.88 | 15.1 | 5.0 | 8.5 | 5.3 | 1.29 | 0.76 |
| 12 × 53 | 6.3 | 149.12 | 14.5 | 4.7 | 8.1 | 5.1 | 1.29 | 0.75 |
| 10 × 57 | 5.9 | 146.78 | 13.7 | 4.8 | 7.7 | 4.8 | 1.29 | 0.81 |
| 10 × 42 | 5.5 | 143.86 | 12.7 | 4.4 | 7.1 | 4.4 | 1.30 | 0.81 |

[1]Equation (2); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 21$ (ft); $E_p = 29{,}000$ (ksi).
Medium clay $E_x = 31.4$ (tsf ft$^{-1}$); $k_h = 107 + 23.4z$ (ksf).

Table 25. $L_f$ values for pinned-headed piles in stiff clays

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 7.5 | 277.13 | 16.9 | 5.9 | 9.4 | 5.9 | 1.26 | 0.79 |
| 14 × 102 | 7.2 | 274.05 | 16.4 | 5.6 | 9.2 | 5.7 | 1.27 | 0.78 |
| 14 × 89 | 6.9 | 270.98 | 15.8 | 5.4 | 8.8 | 5.5 | 1.27 | 0.77 |
| 14 × 73 | 6.6 | 266.88 | 15.0 | 5.0 | 8.4 | 5.2 | 1.28 | 0.76 |
| 13 × 100 | 6.8 | 268.93 | 15.4 | 5.2 | 8.6 | 5.4 | 1.28 | 0.76 |
| 13 × 87 | 6.5 | 265.85 | 14.8 | 4.9 | 8.3 | 5.2 | 1.28 | 0.75 |
| 13 × 73 | 6.2 | 262.78 | 14.2 | 4.6 | 7.9 | 5.0 | 1.28 | 0.74 |
| 13 × 60 | 5.9 | 258.68 | 13.5 | 4.7 | 7.5 | 4.7 | 1.29 | 0.81 |
| 12 × 84 | 6.2 | 262.78 | 14.3 | 4.6 | 8.0 | 5.0 | 1.28 | 0.74 |
| 12 × 74 | 6.0 | 260.73 | 13.8 | 4.8 | 7.8 | 4.8 | 1.29 | 0.80 |
| 12 × 63 | 5.7 | 257.65 | 13.2 | 4.6 | 7.4 | 4.6 | 1.29 | 0.81 |
| 12 × 53 | 5.5 | 254.58 | 12.7 | 4.4 | 7.1 | 4.4 | 1.29 | 0.81 |
| 10 × 57 | 5.2 | 251.50 | 12.0 | 4.2 | 6.7 | 4.2 | 1.30 | 0.81 |
| 10 × 52 | 4.8 | 247.40 | 11.1 | 3.9 | 6.2 | 3.9 | 1.30 | 0.81 |
| 8 × 36 | 4.1 | 239.20 | 9.7 | 3.4 | 5.4 | 3.4 | 1.31 | 0.82 |

[1]Equation (2); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 21$ (ft); $E_p = 29{,}000$ (ksi).
Stiff clay $E_x = 54.5$ (tsf ft$^{-1}$); $k_h = 190 + 41z$ (ksf).

Table 26. $L_f$ values for pinned-headed piles in very stiff clays

| HP | LRFD $L_f$[1] (ft) (1) | $k_e$ (ksf) | $L_c$ (ft) | Chen $L_{fb}$[2] (ft) (2) | Chen $L_{fm}$[3] (ft) (3) | Chen $L_{fs}$[4] (ft) | (3)/(1) | (2)/(1) |
|---|---|---|---|---|---|---|---|---|
| 14 × 117 | 6.5 | 1588.75 | 10.9 | 3.8 | 6.1 | 3.8 | 0.95 | 0.59 |
| 14 × 102 | 6.2 | 1558.25 | 10.6 | 3.7 | 5.9 | 3.7 | 0.95 | 0.60 |
| 14 × 89 | 6.0 | 1527.75 | 10.2 | 3.6 | 5.7 | 3.6 | 0.96 | 0.60 |
| 14 × 73 | 5.7 | 1497.25 | 9.7 | 3.4 | 5.5 | 3.4 | 0.96 | 0.60 |
| 13 × 100 | 5.8 | 1512.50 | 10.0 | 3.5 | 5.6 | 3.5 | 0.96 | 0.60 |
| 13 × 87 | 5.6 | 1482.00 | 9.7 | 3.4 | 5.4 | 3.4 | 0.97 | 0.60 |
| 13 × 73 | 5.3 | 1451.50 | 9.3 | 3.2 | 5.2 | 3.2 | 0.97 | 0.61 |
| 13 × 60 | 5.0 | 1421.00 | 8.8 | 3.1 | 4.9 | 3.1 | 0.98 | 0.61 |
| 12 × 84 | 5.4 | 1451.50 | 9.3 | 3.3 | 5.2 | 3.3 | 0.97 | 0.61 |
| 12 × 74 | 5.2 | 1436.25 | 9.0 | 3.2 | 5.1 | 3.2 | 0.97 | 0.61 |
| 12 × 63 | 5.0 | 1405.75 | 8.7 | 3.0 | 4.8 | 3.0 | 0.98 | 0.61 |
| 12 × 53 | 4.7 | 1375.25 | 8.3 | 2.9 | 4.7 | 2.9 | 0.98 | 0.62 |
| 10 × 57 | 4.5 | 1360.00 | 7.9 | 2.8 | 4.4 | 2.8 | 0.99 | 0.62 |
| 10 × 42 | 4.1 | 1299.00 | 7.3 | 2.6 | 4.1 | 2.6 | 1.00 | 0.62 |
| 8 × 36 | 3.5 | 1238.00 | 6.4 | 2.2 | 3.6 | 2.2 | 1.01 | 0.63 |

[1]Equation (2); [2]eqn (31); [3]eqn (30); [4]eqn (29).
Pinned-headed piles $L_u = 21$ (ft); $E_p = 29{,}000$ (ksi).
Very stiff clay $E_s = 54.4$ (tsf ft$^{-1}$); $k_h = 190 + 41z$ (ksf).

Table 27. Comparisons of $L_f$ values for the fixed-headed piles

| Soil type | Wetness | eqn (27)[1] LRFD method[2] | eqn (28)[1] LRFD method[2] |
|---|---|---|---|
| Loose sand | Moist/dry | 1.57–1.70 | 2.24–2.85 |
| | Submerged | 1.37–1.48 | 1.95–2.48 |
| Medium sand | Moist/dry | 1.45–1.56 | 1.84–2.44 |
| | Submerged | 1.27–1.36 | 1.61–2.12 |
| Dense sand | Moist/dry | 1.41–1.49 | 1.61–2.17 |
| | Submerged | 1.23–1.30 | 1.40–1.89 |
| Soft clay | — | 1.04–1.06 | 1.04–1.26 |
| Medium clay | — | 0.84–0.87 | 0.84–0.88 |
| Stiff clay | — | 0.84–0.87 | 0.84–0.87 |
| Very stiff clay | — | 0.63–0.67 | 0.63–0.67 |

[1]The proposed method.
[2]Equation (2) for piles in clays, eqn (6) for piles in sands.

Table 28. Comparisons of $L_f$ values for the pinned-headed piles

| Soil type | Wetness | eqn (30)[1] LRFD method[2] | eqn (31)[1] LRFD method[2] |
|---|---|---|---|
| Loose sand | Moist/dry | 2.25–2.36 | 1.61–1.87 |
| | Submerged | 1.96–2.05 | 1.40–1.62 |
| Medium sand | Moist/dry | 2.11–2.19 | 1.45–1.64 |
| | Submerged | 1.84–1.91 | 1.26–1.43 |
| Dense sand | Moist/dry | 2.07–2.13 | 1.38–1.53 |
| | Submerged | 1.80–1.85 | 1.20–1.33 |
| Soft clay | — | 1.54–1.58 | 0.95–1.04 |
| Medium clay | — | 1.27–1.31 | 0.75–0.82 |
| Stiff clay | — | 1.26–1.31 | 0.74–0.82 |
| Very stiff clay | — | 0.95–1.01 | 0.59–0.63 |

[1]The proposed method.
[2]Equation (2) for piles in clays, eqn (6) for piles in sands.

Table 29. Adjusting factors (AF) for pile group effects

| $S_p$ (B) | AF |
|---|---|
| $\geqslant 8$ | 1.00 |
| 7 | 0.95 |
| 6 | 0.90 |
| 5 | 0.85 |
| 4 | 0.80 |
| 3 (min) | 0.75 |

the layers covering up to $L_0$ length (up to $K$th layer) must be carried out to determine $k_e$, namely

$$k_e = \frac{3}{L_0^3}\left[\int_{h_0}^{h_1} k_{h1}(L_0 - z)^2\,dz + \cdots + \int_{h_{K-1}}^{h_K} k_{hK}(L_0 - z)^2\right]dz$$

$$= \frac{3}{L_0^3}\sum_{j=1}^{K}\left[A_j L_0^2(h_j - h_{j-1}) + \frac{-2A_j + B_j L_0}{2}L_0(h_j^2 - h_{j-1}^2) + \frac{A_j - 2B_j L_0}{3}(h_j^3 - h_{j-1}^3) + \frac{B_j}{4}(h_j^4 - h_{j-1}^4)\right], \quad (33)$$

where $k_{hj}$ is the horizontal stiffness for the $j$th soil layer (e.g. $k_{h3} = A_3 + B_3 z$), $A_j$ and $B_j$ the soil constants associated with $k_{hj}$, $z$ the coordinate for integration ($h_{j-1} \leqslant z \leqslant h_j$ with $h_0 = 0$, Fig. 3), $K$ the soil layer number dependent of $L_0$ value, and $j$ is the index number ($j = 1$–$K$).

Determination of $k_e$ value is important because it will have significant effect on $L_f$. Solving eqn (33) does involve a complex iterative process. Hence, a computational procedure as detailed in Figs 4–9 (i.e. FORTRAN flow charts) is proposed to compute the inter-dependent $k_e$ and $L_f$ values. The procedure consists of subroutine "PLENGTH" (Figs 4–7), function "LEFIX" (Fig. 8), and function "LEPIN" (Fig. 9). First, the weighted $k_h$, $L_c$ and $k_e$ values are calculated in "PLENGTH". Then, the $L_f$ values are computed by "LEFIX" (for FHP) or "LEPIN" (for PHP). Other notes related to the flow charts (Figs 4–9) are made in the following for clarification:

(1) Matrix symbols are used throughout the flow charts, whenever feasible. For example, the soil layer coordinate, $h_i$ (Fig. 3), is replaced by $h(i)$.

(2) $N$ is the total number of actual soil layers, $uc$ the unit conversion factor, $H$ the total soil depth (Fig. 3), $\Delta H$ the layer thickness [$= h(i) - h(i-1)$], $N_0$ a specific layer number depending on $L_0$ value, and error is the convergence allowance for $L_0$ (error = 1 in or 25 mm). $TL_s$, $TL_m$ and $TL_b$ are the total pile length to be used in the equivalent cantilever model for horizontal stiffness, bending and buckling, respectively.

(3) The initial $k_e$ value is taken as the weighted $k_h$ value, $k_{ave}$, computed by

$$k_{ave} = \frac{1}{H}\sum_{i=1}^{N}\frac{A(i) + B(i)h(i-1) + A(i) + B(i)h(i)}{2} \times [h(i) - h(i-1)]. \quad (34)$$

(4) Ierr is the control code for errors (0: no errors, 1: error from function LEFIX or LEPIN—pile unbraced length being too large). Iquit is the control code for job termination (0: no termination—okay status, 1: quit for undesirable condition—pile being not flexible enough, 2: quit for Ierr = 1—pile unbraced length being too large).

(5) If $L_0$ falls within the layer, say layer $i$, then set the terminating layer depth, $ht(i)$, equal to $L_0$, $N_0 = i$, and $ht(i-1) = h(i)$, where $i = 1$–$N_0$, for calculating $k_e$.

(6) To obtain the more critical $L_f$ values through function "LEFIX" or "LEPIN", two cases concerning the predrilled hole were considered. In the first case (module A in subroutine "PLENGTH", Fig. 5), the predrilled hole is considered part of the lateral support for the pile, and the unbraced pile length denoted as $L_u'(1)$ is equal to the given unbraced length ($L_u$, equaling to zero for fully embedded piles). In the second case (module B in subroutine "PLENGTH", Fig. 6), the predrilled hole is discounted as a lateral support because of the concern of unreliability, and the unbraced pile length represented by $L_u'(2)$ is equal to the given unbraced length plus the predrilled hole depth, namely $L_u'(2) = L_u$ (user specified) + $[h(1) - h(0)]$, Fig. 3.

(7) Function "LEFIX" and function "LEPIN" are based on the proposed method [i.e. eqns (26)–(31)].

(8) Practically, $L_c(1)$ is always greater than $L_c(2)$. If $L_p$ is less than $L_c(1)$, the pile is deemed not flexible enough, and thus represents the undesirable pile condition.

*Comparison with the analytical method*

Comparisons between eqns (26)–(28) and eqns (12)–(14) are shown in Fig. 10 (FHP). While comparisons between eqns (29)–(31) and eqns (18), (19) and (24) are shown in Fig. 11 (PHP) piles. As observed from these two figures, the correlations are excellent. The maximum relative error was only 3% or less. It is therefore recommended that the proposed method for $L_f$ be used because of its accuracy and ease of use.

## NUMERICAL STUDIES FOR PILE FIXITY DEPTHS

All available steel H piles currently used in practice [9] were studied. Young's modulus for pile ($E_p$) is 29,000 ksi. The first important task was to establish the approximate predrilled hole depth as desired for integral abutment construction and required in module B of subroutine "PLENGTH" (Fig. 6). To achieve this task, the LRFD method [i.e. eqns (2) and (6)] in conjunction with the soil properties contained in Tables 2 and 3 were employed to estimate the $L_f$ values, which are summarized in Tables 5 and 6.

From Tables 5 and 6, it is concluded that $R$ [eqn (7)] is no greater than 10.1 ft (HP14 × 117 in soft clay, Table 5) for H piles in clays, and that $T$ [eqn (8)] is no larger than 8.9 ft (HP14 × 117 in submerged loose sand, Table 6) for H piles in sands. For simplicity, it is recommended that the predrilled hole depth be set at 21 ft [to satisfy the necessary condition "$L_u > 2R$" in eqn (2)] for *fully embedded* piles in clays, and 10 ft [to satisfy the necessary condition "$L_u > T$" in eqn (6)] for *fully embedded* piles in sands. Based on past experiences, the practical range of predrilled hole depth has been suggested as 10–25 ft by a number of state departments of transportation. But, no rationale was offered.

The above computational procedure (Figs 4–9, the proposed method) was then applied to determine the fixity depths for the H piles under the various soil conditions. For soil, five layers with equal layer thickness of 20 ft and same type of soils (i.e. all sands or all clays) were considered in the parametric studies. Tables 7–16 summarize the fixity depths for the FHP, and Tables 17–26 summarize those for the PHP with final $L_c$ and $k_e$ values and comparisons. In these tables, the resulting fixity depths from the proposed method are referred as "Chen $L_{fm}$" for bending, "Chen $L_{fb}$" for buckling, and "Chen $L_{fs}$" for horizontal stiffness, while the fixity depth calculated by the LRFD method [eqn (2) or (6)] is called "LRFD $L_f$". Comparisons of $L_f$ values between the LRFD and proposed methods are also concisely shown in Tables 27 (FHP) and 28 (PHP).

## DISCUSSION ON THE RESULTS

Based on the obtained results, the following interesting findings were observed.

(1) With the same soil condition, *smaller piles* resulted in lower $k_e$, $L_c$ and $L_f$ values because of greater flexibility, Tables 7–26. For piles (either FHP or PHP) in sands, the moist/dry condition produced higher (~15%) LRFD $L_f$ values than the submerge condition.

(2) For piles fully embedded in *sands*, changes on $L_u/L_c$ ratios ($L_u$ being 10 ft) were small [0.3–0.5 for loose sands, 0.4–0.7 for medium sands, and 0.5–0.8 for dense sands], which implies small variation on $L_f$ values. However, for piles fully embedded in *clays*, variation on $L_u/L_c$ ratios ($L_u$ being 20 ft) became larger (0.8–1.3 for soft clays, 1.1–1.9 for medium clays, 1.2–2.2 for stiff clays, and 1.9–3.3 for very stiff clays), and hence led to greater fluctuation on $L_f$ magnitudes. It should be noted here that the practical range for $L_u/L_c$ is 0–4.

(3) It was quite interesting to note that $L_f$ varied only slightly with $L_u$ as long as $L_u$ stays in the range of 10–25 ft (the practical range of predrilled hole depth), and that ignoring the predrilled hole depth as part of pile lateral support gave only slightly larger $L_f$ values (representing the more conservative case).

(4) As computed by the proposed method, $L_{fs}$ values were always smaller than the $L_{fm}$s or $L_{fb}$s, Tables 7–26. $L_{fb}$ values were significantly higher than $L_{fs}$ values for the FHP in loose sands, so were $L_{fm}$ values for the PHP in loose sands or clays (any kind). For the FHP in sands or clays, $L_{fs}$ values approached $L_{fm}$s. While for the PHP in medium to very stiff clays, $L_{fs}$ values were close to $L_{fb}$s. The LRFD method does not include the computation on $L_{fs}$.

(5) For piles in sands, LRFD values were significantly (> 17%) lower than the $L_{fm}$ (moment) or $L_{fb}$ (buckling) values computed by the proposed method. This is especially evident (up to ~56%) for larger FHP in looser sands, Tables 27 and 28.

(6) For piles in clays, the variation on $L_f$ values was somewhat irregular. In this case, the LRFD $L_f$ values could be lower or higher than the values computed by the proposed method, depending upon the type of clay and the support condition of pile head, Tables 27 and 28. Contrary to the sand conditions, smaller piles generally had *greater* variation, but only slightly (~4%).

(7) Let us define $DIFF_1 = LRFD\ L_f - L_{fm}$, and $DIFF_2 = LRFD\ L_f - L_{fb}$. $DIFF_1$ was less than $DIFF_2$ for a FHP in sand, approximate to $DIFF_2$ for a FHP in clay, but greater than $DIFF_2$ for a PHP in sand or clay, Tables 27 and 28.

(8) Compared to the PHP, the FHP showed sharper variation for $L_{fb}$. But, such trend was reversed for $L_{fm}$. For piles in sands, on the average the PHP required 43% more for $L_{fm}$, but would require 27% less for $L_{fb}$ than the FHP. While for piles in clays, the average increase and decrease percentages were 50% (for $L_{fm}$) and 9% (for $L_{fb}$), respectively.

(9) The results presented in Tables 7–26 assumed no pile group effects. This is correct when the pile spacing ($S_p$) is eight times the pile dimension (B) or more [10]. If $S_p$ is between 3 B (practical minimum) and 8 B, then pile group effect should be taken into account for $L_f$. To this, *in lieu* of more detailed analysis the factors shown in Table 29 are *suggested* for adjusting the $L_f$ values. As an example, if $S_p = 6$ B, $L_f$ for HP12 × 53 (fixed-headed) in moist/dry sand can be adjusted to 11.8 ft (= 10.6/0.90, Tables 9 and 29).

## SUMMARY AND REMARKS

Various methods for determining the depths to fixity required in an equivalent cantilever pile model are described and discussed in detail in the above. A numerical procedure for determining the effective horizontal stiffness ($k_e$) and fixity depths for multi-layered soil is proposed.

The LRFD method [eqns (2) and (6)], currently adopted in practice, is only valid when the unbraced pile length exceeds 10 ft for piles in sands or 21 ft for piles in clays. These lengths would be required predrilled hole depths for fully embedded piles.

While no single method gives consistently more conservative results, the proposed method appears more favorable because it is more general and is supported by test results. Tables 7–29 can be served as the design tables for use in practice. The value of fixity depth will certainly affect pile strengths and design, and this issue is addressed in Part II of this paper.

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## APPENDIX

### CONVERSION OF UNITS

1 ft = 300 mm
1 ksi = 7 MPa
1 ksf = $4.8 \times 10^4$ Pa
1 tsf = $9.6 \times 10^4$ Pa
1 tst $ft^{-1}$ = 320 Pa $mm^{-1}$.