Limits of Applicability of Conventional and Adaptive Pushover Analysis for Seismic Response Assessment

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ABSTRACT

In this report, the applicability of conventional and advanced inelastic static (pushover) analysis for seismic response assessment is investigated. A methodology is first suggested for evaluating the performance of pushover methods, based on a quantitative measure for the difference in response between pushover and inelastic dynamic analysis which is deemed to be the most accurate, but still expensive numerical method available. This methodology is applied on a set of eight different structural systems, covering various levels of irregularity in plan and elevation, structural ductility and directional effects. An extensive series of pushover analysis results, monitored on various structural levels is presented and compared to inelastic dynamic analysis under various strong motion records. General conclusions on the applicability of inelastic static analysis for seismic response assessment are finally discussed.

1. INTRODUCTION

1.1 Background

Conventionally, seismic assessment and design has relied on linear or equivalent linear (with reduced stiffness) analysis of structural systems. In this approach, simple models are used for various components of the structure, which is subjected to seismic forces evaluated from elastic or design spectra, and reduced by force reduction (or behavior) factors. The ensuing displacements are amplified to account for the reduction of applied forces. This procedure, though simple and easy to apply in the design office environment, suffers from the following shortcomings :

- The force reduction factors recommended in codes of practice are approximate and do not necessarily represent the specific structure under consideration.
- When critical zones of a structure enter into the inelastic range, the force and deformation distribution change significantly. This change is not represented by a global reduction of forces.
- The mechanism that will most likely perpetuate collapse is unlikely to be that represented by the elastic action and deformation distribution.
- The global and particularly the local distribution of deformations in the inelastic range may bare no resemblance to those in the elastic range. The same applies to the values of deformations, not just the distribution.

As a consequence of the above, the reduced forces - amplified deformations linear elastic approach fails to fit within the principle of failure mode control, which is part of performance based assessment and design. This in turn has lead to an increase in the use of inelastic analysis as a more realistic means of assessing the deformational state in structures subjected to strong ground motion.

Static and dynamic analyses use the same material constitutive relationships, with the only exception being static monotonic analysis that does not require unloading - reloading models. Both use principles of equilibrium and compatibility, with the difference being that the dynamic equilibrium of forces includes damping and inertial effects. Both make use of iterative procedures to arrive at convergent solutions. Finally, whereas in static inelastic analysis the variable is the current level of displacement or force, that in dynamic analysis is time. Therefore the four differences that may affect the difference in the level of complexity and computing resource requirements are:

1. Static monotonic analysis requires only monotonic constitutive models.
2. Dynamic analysis requires treatment of structural damping and mass distribution.
3. Static analysis to collapse is repeated as many times as the deformation causing collapse divided by the displacement increment necessary for convergence; this is likely to be in the tens of steps.
4. Dynamic analysis is a static analysis repeated as many times as the duration of the earthquake divided by the time step for response history analysis; this is likely to be in the thousands of time increments.

The outcome of the above discussion leads to the conclusion that static analysis requires simpler models, representation of stiffness and strength only and a fraction of the number of analyses, compared to dynamic analysis. This is the underlying reason for the increased use of pushover analysis in the industry, and the inclusion of static inelastic methods in assessment guidance notes (e.g. FEMA 273/274 and sequels) as well as modern design codes (e.g. the new draft of Eurocode 8).

1.2 Objectives and Organization

With the proliferation in the use of static inelastic (pushover) methods for seismic assessment and design comes controversy. Many researchers have contributed developments to enhance the performance the pushover technique (e.g. Freeman et al, Bracci et al, Chopra et al amongst others). They, including the authors of this report, advocate the use of pushover, in its various forms, in lieu of dynamic analysis. On the other hand, resistance to the use of pushover analysis comes from two opposing ends of the complexity spectrum. Advocates of simplicity legitimately state that the single load distribution pushover, as discussed in Chapter 2 of this report, fails to capture the actual behavior, and that more advanced versions are too complex for practical applications. Others state, also legitimately, that inelastic dynamic effects cannot be captured by any static method, hence full dynamic analysis is necessary. Work in the literature, with very few exceptions, has focused on developing pushover techniques without assessing their performance comprehensively. This would require application of static pushover to a wide range of structures, ranging from low to high rise, from regular to highly irregular, subjected to a large number of earthquake records covering a wide range of magnitudes, distance, site condition and source mechanism. The objective of this report is to make a contribution to the detailed assessment of static pushover methods, including conventional and adaptive, as compared to inelastic response history analysis. In this respect, the important features of the current study that add to the existing technical literature are that it deals with a wide range of structural forms subjected to strong-motion records of different characteristics and monitored on the local, intermediate and global levels. These features are summarized in the following:

The Sample Structures:

The structures used for this study cover a wide range of variation in characteristics to help in deriving some generally - or widely - applicable conclusions. They have the following attributes:

- Regular structure design to modern seismic design codes
- Structures with various levels of intentional ductility
- Structures that are high irregular
- Structures with no seismic design provisions
- Shear-frame dominated response (frame systems)
- Cantilever dominated response (frame-wall systems)

Whereas they are specific structures, conclusions from their response to a number of earthquake input motions may be considered sufficiently indicative of the response of a much larger class of building.

The Assessment Criteria :

A complete set of criteria is used to monitor the response, on all local and global levels. These are :

- Global displacement
- Inter-story displacement at all floors
- Moment-curvature at critical sections
- A new measure of the difference between static and incremental dynamic analysis curves.

The Input Motion :

The input motion set used is reasonably varied to provide the necessary variability but without masking issues of structural response by using records that depart from the norm to considerable degrees. Their characteristics include the following features :

- Short period high amplification records
- Long period high amplification records
- Code-compatible synthetic records

The above are deployed to answer, at least in part, the central question of whether pushover analysis of various levels of complexity is capable of replacing dynamic analysis as a reliable assessment and design tool. The complementary question is what the limits of applicability of the method, in terms of structural and input motion characteristics, are. Towards this end, the report is divided into six chapters, dealing with (1) Introduction, preliminaries and background, (2) Review of static pushover formulations and conclusions from previous work by other researchers as well as the writers, (3) Description of the structures, input motion, analysis program and assessment criteria, (4) Results of all static and dynamic analyses, (5) Evaluation of results and (6) Conclusions.

2. CONVENTIONAL AND ADVANCED PUSHOVER ANALYSIS

2.1 Overview

In this chapter, the characteristics of both conventional and advanced approaches for inelastic static (pushover) analysis are presented, and comprehensively compared. The background to the decisions taken in executing the analyses presented in subsequent sections is given. A brief review of previous work on this subject is also given.

2.2. Review of pushover analysis development

The use of inelastic static analysis in earthquake engineering is traced to the work of Gulkan and Sozen (1974) or earlier, where a single degree of freedom system is derived to represent the multi-degree of freedom structure via an equivalent or 'substitute' structure. The load-displacement curve of this substitute to the real structure is evaluated by either finite element analysis or hand calculation to obtain the initial and post-yield stiffness, the yield strength and the ultimate strength. Simplified inelastic analysis procedures for multi-degree of freedom systems have also been proposed by Saiidi and Sozen (1981) and Fajfar and Fischinger (1988). Therefore, pushover analysis per se is not a recent development. However, this review is concerned with multi-degree of freedom inelastic analysis of complex structures, which is relatively recent.

There are several publications that review the advantages and disadvantages of pushover analysis, with varying degrees of success. They all, however, utilize global response parameters, namely top displacement versus base shear. Lawson *et al.* (1994) discuss in some detail the range of applicability, the expected realism for various structural systems and highlight the difficulties encountered. The latter study is both conceptual and applied, rendering it specifically valuable. In the course of describing recent trends in seismic design, Krawinkler (1995) discusses pushover analysis as a prelude to capacity spectrum applications. The author mentions a contentious point, which is that 'in most cases the normalized displacement profile at a first estimate of the target displacement level is utilized for these (defining the shape vector) purposes'. Another important issue raised in the latter review is that 'it must be emphasized that the pushover analysis cannot disclose performance problems caused by changes in the inelastic dynamic characteristics due to higher mode effects'. This is true if the current (as in 1995) techniques are assessed, but is not an insurmountable problem, as discussed in the current report and the earlier work of Bracci *et al.* (1995) and the authors of this report. Several cases of success of the pushover method were reported by Faella (1996), who also pointed towards difficulties with static-dynamic comparisons when the strong-motion input is rich in long period amplifications.

Attempts at improving the procedure have been made, with varying degrees of rigor and success. The simplest and most pragmatic of which is the work of Sasaki *et al*. (1998). This comprises running several pushover analyses under forcing vectors representing the various modes deemed to be excited in the dynamic response. If the individual pushover curves, converted to spectral displacement-spectral acceleration space using the dynamic characteristics of the individual modes are plotted alongside the composite spectra, it becomes apparent which mode would be the cause of more damage and where is the damage likely to occur. The procedure is intuitive, and does indeed identify potential problems that conventional single mode pushover analysis fails to point out. It, however, falls short of the work of Bracci *et al*. (1997), which is the most recent in-depth study of pushover analysis.

The pros and cons of the procedure were also discussed by Krawinkler and Seneviratna (1998). Amongst many other interesting comments, the authors stress that the most important shortcoming of the procedure is the definition and invariance of the applied load vector. The significance of defining the target displacement (required for evaluating structural adequacy under a specific earthquake) is considered secondary to the load vector definition and control. Whereas the authors report on successful pushover cases, they emphasize problem areas by discussing the response of a tall (20 story) structure, and a structure with a full-height wall. In the former case, the errors are due to the omission of higher mode effects, whilst the latter demonstrates the difficulties encountered with effect of concentrated local demand (base of the wall) on force distribution. A short review by Tso and Moghadam (1998) concluded that fixed load patterns in pushover analysis are limiting, but newly proposed variable load patterns are not sufficiently verified as a superior option.

Kim and D'Amore (1999) set out to assess pushover analysis in comparison with inelastic time-history procedures. They concluded that not all analyses of the same structure under a set of distinct earthquake records are predicted by pushover analysis, a rather obvious conclusion that did not require inelastic dynamic analysis to prove. The interaction between the continuously-changing dynamic characteristics of the inelastic multi-degree of freedom structure with the various frequencies of a set of natural records cannot possibly be duplicated by a single pushover analysis under a predefined and fixed transverse load or displacement vector. But, again, this problem may have a solution.

An adaptive procedure is described in the paper by Bracci *et al*. (1997), and attributed to a previous publication by Reinhorn and Vladescu. This comprises starting the analysis assuming a certain force distribution, usually triangular. Loads imposed in subsequent increments are calculated from the instantaneous story resistance and the base shear in the previous step.

This procedure is applied in the context of defining the moment curvature relationship of the various members as an input parameter, and is intended, to capture the effect of local mechanisms. It does not account for higher mode contribution. The procedure was implemented in the dynamic analysis package IDARC (Kunnath *et al*., 1992) and demonstrated in the paper to give accurate results for the structure considered. However, numerical tests conducted by Lefort (2000) showed that the above procedure grossly under-estimate the strength, compared to inelastic dynamic analysis using IDARC, by up to 60% for a regular 10 story structure. Peculiarly, conventional pushover with triangular or uniform load distribution gave results far superior to the above adaptive method.

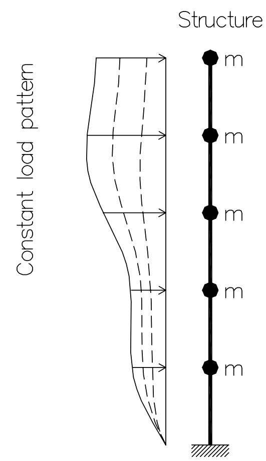
Work undertaken by the writers and their co-workers has developed a robust procedure for adaptive pushover analysis that is shown to be superior to, or at worst as good as, conventional pushover. Formulations given by Papanikolaou (2000), subsequent developments by Antoniou (2003) and an overview by Elnashai (2002) detail this fiber-based, self-adjusting adaptive approach. This is described in the subsequent sections and previous results obtained from idealized structures are highlighted.

A similar study by Antoniou and Pinho (2004), also question the applicability of conventional and adaptive pushover methods in predicting the horizontal capacity of reinforced concrete structures, compared to inelastic dynamic analysis. These three analysis approaches were applied on a set of different concrete structures with varying structural properties under various strong motion records. It was mainly concluded that the estimation of structural deformation patterns were poorly predicted by both types of pushover analysis. Several other aspects of this study are met and discussed in the present report in subsequent chapters.

2.3 Conventional pushover analysis formulation

Conventional pushover analysis is the nonlinear incremental-iterative solution of the equilibrium equation **KU** = **P** in a finite element formulation, where **K** is the nonlinear stiffness matrix, **U** is the displacement vector and **P** is a predefined load vector applied laterally over the height of the structure in relatively small load increments . This lateral load can be a set of forces or displacements that have a necessarily constant ratio throughout the analysis (fixed pattern). At the end of each iteration, the reaction vector ($\mathbf{P}^e$) of the structure is calculated from the assemblage of all finite element contributions. The out-of-balance forces are iteratively re-applied until convergence to a specified tolerance is reached (Bathe, 1982) :

$$\Delta\mathbf{U} = \left[\mathbf{K}_T\right]^{-1} \cdot \left(\lambda \cdot \mathbf{P}_0 - \mathbf{P}^e\right) \qquad (2.1)$$



where :

$\Delta\mathbf{U}$ is the calculated displacement increment within an iteration
$\mathbf{K}_T$ is the current nonlinear (tangent) stiffness matrix
λ is the load factor within the corresponding load increment
$\mathbf{P}_0$ is the initial load
$\mathbf{P}^e$ is the equilibrated load (reaction) of the previous iteration

$$\mathbf{P}^e = \sum \int_V \mathbf{B}^T \cdot \sigma_{NL} \cdot dV \qquad (2.2)$$

where :

$\mathbf{B}$ is the strain-displacement matrix of each element.
σ_{NL} is the element nonlinear stress vector as determined by its material constitutive law.

The procedure continues either until a predefined *limit state* is reached or until structural collapse is detected. This target limit state may be the deformation expected for the design earthquake in case of designing a new structure, or the drift corresponding to structural collapse for assessment purposes. Furthermore, it is presumed that the finite element code has been sufficiently verified, so that numerical collapse, as opposed to structural, is not operative. Generally, this procedure allows tracing the sequence of yielding and failure on the member and structure level, as well as the progress of the overall capacity curve of the structure (figure 2.1).

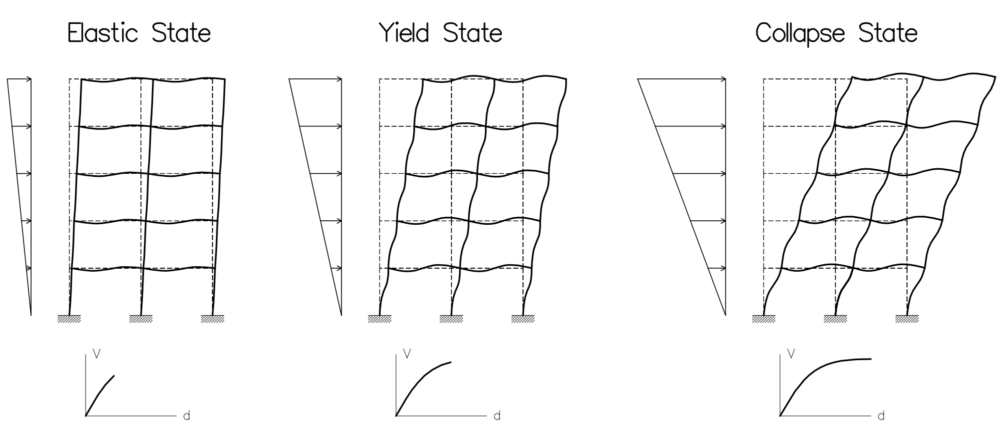


Figure 2.1 Yielding sequence through conventional pushover analysis.

The critical parameters to define the characteristics of the conventional pushover analysis are the lateral load nature (forces or displacements), its distribution pattern along the height of the structure (triangular, uniform etc) and its magnitude. The number of load steps, the convergence criteria and the iterative strategy also play a significant role in the effectiveness and reliability of the analysis.

2.4 Extension to advanced pushover analysis

The necessity of implementing advanced pushover approaches such as the adaptive pushover analysis applied in this analytical study is due to many fundamental deficiencies of conventional pushover approach, compared to inelastic dynamic analysis:

- Generally, pushover analysis implies a separation between structural capacity and earthquake demand. However, many research findings have established an interconnection between structural capacity and earthquake demand. Moreover, nonlinear structural behavior is *load path* dependent and separation between loading input and structural response is not always adequate.

- This procedure also assumes that structural damage is a function only of the lateral deformation of the structure, neglecting *duration* effects and *cumulative energy* dissipation demand. It is generally accepted that damage is a function of both deformation and energy and hence the applicability of pushover analysis is too simplistic, particularly for nonductile structures which exhibit severely pinched hysteretic behavior.

- Static pushover analysis neglects dynamic effects. Hence, during an earthquake, the inelastic structural behavior can be described by balancing the dynamic equilibrium at every time step. As pushover analysis focuses only on the strain energy of the structure during a monotonic static push, it neglects other sources of energy mainly associated with dynamic components of forces such as *kinetic* energy and *viscous damping* energy.

- The conventional pushover analysis procedure does not account for the progressive changes in the *modal properties* during nonlinear yielding and cracking in the structure which leads also to *period elongation* and hence different *spectral amplifications*. This is due to the constant lateral load pattern used, which ignores the potential redistribution of inertia forces and higher mode effects, as yielding and cracking governs the inelastic structural behavior.

It is clearer now that conventional pushover analysis, being a static analysis, lacks many features of its dynamic counterpart which may be crucial in certain cases. However, it provides the engineer with an efficient alternative not only to expensive inelastic dynamic analysis but also to standard seismic code practice. Consequently, some possible developments aimed at enhancing conventional pushover analysis are suggested below:

- A combination of pushover analysis with *fiber models* where no prior assumptions are made on the behavior of the member, and where the moment-curvature response is derived from the material characterization.

- Adaptive pushover analysis which takes into account the current level of local resistance or by *mode interaction* at instantaneous states of inelasticity and updates the forcing function accordingly. Moreover, spectral amplifications due to the elongated instantaneous inelastic periods of the structure can be also accounted for.

Several attempts at adapting the force distribution to the state of inelasticity are described in the literature. The first attempt to improve the conventional static pushover procedure was the method of Sasaki *et al.* (1998). According to this method, several pushover analyses are run for the same structure with force patterns representing the various modes of the structure. A combination of these separate solutions is then performed to assess the overall structural response (figure 2.2).

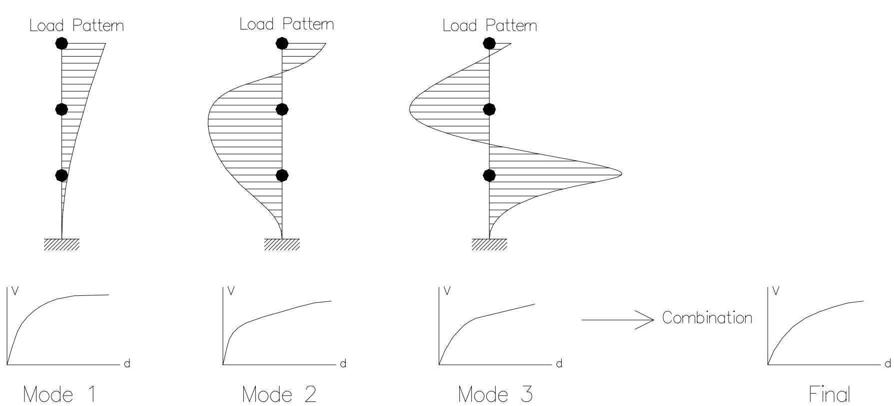


Figure 2.2 Method by Sasaki et al. (1998)

Bracci *et al*. (1997) and Lefort (2000) used the inelastic story forces of the previous equilibrated load step to update the lateral load pattern (figure 2.3). The story force distribution is obtained either by adding an increment of the new force vector to that existing from a previous step or by a new set of forces accounting for the current state of resistance distribution. The researchers have reported varying degrees of success.

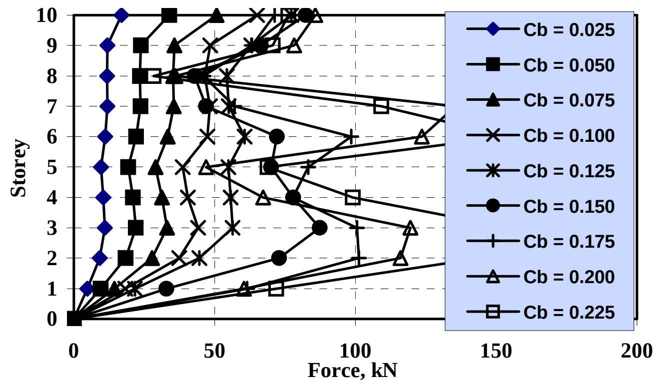


Figure 2.3 Adaptive pushover using story forces to update the lateral load pattern

In this analytical study, an adaptive pushover approach (Papanikolaou, 2000) which takes into account mode interaction and spectral amplifications was adopted. According to this approach, the lateral load *pattern* is not kept constant during the analysis but it is continuously updated, based on a combination of the instantaneous *mode shapes* and/or *spectral amplifications* corresponding to the inelastic periods of the structure (figure 2.4).

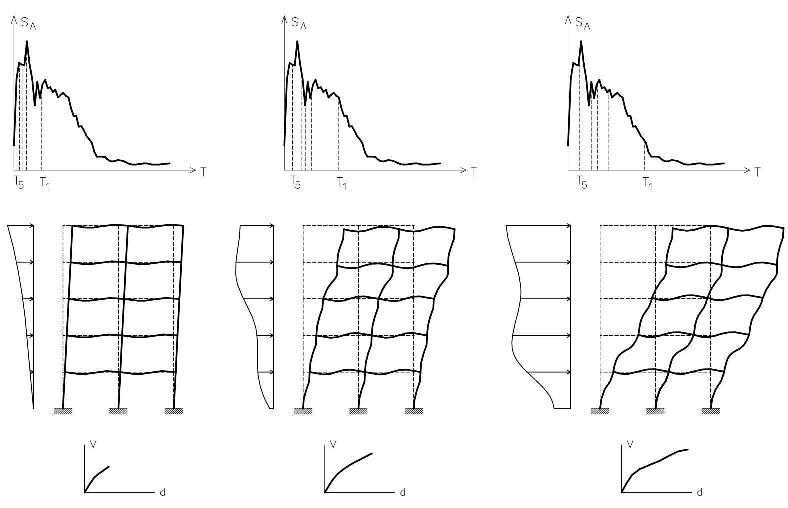


Figure 2.4 Adaptive pushover using modal combinations

These mode shapes are determined by performing eigenvalue analysis (using a Lanczos eigenvalue solver) operating on the current *tangent* stiffness matrix ($\mathbf{K}_T$) and the spectral amplifications are calculated by numerical integration of the strong-motion record. These ingredients can form the new lateral load pattern as described below. A flow chart of the procedure is shown in figure 2.5.

The modal load for mode i, applied on the j degree of freedom is defined by :

$$F_j^i = \nu^i \cdot \Phi_j^i \cdot m_j \cdot (S_A^i) \tag{2.3}$$

where :

ν^i is the modal participation factor of mode i , $\nu^i = \dfrac{\Phi^i \cdot \mathbf{M} \cdot \delta}{\Phi^{i^T} \cdot \mathbf{M} \cdot \Phi^i}$

(Φ_i is the eigenvector for mode i, **M** is the mass matrix and δ a unit vector)

F^i_j is the element of the i-mode eigenvector, referring to the j degree of freedom

m_j is the lumped mass on the j degree of freedom

S^i_A is the spectral amplification of mode i

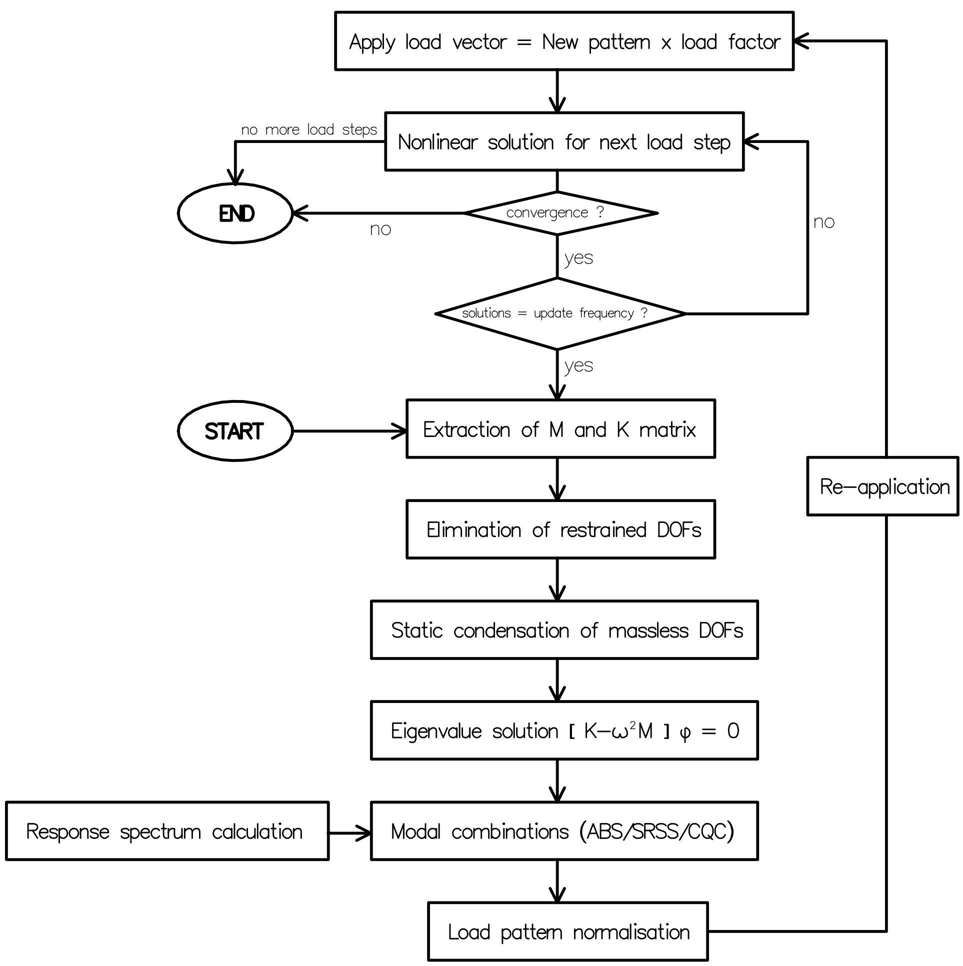


Figure 2.5 Flow chart of the adaptive pushover method

The contribution of S^{i}_{A} is indicated in parenthesis in equation 2.3 because it is possible omit it in the adaptive pushover analysis, so that the lateral load pattern can be defined regardless of the earthquake input, depending only on the mode shapes of the structure. This is the case in the present analytical study for the following reasons :

- Spectral amplifications refer to an elastic response spectrum derived from a single degree of freedom pendulum and hence it is deemed not suitable for application to multi-degree-of-freedom structures exhibiting inelastic behavior.

- It is not realistic to assume a constant damping factor to derive the elastic spectrum when damping is strongly related to inelastic structural response and especially structural ductility.

- Numerical instabilities may occur when there are sudden peaks in the elastic response spectrum which alter abruptly the instantaneous lateral force distribution.

After defining the lateral load profiles for all different modes, a modal combination (SRSS, CQC or ABSolute) is carried out to yield the new load vector. The last step before the re-application of this lateral load vector is normalization and multiplication by the current loading level (product of nominal load and the current load factor). Consequently, the *base shear* remains independent of the load pattern, being controlled only by the current loading level. Thus, the magnitude of the pushover load is still applied incrementally, as performed in the conventional pushover approach.

Finally, it is necessary to point out an important feature of the analysis package which was adopted in the pushover analyses of the present investigation. It is the multi-phase (automatic response) analysis, which switches to displacement control *after* a possible convergence failure of the conventional force control method (usually occurs when the strength peak in the pushover curve is reached). Following this development, it is possible for the capacity curve to continue in the post-peak force resistance range, thus potential softening of the structure (descending branch) may be investigated and the overall ductility may be assessed (figure 2.6).

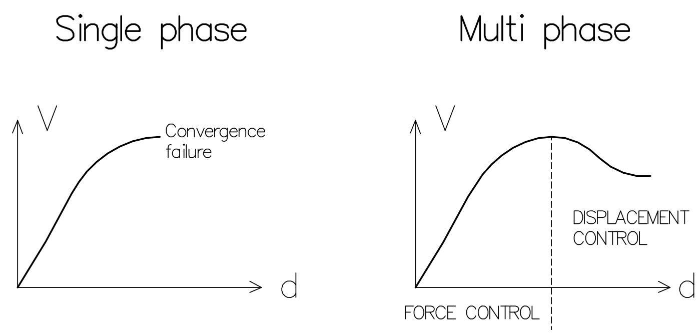


Figure 2.6 Single versus multi-phase solution

2.5 Previous comparisons between conventional and adaptive pushover analysis

The adaptive pushover method is expected to perform as good as or in most cases better than its conventional counterpart where stiffness and strength irregularities (geometry, soft story) and mass eccentricities and concentrations (poor construction, excess storage) are imposed on the structure during design or assessment. Figure 2.7 shows a comparative assessment (Papanikolaou, 2000) of conventional pushover with uniform and triangular lateral load patterns, adaptive pushover with and without spectral amplifications and incremental dynamic analysis results (described in paragraph 3.4). The structural systems under consideration were various idealized DOF (shear) structures. The response parameter used was the base shear versus global drift (displacement difference between top and base of the structure divided by the total height).

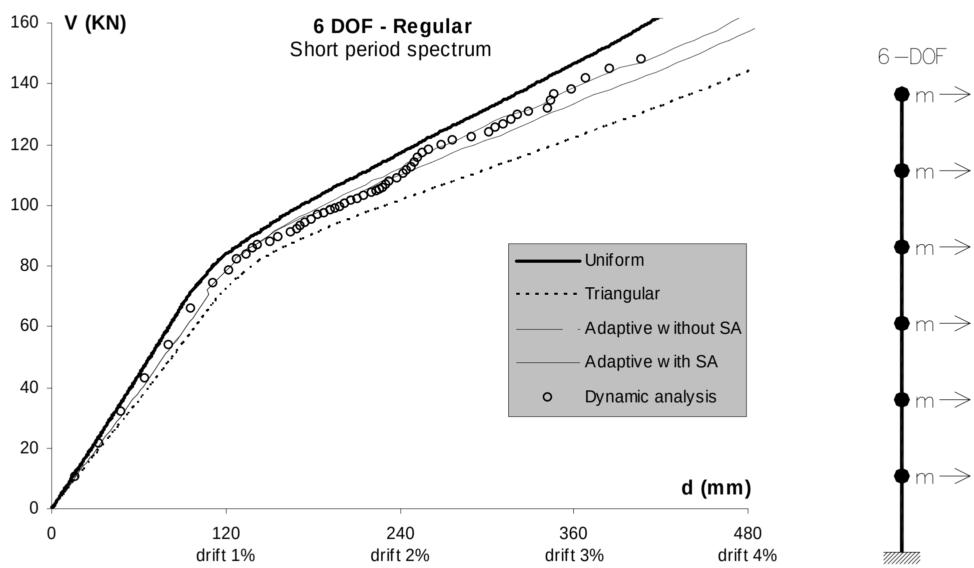


Figure 2.7 Comparative plot among conventional pushover, adaptive pushover and dynamic analysis

The suggested adaptive pushover approach has the capability to capture modal and spectral contributions throughout the inelastic range, by operating directly on the instantaneous tangent stiffness matrix. When compared to inelastic dynamic analysis, although being a static analysis, it successfully simulates the structural response, especially at low drift levels (usual structural response under moderate earthquake events) but also at higher levels under certain circumstances (structural and strong motion regularity).

However, there are cases when the dynamic response of the structure diverges from the smooth 'pushover' shape, especially at higher drift levels (figure 2.8). This is due not only to potential higher mode effects and sensitivity to the record frequency content and/or high frequency peaks emanating from finite element idealizations, but also to the time integration scheme parameters and the unidentified relationship between real hysteretic and equivalent viscous damping. Nevertheless, even in the above cases, the adaptive pushover approach still performs significantly better than the conventional approach, especially at lower drift levels.

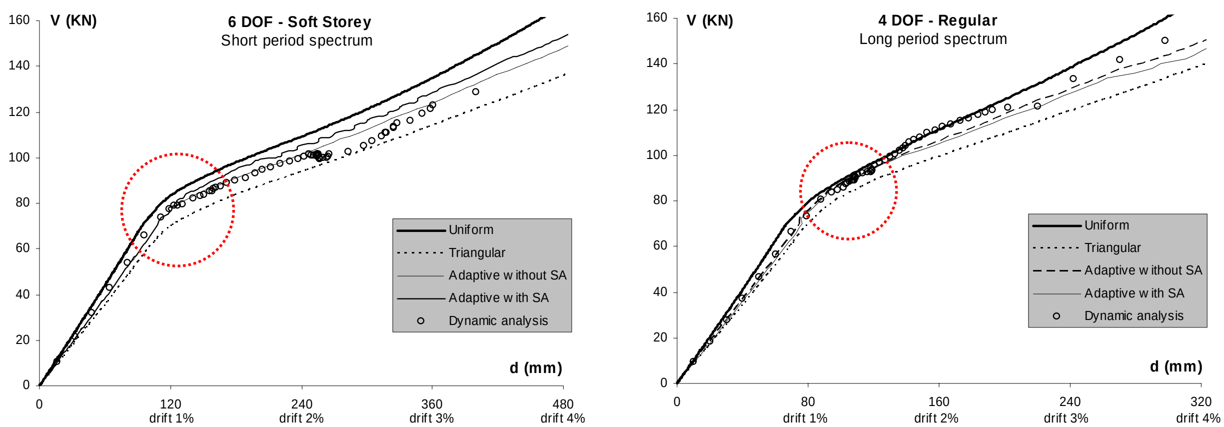


Figure 2.8 Dynamic response irregularities

Displacement profiles, for different load levels and different pushover methods, are depicted in figure 2.9. It is apparent from these plots that the adaptive pushover response lies between the two conventional pushover solutions. At early loading stages, the adaptive pushover displacement profile is closer to conventional pushover with triangular distribution and incrementally approaches the profile of conventional pushover with uniform distribution (attracting equal forces at all story levels), favouring a fundamental mode (cantilever) behavioral pattern.

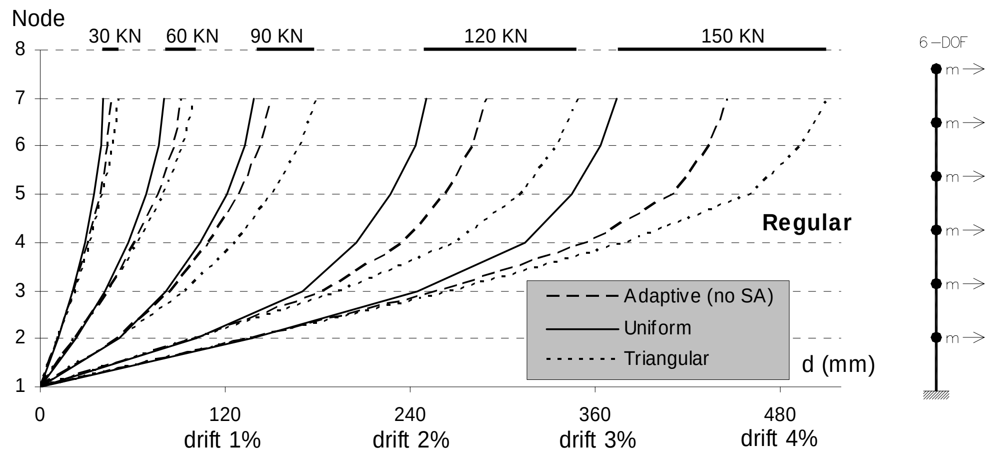


Figure 2.9 Displacement profiles

Another confirmation of the aforementioned shifting from triangular to uniform force distribution, which is captured successfully by the adaptive pushover analysis, is shown by the story shear versus interstory drift plot of figure 2.10. There is a drastic change from a distributed response mode to a SDOF behavior (cantilever), when the first story reaches its yield limit. The observed descending branch is due to possible stress redistribution after first story yielding.

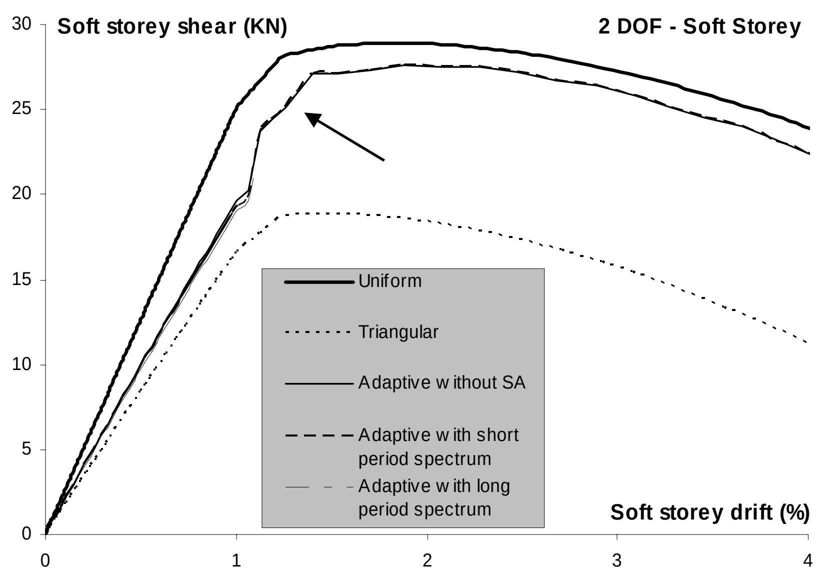


Figure 2.10 Story shear versus drift plot for the soft story

It is noteworthy that the additional complexity required to perform adaptive pushover analysis is considerable, in terms of accessing a robust eigenvalue solver, efficiently updating the applied force vector and switching if necessary to a fixed-distribution displacement control past the peak point on the load-displacement curve. However, the onus of these complications is on the software developer, not the user. The software package used in this analytical study, Zeus-NL (Elnashai, Papanikolaou and Lee, 2002-2004), fulfills all these requirements as described in the subsequent chapter. The only added complexity to the user of adaptive pushover is the requirement of a reasonable mass representation of the weight of the structure.

3. ANALYTICAL INVESTIGATION

3.1 Overview

In this chapter, all the components of the present analytical investigation are described in detail. These include analysis software and methods, structural models of the building portfolio, strong motion input and response evaluation parameter.

3.2 Analysis software and Methods of analysis

The software package used in the present investigation is Zeus-NL (Elnashai, Papanikolaou and Lee 2002-2004) developed at the Newmark Laboratories of the University of Illinois at Urbana-Champaign. This is an analytical tool for finite element analysis of reinforced concrete, composite and steel space frames. It is an advancement of the analysis packages ADAPTIC (Izzuddin and Elnashai, 1989) and INDYAS (Elnashai *et al.* 2000), developed at Imperial College, London.

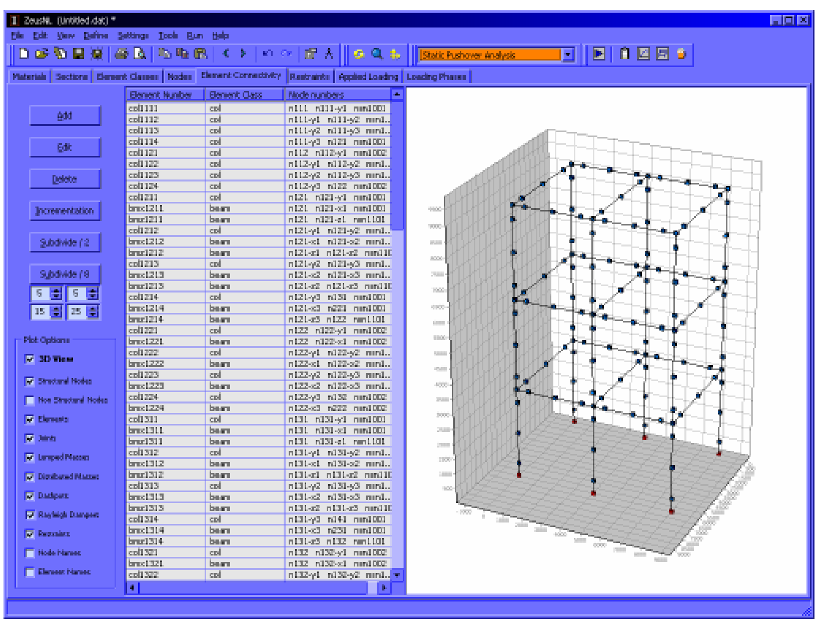


Figure 3.1 The Zeus-NL program

Zeus-NL performs nonlinear analysis using a layered response approach which accounts for the spread of inelasticity along the element length and across the section depth. It is also capable of predicting the large frame displacements by imposing equilibrium in the deformed state of the structure and hence it can represent geometrical nonlinearity and P-Δ effects. It also has the ability to perform eigenvalue, static pushover (as described in the previous chapter), static time-history and dynamic analysis, as follows:

• *Eigenvalue analysis* : The efficient Lanczos algorithm is used for the evaluation of the structural natural frequencies and mode shapes.

• *Static pushover analysis (conventional and adaptive)*: In the conventional pushover analysis, the applied loads (displacements, forces or both) vary proportionally according to a predefined pattern. The post-peak response is obtained with different displacement control procedures. In adaptive pushover, the applied load pattern is not kept constant, but is continually updated in order describe in a more accurate manner the stiffness degradation and the period elongation of the structural system.

• *Static time-history analysis* : The applied loads vary independently in the pseudo-time domain, according to a prescribed load pattern.

• *Dynamic analysis* : The applied load is accelerations or forces at the supports of the structures, according to a strong motion record. Both synchronous and asynchronous excitation can be modeled. The Hilber-Hughes-Taylor or Newmark integration algorithms may be employed.

In the present analytical investigation, all the models considered were analyzed using conventional pushover (with triangular lateral load distribution), adaptive pushover (without spectrum scaling), and inelastic dynamic analysis (with HHT integration and various input motions). The latter was adopted for comparison purposes because it is considered to be the most advanced and precise analysis procedure available.

The Zeus-NL materials library contains a set of uniaxial models for steel and concrete reinforced concrete described briefly below:

i) Bilinear steel model with kinematic strain-hardening.
ii) Tri-linear concrete model. It is a simplified concrete model for uniaxial modeling.
iii) Constant confinement concrete model.
iv) Variable confinement concrete model. It accounts for variable confinement effects, which are influenced by the confined area within the hoops, hoop size, material and spacing.

The section library features a large number of steel, reinforced concrete and composite sections, including solid, hollow and jacket sections. The elements library of Zeus-NL includes the following options:

i) 3D cubic elastoplastic beam/column element. This element was used for modeling the structural members of the analyzed models.
ii) 3D joint element, exhibiting various load-displacement formulations (from linear to hysteretic flexure or shear with varying axial load). The linear joint elements were extensively used to describe the beam-column connections of the analyzed models.
iii) Lumped (concentrated) mass element. It is used in dynamic, eigenvalue, and adaptive pushover analysis for modeling point structural masses.
iv) Cubic distributed mass element. It is used for modeling uniformly distributed mass.
v) Dashpot (concentrated) viscous damping element.
vi) Rayleigh distributed damping element.

Contributions and verifications of the Zeus-NL finite element code in its present or previous forms of ADAPTIC and INDYAS have been carried out by a large number of researchers on MSc and PhD theses in the past and hence its stability and robustness has been confirmed throughout. Consequently it is deemed appropriate for the present analytical investigation on the limits of applicability of conventional and adaptive pushover analysis for seismic response assessment.

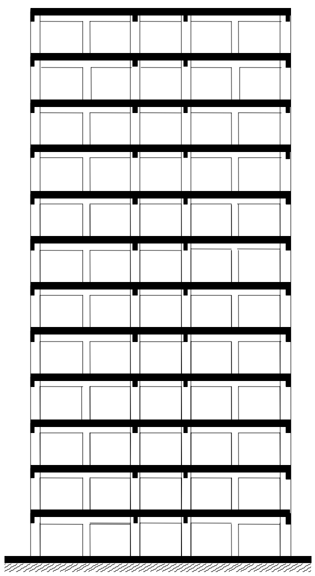
3.3 Models considered

Eight structural models were used in the present study, chosen in order to cover various levels of irregularity in plan and elevation, structural ductility, in-plane stiffness and directional effects. They are also intended to represent buildings with and without seismic design provisions, hence coverage of both old existing and modern structures is provided.

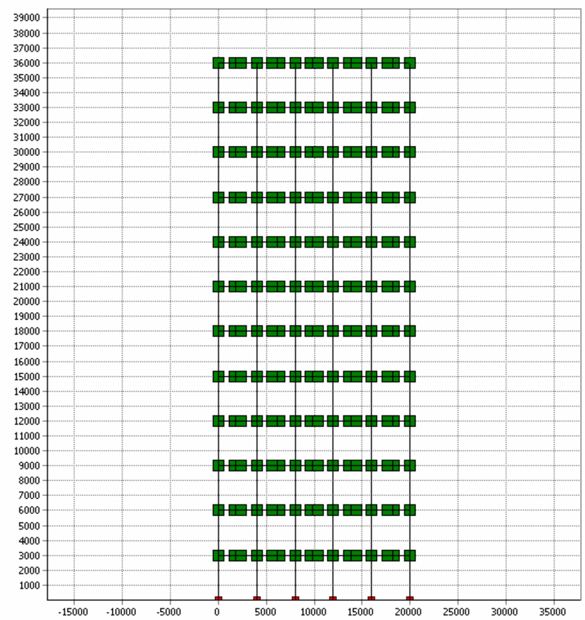
The first six models were buildings designed according to the Eurocode 8 recommendations (Fardis *et al*, 1994, Mwafy and Elnashai, 2000), featuring the following characteristics :

- **Regular structures** : two models, designed for high (H) and low (L) ductility recommendations and detailing according to Eurocode 8, under design accelerations of 0.30g and 0.15g respectively (figure 3.2). Apart from their structural regularity, these structures were selected for their long natural periods and the possibility of exhibiting higher mode effects during the inelastic process.

Actual structure



FE modeling and mass distribution



2D reinforced concrete frame
with regularity in plan and elevation.

Structural ductility : High / Low
Design acceleration : 0.30g / 0.15g

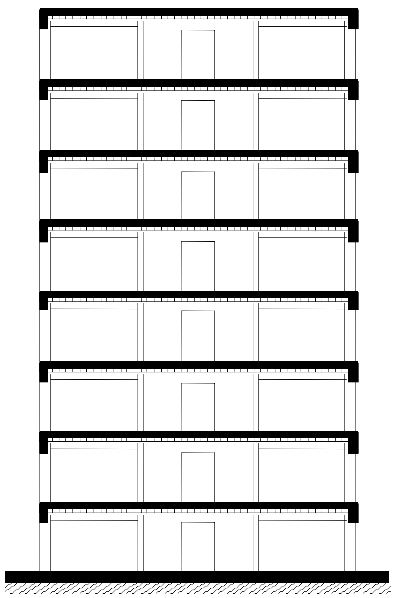
Concrete strength : f_c = 30 MPa
Steel yield strength : f_y = 585 MPa

Stories :	12
Total height :	36.00 m
Typical story height :	3.00 m
Total length :	20.00 m
Number of nodes :	584
Number of elements :	624 (conv. pushover)

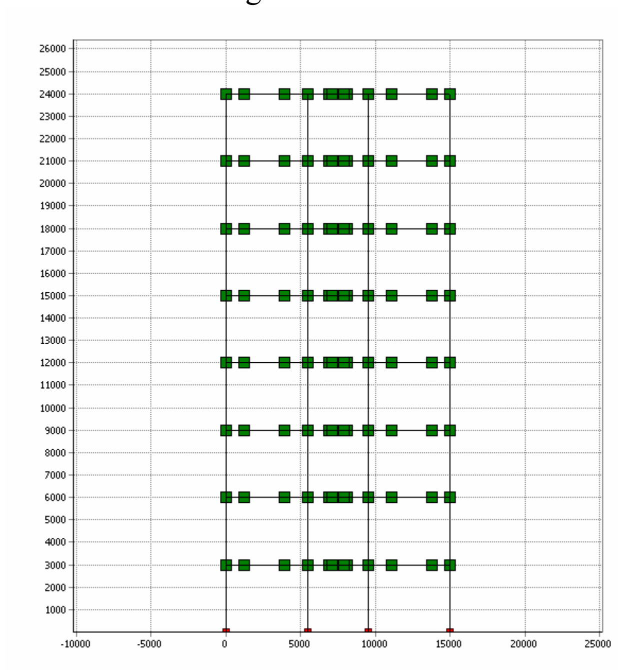
Figure 3.2 Regular structure modeling and features

• **Frame-wall structures** : two models, designed with high (H) and low (L) ductility recommendations with a design acceleration of 0.30g and 0.15g respectively (figure 3.3). These structures were selected because they may exhibit a cantilever (single mode) instead of frame behavior and hence be dominated by the fundamental mode. Damage at the base of the wall is likely to lead to a uniform force distribution, thus its analysis would test the procedures employed.

Actual structure



FE modeling and mass distribution



2D reinforced concrete frame with a shear wall core.

Structural ductility : High / Low
Design acceleration : 0.30g / 0.15g

Concrete strength : f_c = 30 MPa
Steel yield strength : f_y = 585 MPa

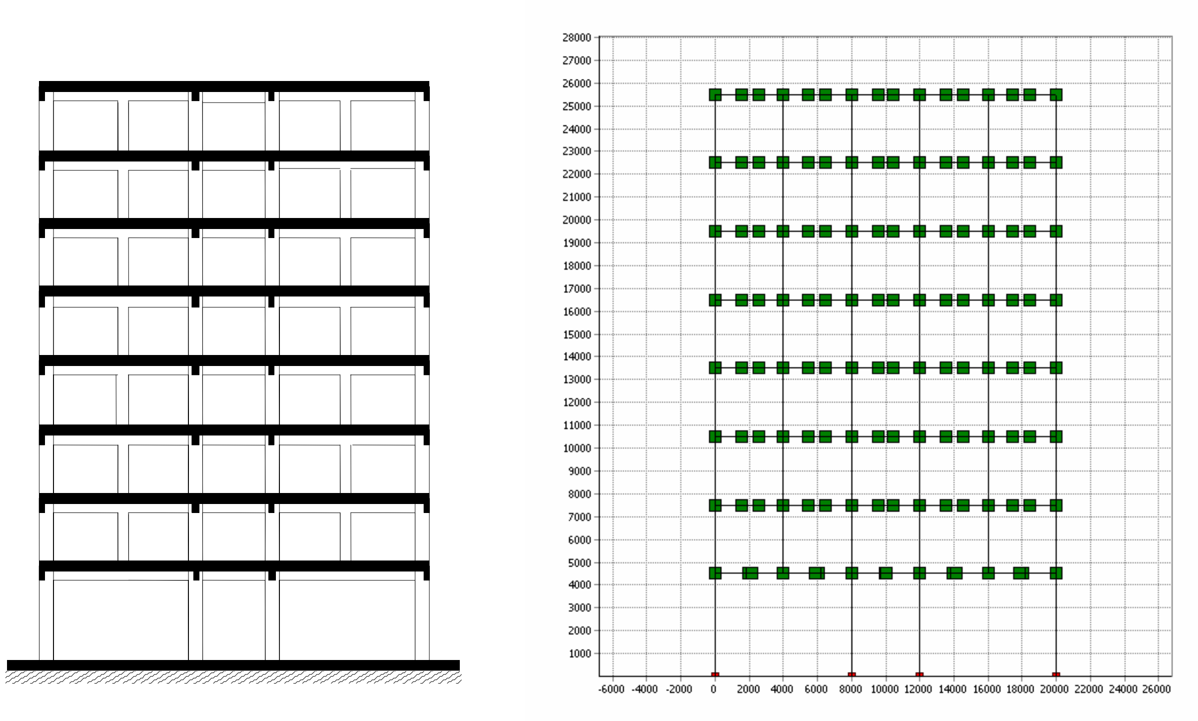
Stories :	8
Total height :	24.00 m
Typical story height :	3.00 m
Total length :	15.00 m
Number of nodes :	238
Number of elements :	248 (conv. pushover)

Figure 3.3 Shear wall structure modeling and features

• **Irregular structures** (in elevation) : two models, designed with high (H) and low (L) ductility recommendations with a design acceleration of 0.30g and 0.15g respectively (figure 3.4). These structures were selected for their irregularity in elevation and hence the possibility of developing soft story mechanisms and attract higher mode effects.

Actual structure

FE modeling and mass distribution



2D reinforced concrete frame
with irregularity in elevation.

Structural ductility : High / Low
Design acceleration : 0.30g / 0.15g

Concrete strength : f_c = 30 MPa
Steel yield strength : f_y = 585 MPa

Stories : 8
Total height : 25.50 m
Typical story height : 3.00 m (first : 4.50 m)
Total length : 20.00 m

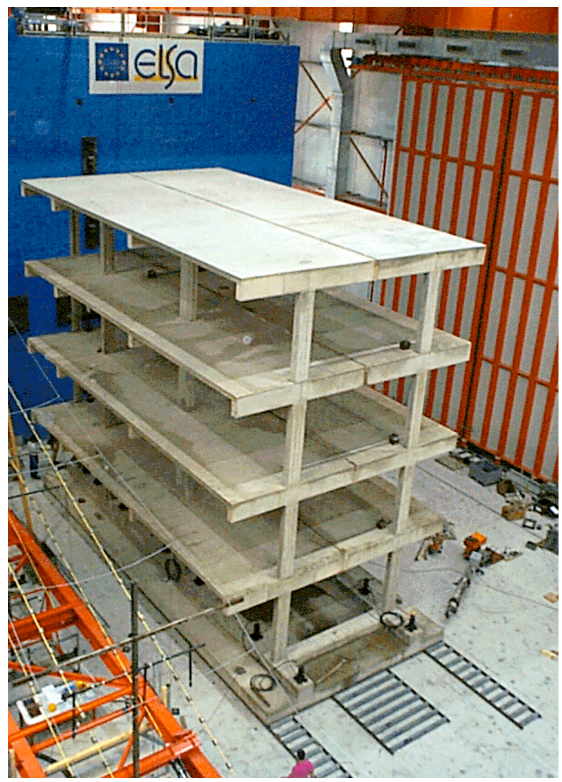
Number of nodes : 390
Number of elements : 414 (conv. pushover)

Figure 3.4 Irregular structure modeling and features

• **ICONS frame**: One model based on the full scale structure (ICONS) tested in Ispra, Italy in May 1999 (figure 3.5). It was selected for its strong irregularity in plan and its varying in-plane stiffness.

The structure were designed by Carvalho et al. [1999] with the objective of representing design and construction practice in many Southern European and Mediterranean countries in the 1950's and 60's. The design procedure followed regulation requirements that were in use then, and made use of materials typically employed at the time. A detailed description of the structure may be found in Carvalho et al. [1999]. The structure has been extensively studied analytically (Pinho, 2001, Pinho and Elnashai, 2002).

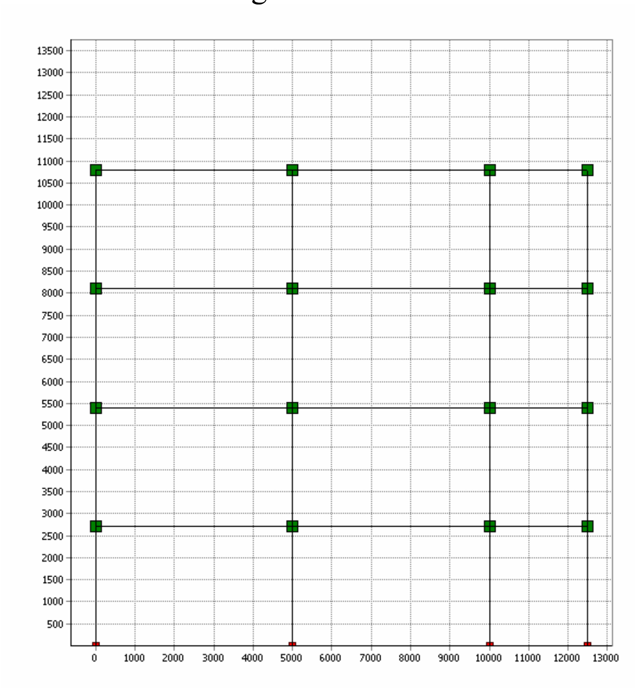
During construction



Inside the ELSA testing facility, Ispra

Figure 3.5 Photo of the ICONS structure during construction and inside the Ispra testing site, Italy.

FE modeling and mass distribution



2D reinforced concrete frame
with irregularity in plan.

Stories :	4
Total height :	10.80 m
Typical story height :	2.70 m
Total length :	12.50 m

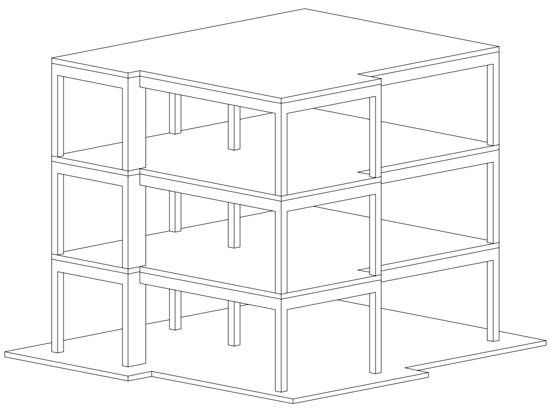
Number of nodes :	150
Number of elements :	152 (conv. pushover)

Concrete strengths : f_c = 13.6 to 21.7 MPa
Steel yield strength : f_y = 343.6 MPa

Figure 3.6 ICONS frame modeling and features

The SPEAR frame: A 3D frame based on a full scale structure tested in 2003 within the European network 'Seismic Performance Assessment and Rehabilitation (SPEAR)'. It features irregularities both in plan and elevation (figure 3.7). The frame was designed by Fardis (2002). Similar to the ICONS frames, this structure was designed according to outdated design codes, and with no seismic design provisions. It will be constructed from weak concrete and smooth bars. It has heavily imbalanced stiffness in the two orthogonal directions as well as large eccentricity in plan and irregularity in elevation. It differs from the ICONS frame in being a 3D structure with potential torsional effects.

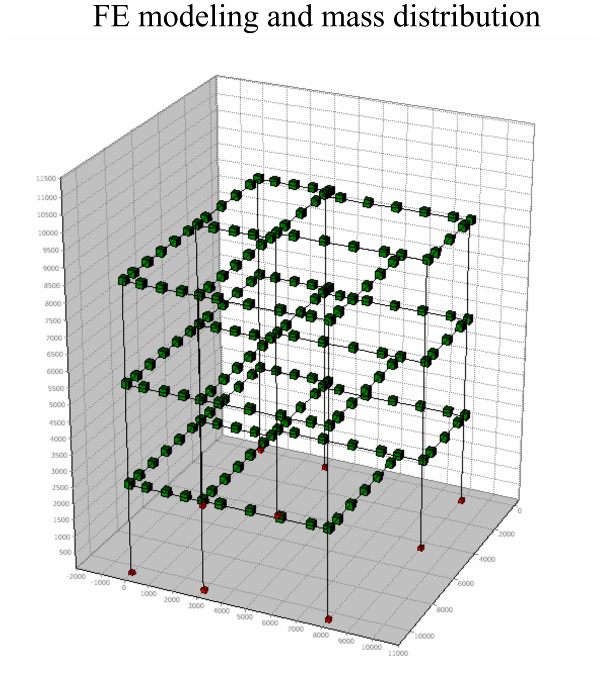
Actual structure



3D reinforced concrete frame
with irregularity in plan and elevation.

Stories :	3
Total height :	8.75 m
Typical story height :	3.00 m (first : 2.75 m)
Total length (x) :	9.00 m
Total length (y) :	10.375 m

FE modeling and mass distribution



Number of nodes :	366
Number of elements :	384 (conv. pushover)

Concrete strength : f_c = 20 MPa
Longitudinal steel strength : f_y = 400 MPa
Transverse steel strength : f_y = 220 MPa

Figure 3.7 SPEAR frame modeling and features

3.4 Dynamic analysis and input motion

In order to investigate the applicability of conventional and adaptive pushover analysis for seismic response assessment, it was deemed necessary to compare the pushover results with inelastic dynamic analysis, as stated earlier in this chapter. An efficient way to accomplish the task of comparing these two very different analysis methods in principle is to employ the incremental dynamic analysis (IDA) technique (Mwafy and Elnashai, 2000 and others). According to this approach, the structural systems under consideration are excited by the same strong motion input, scaled to different PGA values. For every scaling factor, the maximum response parameters (load-displacement, moment-curvature etc.) are plotted on a two dimensional chart, similarly to pushover analysis (figure 3.8)

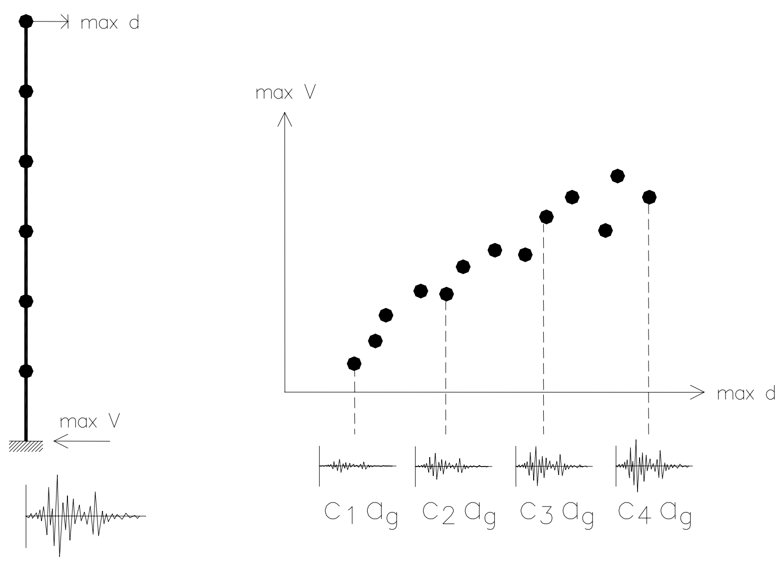


Figure 3.8 Implementation of the incremental dynamic analysis technique (IDA)

The selection of the dynamic response absolute maxima (maximum deformation and maximum action) is an issue which may require further discussion, since they may occur in different time instants. Nevertheless, it is herein assumed that this approach reflects the monotonic pushover response of the structure, where response maxima are simultaneous. Another advantage is that the use of absolute maxima sets a safe upper bound on the structural response. To capture these maxima, an analysis utility was developed in the analysis package Zeus-NL, which allows the user to run a series of dynamic analyses by automatically scaling the input motion, collecting the appropriate results for each run and plotting the incremental dynamic analysis curve (figure 3.9).

The user can choose to monitor base shear vs. global drift, story shear vs. interstory drift or element moment vs. section curvature, features which are described in detail in the subsequent section.

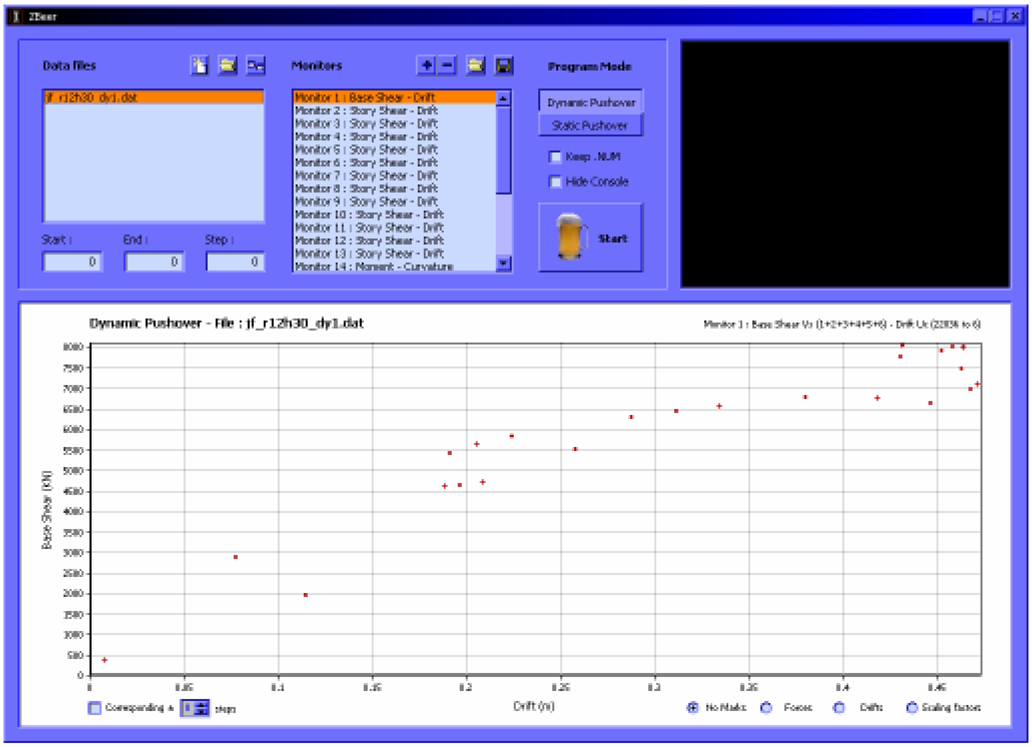


Figure 3.9 Utility developed for IDA generation

Four different strong motion records were applied to the above eight structural systems, selected because their origin and frequency content : Two artificial records, one representing the European earthquake of a 975 year return period (E975) and another derived from the Eurocode 8 design code response spectrum (EC8). In addition, two natural records, one with low frequency content (Emeryville, Loma Prieta earthquake, 1989) and another with high frequency content (Santa Monica, Northridge earthquake, 1994), were selected, providing high amplifications in the short and long period range, respectively. All four records were normalized to a PGA of 0.3g in order to be comparable during the variable scaling of the IDA approach. Twenty five runs for each record were performed, with a scaling factor from 0.2 to 5.0 (up to 1.5g of PGA) with a step of 0.2. The accelerograms and elastic response spectra are depicted in figures 3.10 to 3.13.

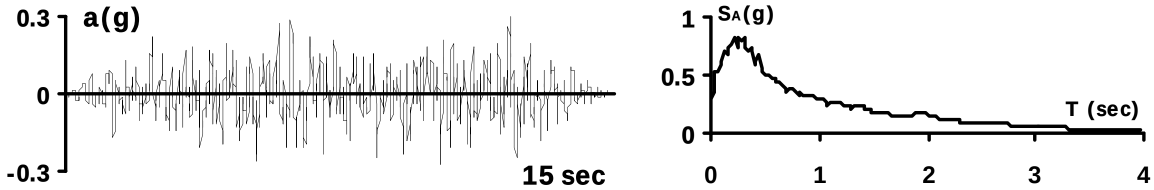


Figure 3.10 Scaled accelerogram and response spectrum of the E975 record

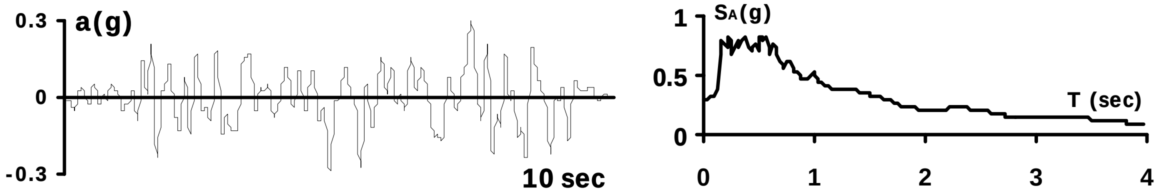


Figure 3.11 Scaled accelerogram response spectrum of the EC-8 record

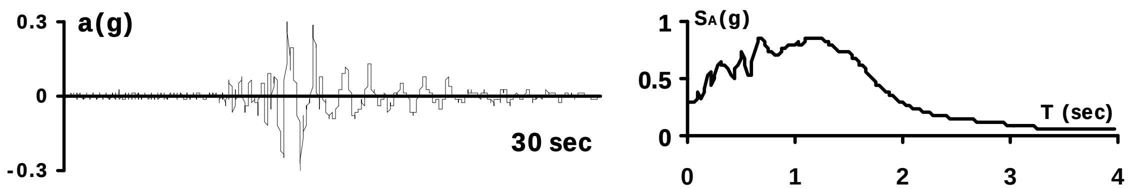


Figure 3.12 Scaled accelerogram and response spectrum of the Emeryville record

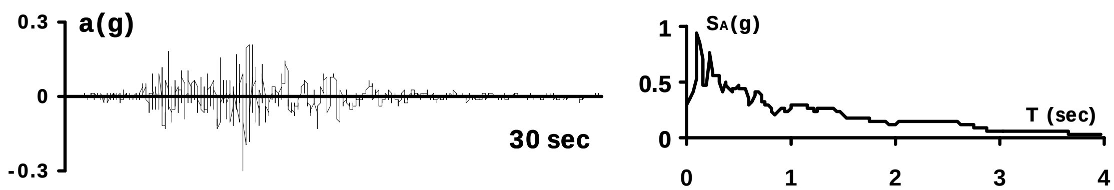


Figure 3.13 Scaled accelerogram and response spectrum of the Santa Monica record

3.5 Response measures

The response evaluation parameters were selected to achieve monitoring not only the global, but also the local response of the structural systems analyzed. Consequently, apart from the usual practice of monitoring base shear versus global drift, also story shear versus interstory drift and element moment versus section curvature were included in the evaluation of pushover (conventional and adaptive) and inelastic dynamic analysis of the eight structural systems under consideration.

• **Global level:** The first monitored quantity was the base shear (V) versus top displacement (d) (figure 3.14). Horizontal forces (V_i) of the support nodes were added and plotted against the horizontal displacement of the top floor. In dynamic analysis though, the support displacement is subtracted from the top displacement in order to establish the global drift of the structure.

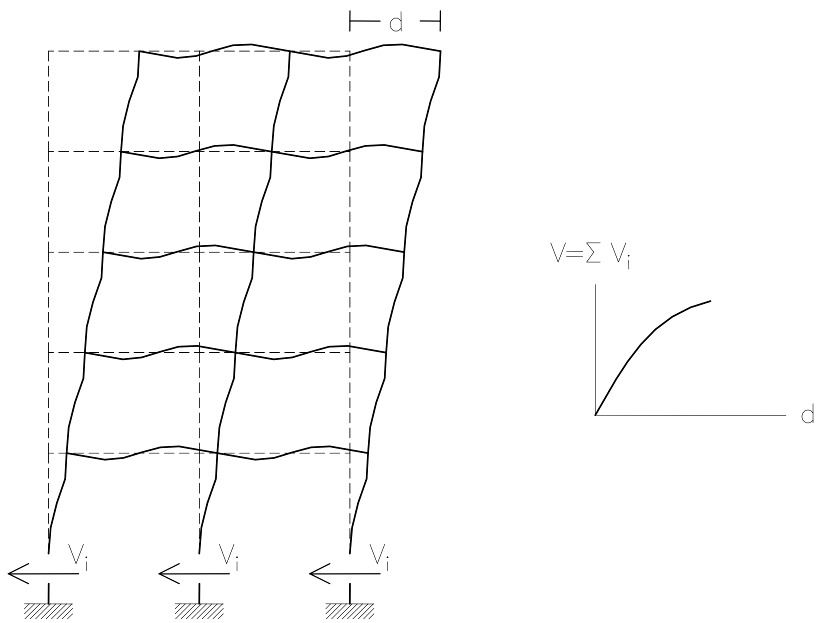


Figure 3.14 Base shear versus global drift monitoring

• **Story level:** The second performance indicator selected was the story shear (V^s) versus interstory drift (d^s) (figure 3.15). The story force (V_i^s) was derived by adding all individual element shear forces (V_i^s) which are equal to $(M^t-M^b)/\ell$, where M^t and M^b are the top and bottom element moments respectively and ℓ the element length.

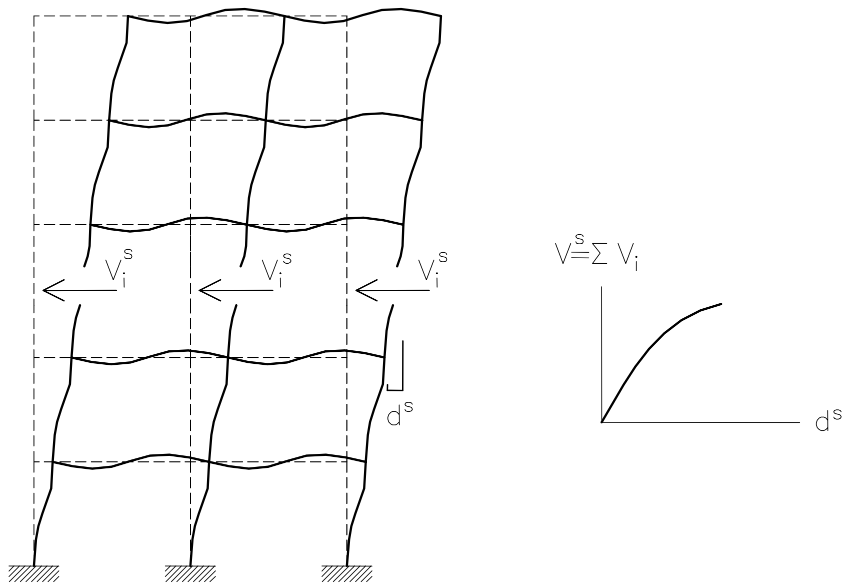


Figure 3.15 Story shear versus interstory drift monitoring

• **Section level**: The final monitor used in this analytical investigation accounts for the local behavior (section level) during inelastic static and dynamic analysis through plotting the element moment (M) versus curvature (κ) (figure 3.16). The latter is derived from $(\varepsilon_t - \varepsilon_b)/h$ where ε_t and ε_b are the top and bottom layer strains respectively and h the height of the section.

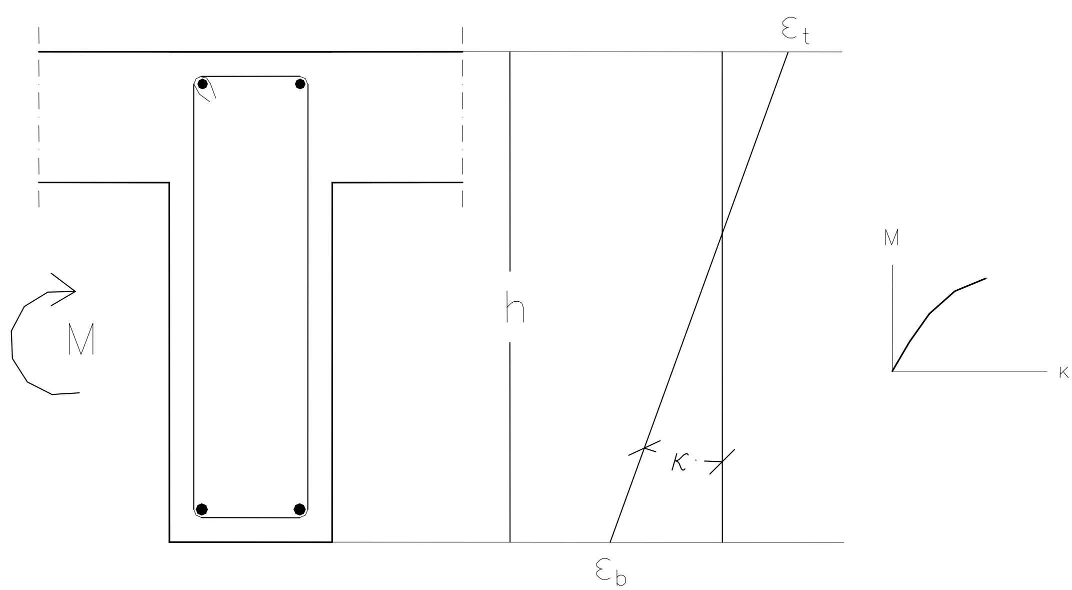


Figure 3.16 Element moment versus section curvature monitoring

3.6 Capacity Curve Discrepancy Factor

The present study have surfaced the need for defining a measuring quantity for the difference between inelastic static and dynamic analysis, in the form of a simple percentage number. The Capacity Curve Discrepancy Factor (CCDF) parameter is a numerically simple but yet efficient way to define the difference between the ordinates of a single pushover curve compared to a set of IDA points, both emerging from the same structural model.

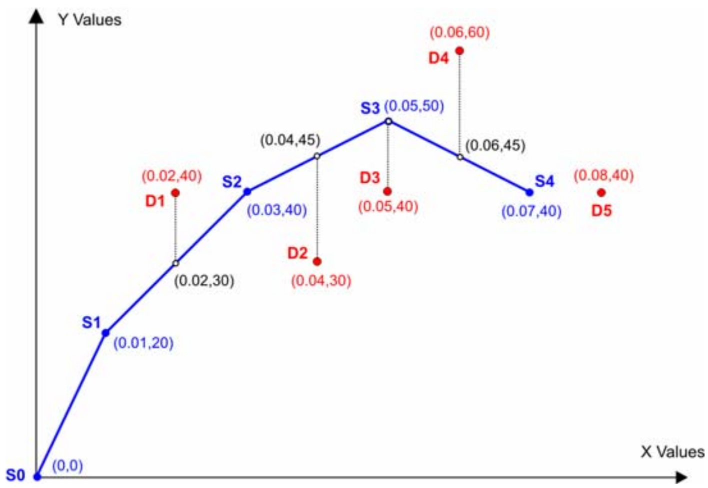


Figure 3.17 Definition of the Capacity Curve Discrepancy Factor (CCDF)

Consider the pushover curve S0-S4 and the set of dynamic points D1-D5 of figure 3.17. The coordinates of the vertical projection of each IDA point on the pushover curve are calculated by linear interpolation between neighboring pushover points. Points with no projections, like D5, are ignored. The percentile difference *d* between each projection point (static analysis) and the corresponding dynamic point is calculated as follows :

$d_{Di} = abs\ (Y_{Pi} - Y_{Di}) / Y_{Di}$, i = 1 to 4 (D5 ignored) (4)

The CCD value is the average of the above difference values :

$CCD = \Sigma d_{Di} / 4 = 31.25\ \%$ (5)

Averaging all differences suggests that equal weight has been assigned to each dynamic point. This is deemed realistic only when the scaling step of the record remains constant throughout the analysis, which is recommended and always the case in this study. The formula $(Y_{Pi} - Y_{Di}) / Y_{Di}$ was selected, because the reference response (denominator Y_{Di}) is the dynamic response (comparing static to dynamic, not the opposite). For easy and fast application, the CCDF method was implemented as an additional tool in the used software package. An application of the CCDF method is shown in the simple case study of figure 3.18.

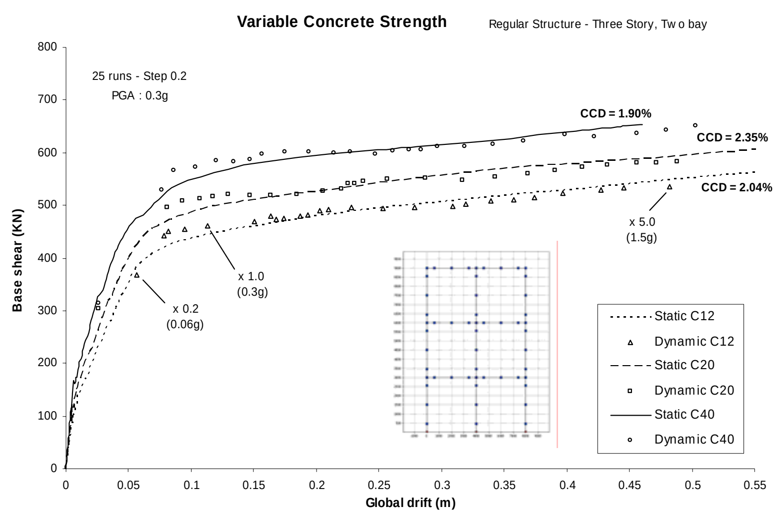


Figure 3.18 Application of the Capacity Curve Discrepancy Factor (CCDF)

4. ANALYSIS RESULTS

This chapter includes a complete review of the analysis results. For each one of the considered structures, analysis results are presented for all monitored response indicators.

The analyses were performed using Zeus-NL version 1.5. The parameter used to compare the results between static inelastic (pushover) and incremental dynamic analysis curves was the Capacity Curve Discrepancy Factor (CCDF). As was stated in the previous chapter, this is a numerically simple but yet efficient way to define the difference between the ordinates (forces or moments) of a single pushover curve compared to a set of IDA points, both emerging from the analysis of the same structural system, using a simple averaged difference between the curves across the common load-displacement (or moment-curvature range).

4.1. Regular Structures

Table 4.1 summarizes the difference values between pushover and incremental dynamic analysis for regular structures.

HIGH DUCTILITY REGULAR STRUCTURE

Monitoring Point	Feature	Dy-1 E-975			Dy-2 EC-8			Dy-3 Emerville			Dy-4 Sta Monica		
		Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform)	Reduction (-) or Increment in the error	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform)	% Error above or below the Conventional
Point 1	Base Shear vs. Total Drift	18.14%	13.22%	-27.12%	7.19%	7.85%	9.18%	7.04%	6.13%	-12.93%	6.25%	1.85%	-70.40%
Point 2	Story 1 - Shear vs. Drift	9.28%	7.63%	-17.78%	8.91%	7.08%	-20.54%	6.33%	3.50%	-44.71%	6.99%	5.33%	-23.75%
Point 7	Story 6 - Shear vs. Drift	3.15%	4.05%	28.57%	2.87%	3.85%	34.15%	4.35%	3.86%	-11.26%	2.82%	4.08%	44.68%
Point 20	Story 1 - Rightmost column Moment vs. Curvature	2.54%	3.35%	31.89%	2.05%	1.90%	-7.32%	3.87%	3.17%	-18.09%	2.16%	1.48%	-31.48%

LOW DUCTILITY REGULAR STRUCTURE

Monitoring Point	Feature	Dy-1 E-975			Dy-2 EC-8			Dy-3 Emerville			Dy-4 Sta Monica		
		Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	Reduction (-) or Increment in the error	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional
Point 1	Base Shear vs. Total Drift	19.15%	12.51%	-34.67%	10.71%	6.79%	-36.60%	5.43%	1.74%	-67.96%	7.05%	4.97%	-29.50%
Point 2	Story 1 - Shear vs. Drift	11.63%	9.69%	-16.68%	11.36%	9.53%	-16.11%	3.66%	1.62%	-55.74%	6.64%	4.92%	-25.90%
Point 7	Story 6 - Shear vs. Drift	3.51%	4.49%	27.92%	3.38%	5.63%	66.57%	4.67%	8.15%	74.52%	4.20%	4.59%	9.29%
Point 20	Story 1 - Rightmost column Moment vs. Curvature	2.76%	3.39%	22.83%	1.91%	2.29%	19.90%	3.91%	3.03%	-22.51%	1.85%	1.63%	-11.89%

Table 4.1 Capacity curve discrepancy factors (CCDF), regular structures (High and low ductility)

4.1.1. Global level

As shown in table 4.1, ***at the global level***, adaptive pushover in general improves the quality of the results, since the difference values for the conventional pushover ranges from around 6.3 % to 18.1 % for the high ductility structure and from 5.4 % to 19.1 % for the low ductility structure, while for adaptive pushover, difference values drop to 1.9 % - 13.2 % and 1.7 % - 12.5 % for the high and low ductility structures respectively.

In order to obtain a better overview of the global response, figures 4.1 and 4.2 show a detailed comparison between the dynamic analyses and the pushover curves for each one of the considered strong motion records in the high and low ductility structures.

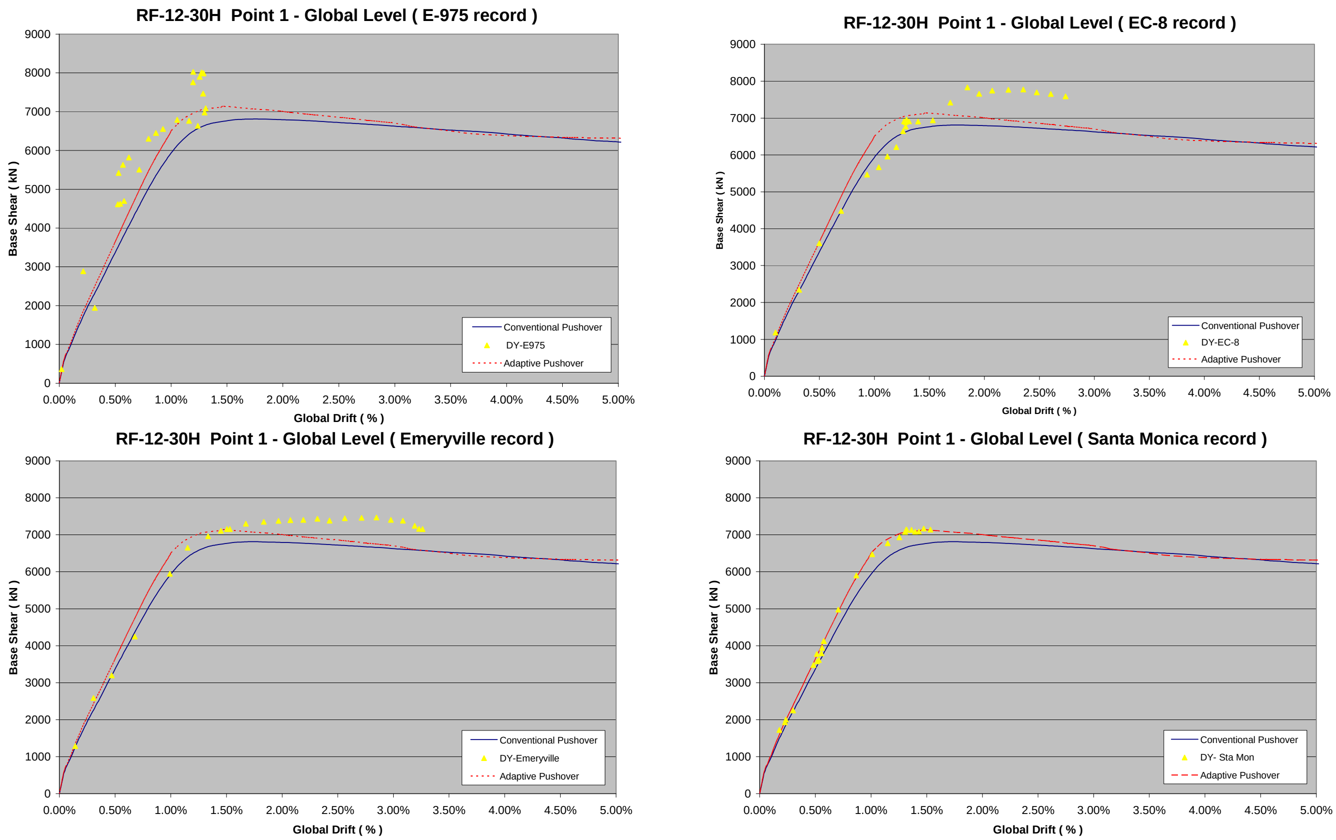


Figure 4.1 Response at the global level, regular structure (high ductility)

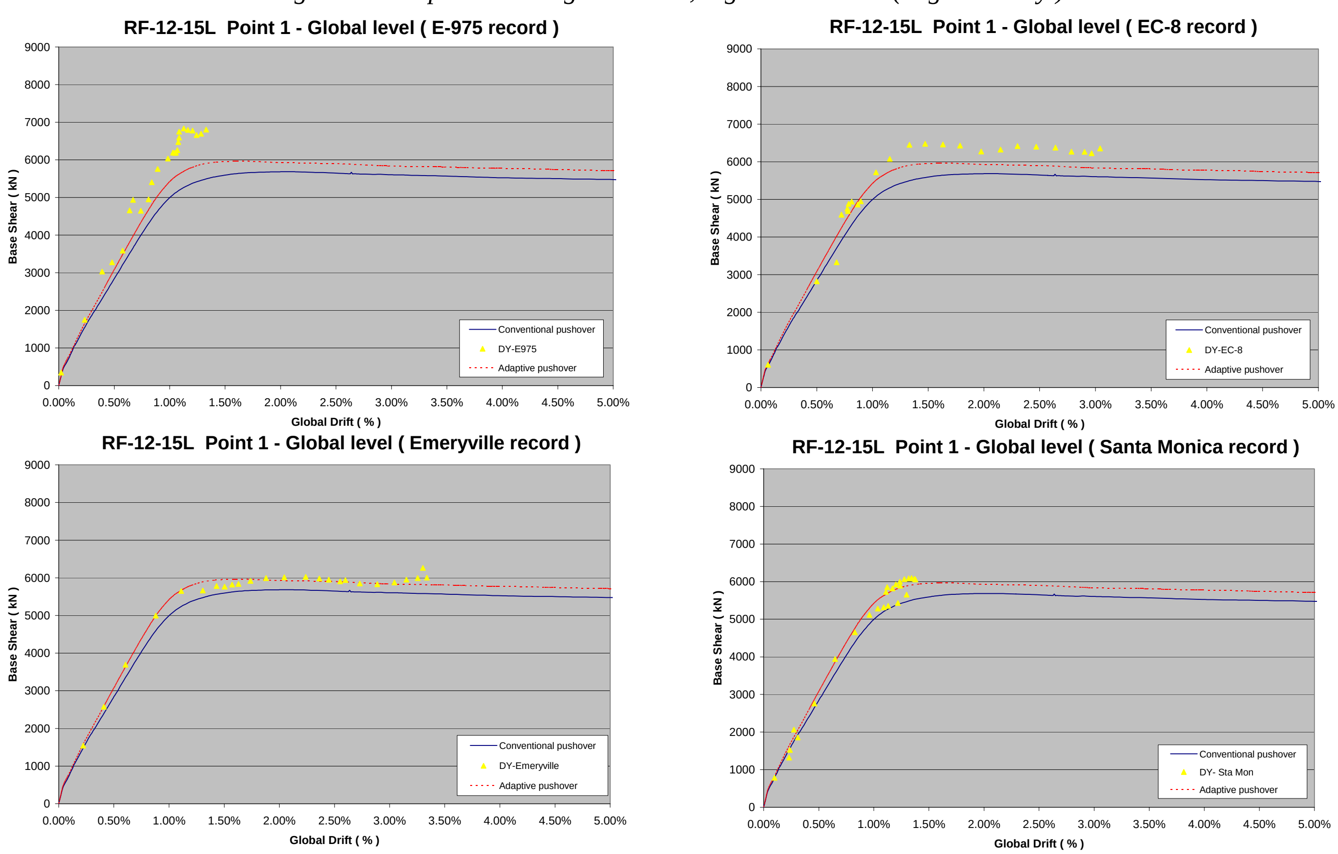


Figure 4.2 Response at the global level, regular structure (low ductility)

From figure 4.1, it is observed that adaptive pushover is closer to the dynamic response when the structure is in the elastic range or the initial part of the inelastic response (in this case, for global drifts around 2.5 %). For higher levels of inelastic response, there is no marginal improvement in the results using the adaptive approach.

This last observation is due the fact that in the high inelastic range the analysis solver is not able to find real solutions to the eigenvalue problem, and hence the load vector cannot be updated in the subsequent steps. Consequently, no improvement in the results could be introduced using the adaptive approach.

Larger differences are observed for both methods when the structures are subjected to artificial records. There is also a trend for both pushover approaches to underestimate the dynamic response, especially when the structure is in the high inelastic range.

The figure 4.2 illustrates that the same trends apply for the low ductility structure as well, indicating that for this type of bi-dimensional regular structures, ductility is not a factor that modifies the behavioral trends at the global level.

4.1.2. First story level

As shown in the table 4.1, adaptive pushover behaves slightly better at the first story for both types of structures. In the high ductility structure, the difference between conventional pushover and dynamic analyses ranges from around 6.3 % to 9.3 % and for the adaptive pushover the difference is between 3.5 % and 7.6 % respectively.

For the low ductility structure the difference between the conventional pushover and dynamic analyses ranges from 3.7 % to 11.6 % and for the adaptive pushover lies between 1.6 % and 9.7 %.

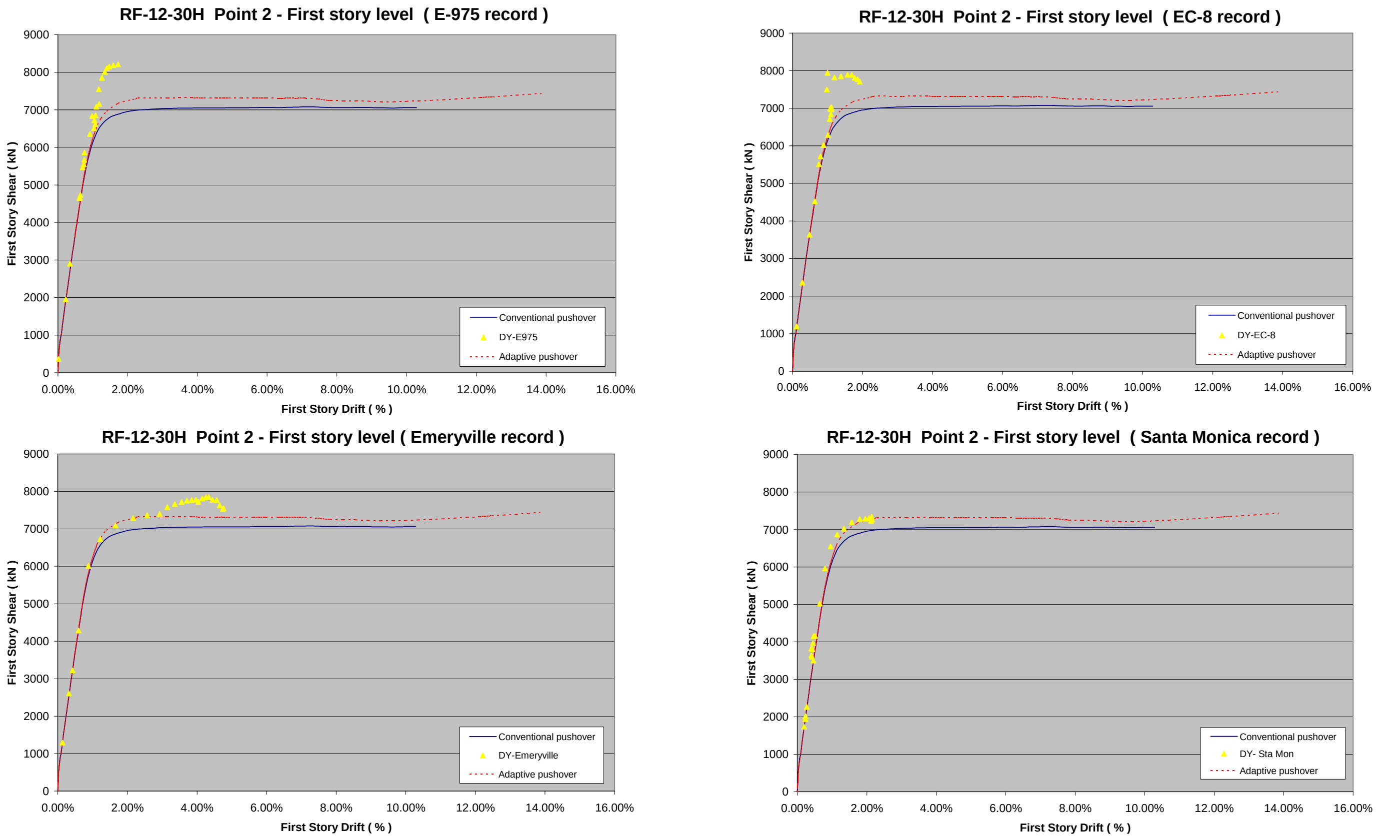


Figure 4.3 Response at the first story level, regular structure (high ductility)

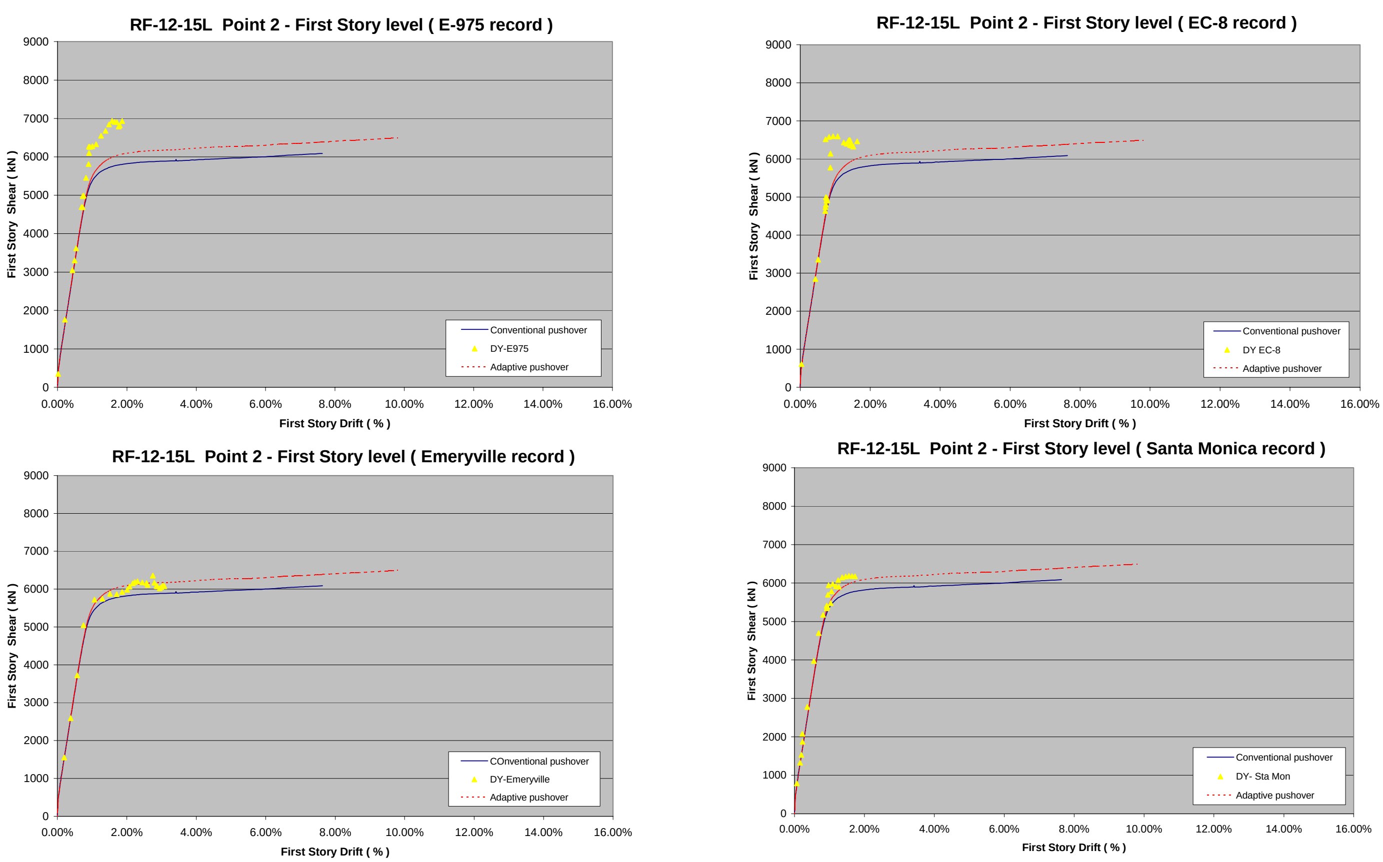


Figure 4.4 Response at the first story level, regular structure (low ductility)

Figure 4.3 shows a detailed graphical comparison between the dynamic analyses and pushover curves for the high ductility regular structure. It is clear again that the correspondence between the dynamic analyses and the pushover curves is better for natural records than artificial ones. On the other hand, it can be observed that adaptive pushover converges to a fixed load pattern for high inelastic deformation.

Considering pushover curves as indicators of the structural capacity, the results show that the adaptive technique produces slightly less conservative results than the conventional one, especially in the inelastic range. There is not a big difference; however it is an indicator of how the adaptive procedure produces a resultant of the lateral forces closer to the base of the structure than the one produced by the fixed triangular pattern. This trend had been already observed in the response at the global level.

The same trends are observed for the low ductility structure in figure 4.4. In the same way that for the response at the global level, ductility doesn't modify the general tendencies observed in the response at the first story level for this type of structure.

4.1.3. Middle story level

Contrary to the response at the global and at the first story levels, at the middle story level adaptive pushover losses its superiority. As is shown in table 4.1 and in figures 4.5 and 4.6, opposing to the trend observed in the previous sections, in this case the adaptive pushover produces higher difference rates than the conventional one and shows a lower structural capacity that seems too conservative compared to the dynamic analysis results.

Indeed, as appears in the figure 4.5 for the high ductility structure the adaptive pushover shows that the middle story softens in a brittle way reaching a lower total deformation, which is not reflected in the dynamic response. This trend is also observed in the low ductility structure but in this case the brittle failure doesn't occur.

For the high ductility structure the difference between the conventional pushover and dynamic analysis ranges from 2.8 % to 4.3 % and for the adaptive pushover the differences are between 3.9 % and 4.1 %.

For the low ductility structure the difference between conventional pushover and dynamic analysis ranges from 3.4 % to 4.7 % and for the adaptive pushover lies between 4.5 % and 8.2 %.

These results suggest that adaptive pushover technique imposes higher inelastic deformations at this level and hence has a tendency to overestimate the structural damage. This behavior has also been observed in the recent work by Antoniou and Pinho (2004).

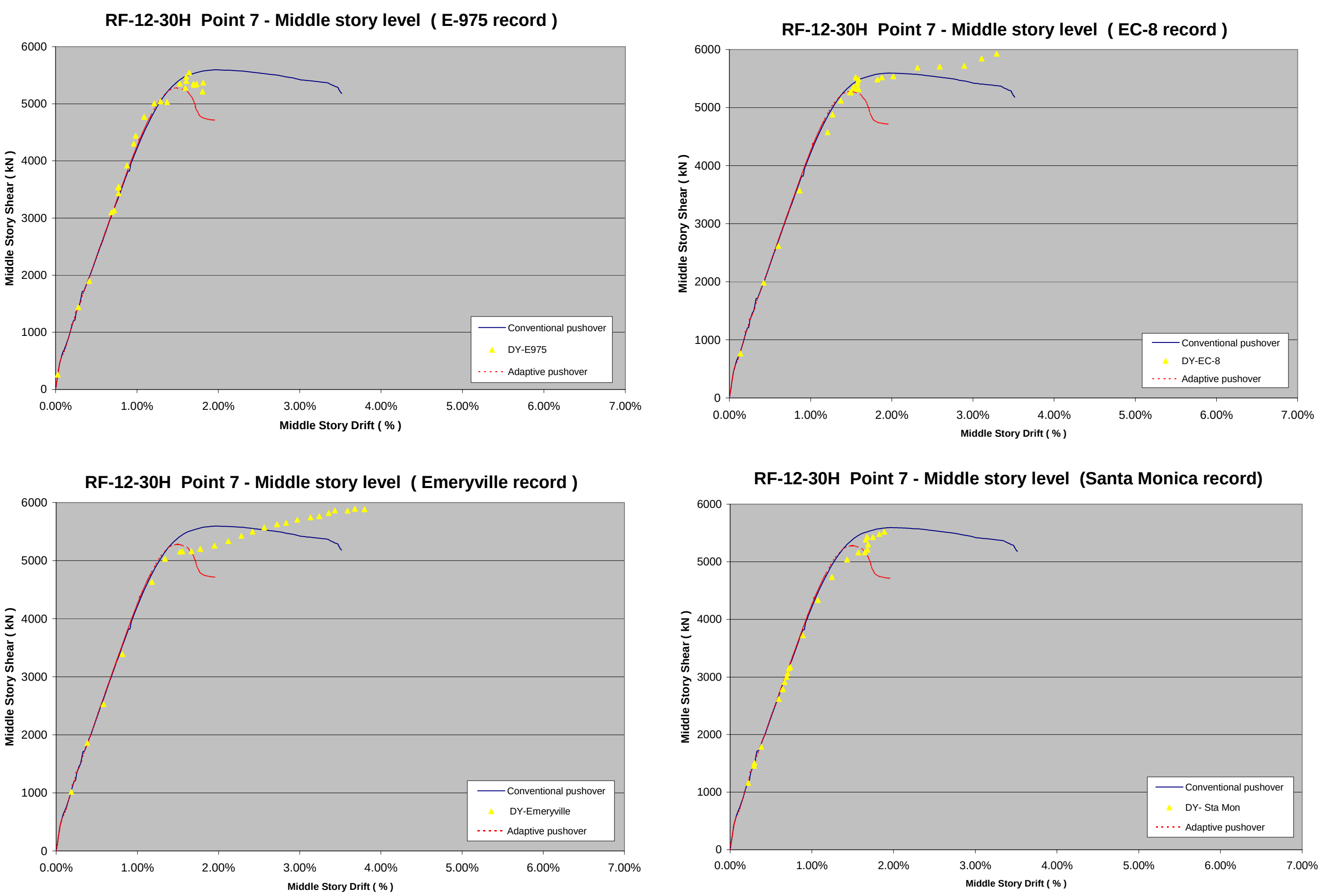


Figure 4.5 Response at the middle story level, regular structure (high ductility)

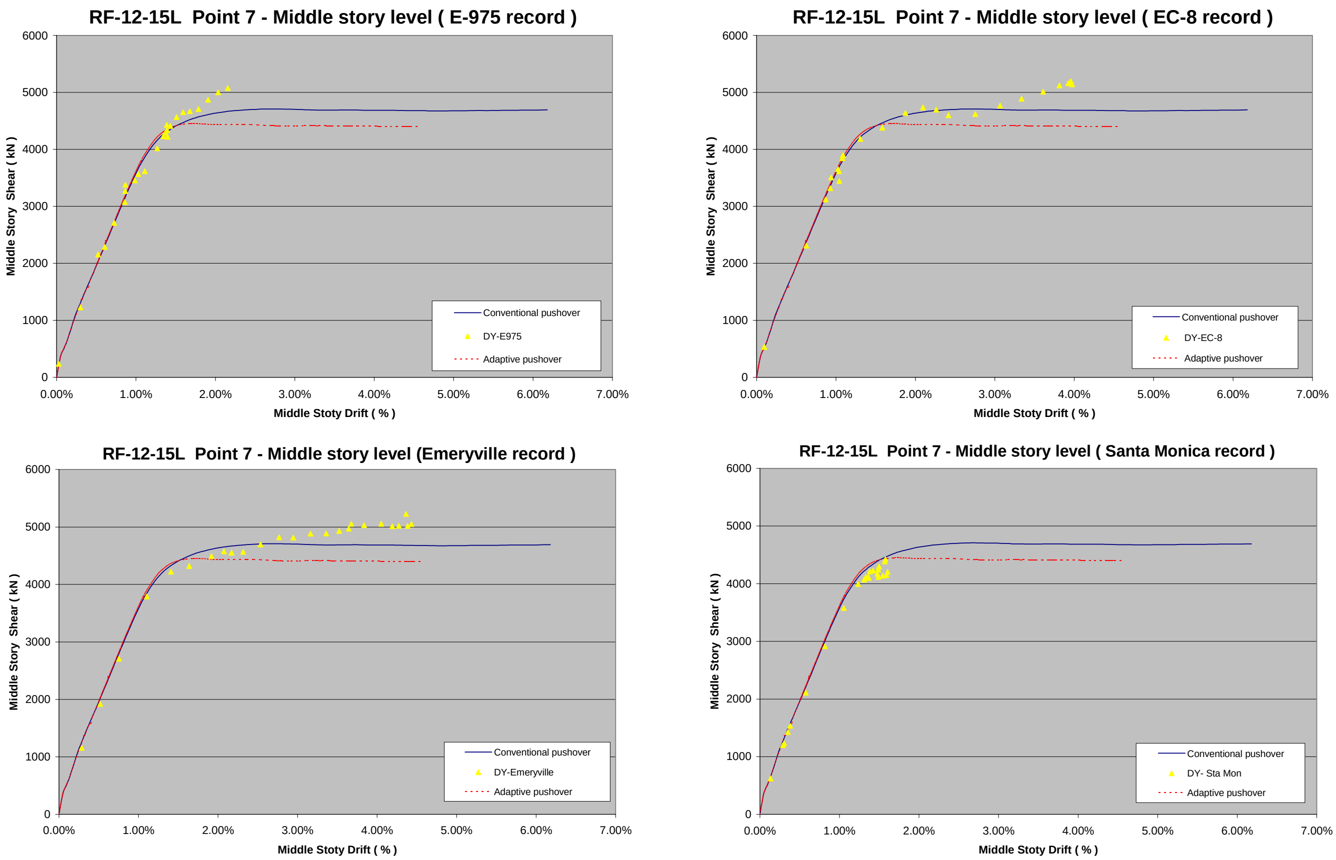


Figure 4.6 Response at the middle story level, regular structure (low ductility)

In order to explain the reasons behind these results, the methodology used to combine the modal forces in order to update the lateral load profile should be considered. Since the SRSS and the CQC rules are unable to represent the sign inversion in the force vector modal decomposition, the constant augment in the applied lateral forces in not strictly realistic, and sometimes will produce very high loads especially in the intermediate stories. As suggested by Antoniou and Pinho (2004) for future developments a more refined methodology of modal force combination should be employed, for example a weighted vectorial addition of each mode contribution.

Finally it is important to note that in the above case is not so clear that the artificial accelerograms produce a better agreement between incremental dynamic analysis and pushover curves. Moreover, the lower differences for adaptive pushover in the high ductility structure result from the lack of comparison points rather than from the accuracy of the analysis, since as it was previously mentioned the capacity curve discrepancy factor (CCDF) just compares the IDA points in the common deformation range.

4.1.4. Section level

At the **section level**, both pushover methods (adaptive/conventional) and dynamic analyses results are almost identical for both structures (high and low ductility) and under all strong motion records.

In the high ductility structure, the difference between conventional pushover and dynamic analyses ranges from 2.1 % to 3.9 % and for the adaptive pushover the differences are between 1.5 % and 3.4 %.

For the low ductility structure, the difference between conventional pushover and dynamic analysis ranges from 1.9 % to 3.9 % and for the adaptive pushover the differences are between 3.4 % and 1.6 %.

Figures 4.7 and 4.8 show in detail the results for the high and low ductility structures. Since the relation between moment and curvature is a property of the section, in this case the difference between the dynamic analysis and the pushover curves is produced by the methodology used to plot the points of the dynamic analysis, using the maximum moment and the maximum curvature.

Nevertheless, these results are useful to show how the static analyses handle a similar level of axial forces, which is the only factor able to modify this relation for a fixed section.

Finally, figures 4.9 and 4.10 summarize the difference values presented in this section for regular frame structures.

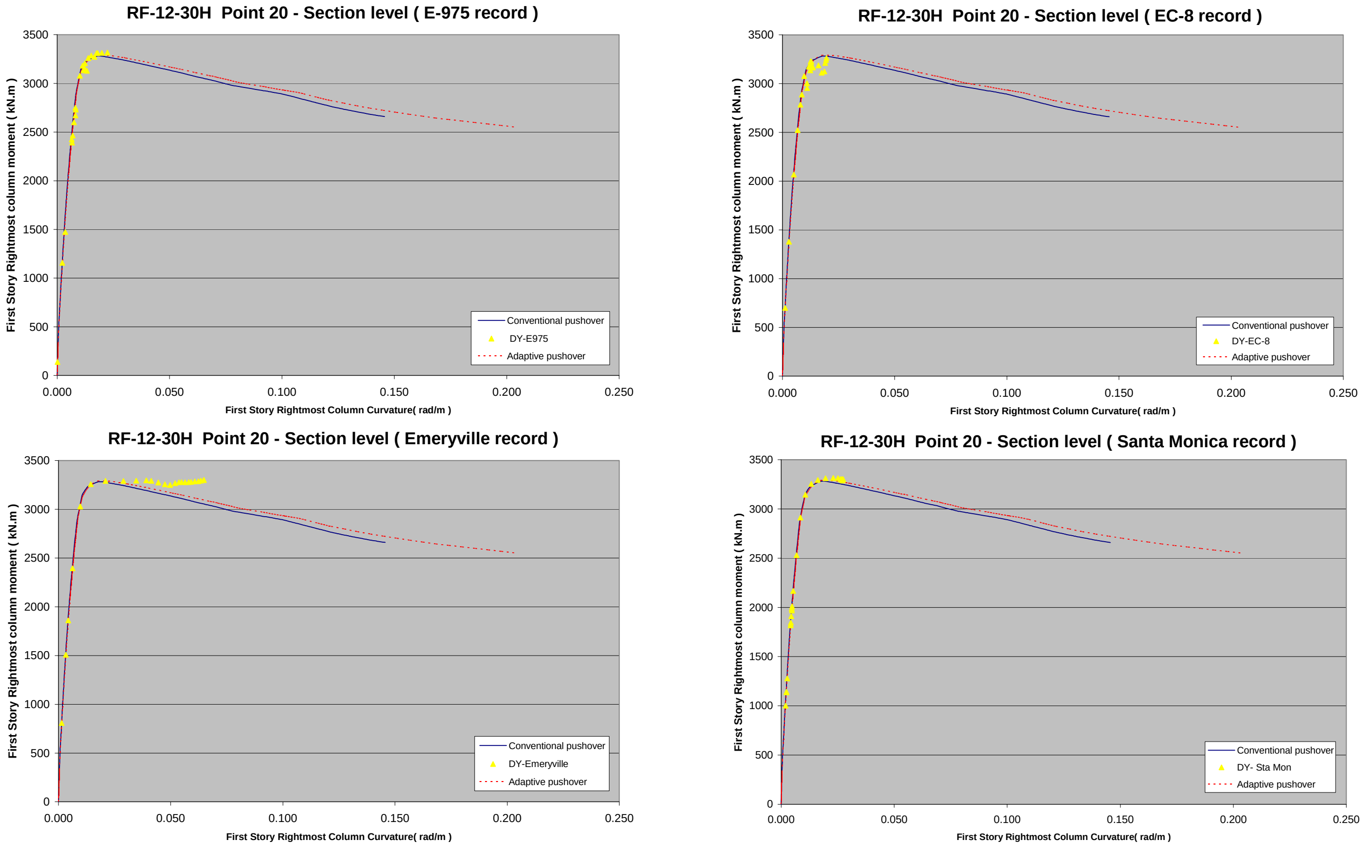


Figure 4.7 Response at the section level, regular structure (high ductility)

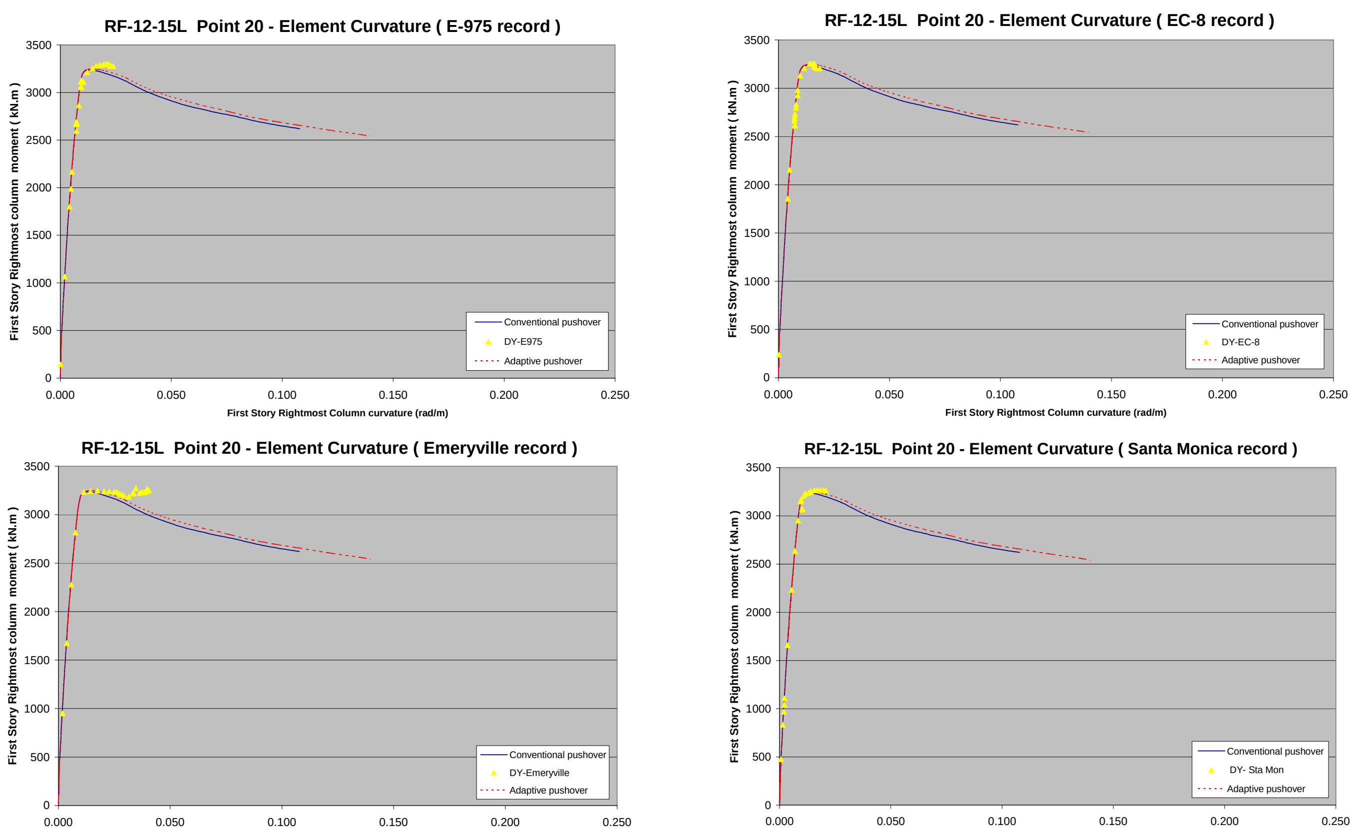


Figure 4.8 Response at the section level, regular structure (low ductility)

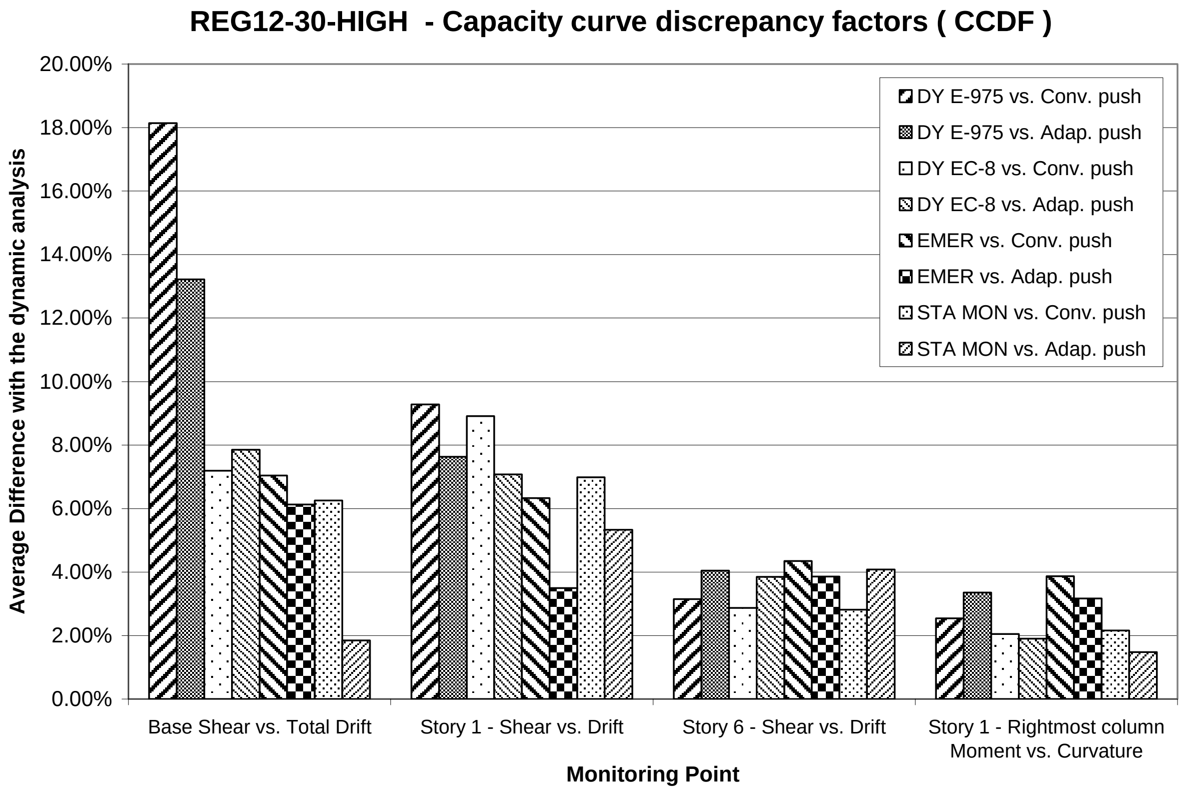


Figure 4.9 Results summary, regular structure (high ductility)

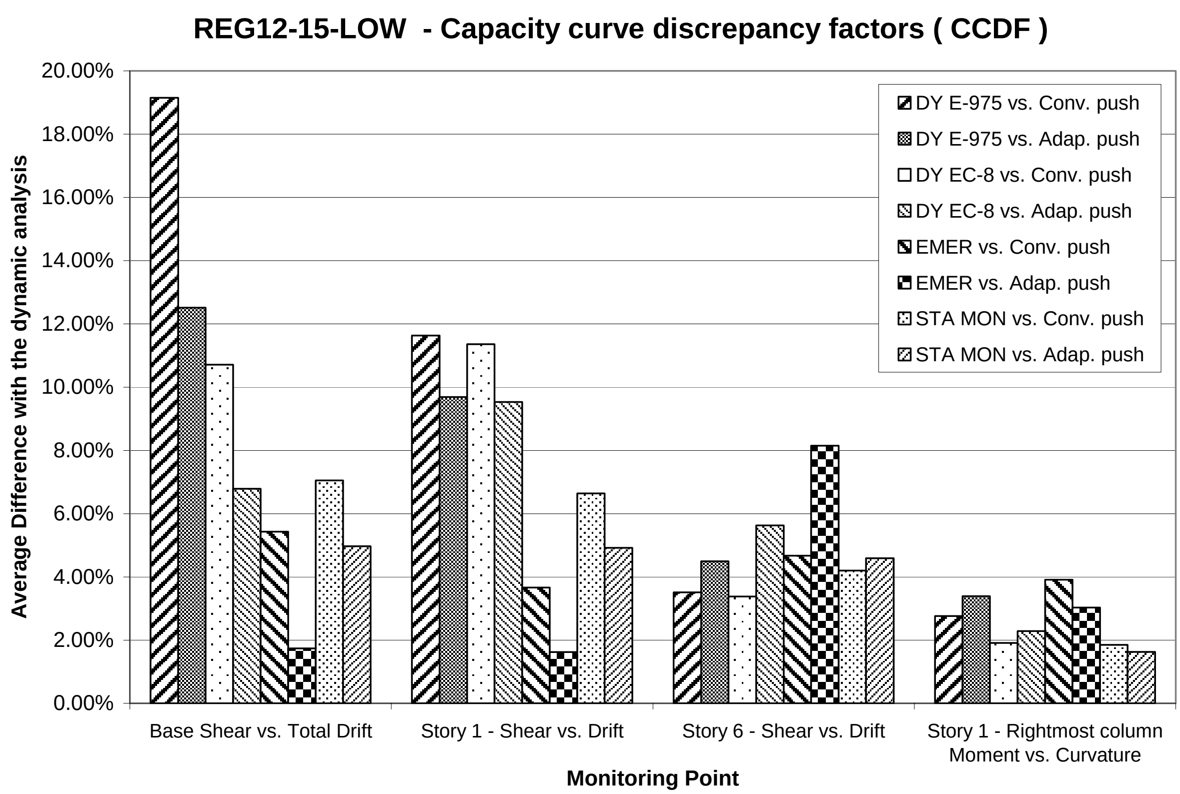


Figure 4.10 Results summary, regular structure (low ductility)

4.2. Frame - Wall Structures

Table 4.2 summarizes the difference values between pushover and incremental dynamic analysis for frame-wall structures.

HIGH DUCTILITY SHEAR WALLS STRUCTURE													
		Dy-1 E-975			Dy-2 EC-8			Dy-3 Emerville			Dy-4 Sta Monica		
Monitoring Point	Feature	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	Reduction (-) or Increment in the error	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional
Point 1	Base Shear vs. Total Drift	11.00%	6.67%	-39.36%	10.61%	7.01%	-33.93%	7.09%	5.06%	-28.63%	6.82%	5.70%	-16.42%
Point 2	Story 1 - Shear vs. Drift	12.43%	10.47%	-15.77%	10.34%	7.64%	-26.11%	6.68%	3.93%	-41.17%	7.17%	5.95%	-17.02%
Point 5	Story 4 - Shear vs. Drift	4.86%	5.38%	10.70%	4.74%	4.75%	0.21%	7.68%	7.58%	-1.30%	2.23%	2.40%	7.62%
Point 16	Story 1 - Rightmost column Moment vs. Curvature	8.10%	9.64%	19.01%	10.03%	10.37%	3.39%	13.39%	14.31%	6.87%	5.59%	5.61%	0.36%

LOW DUCTILITY SHEAR WALL STRUCTURE													
		Dy-1 E-975			Dy-2 EC-8			Dy-3 Emerville			Dy-4 Sta Monica		
Monitoring Point	Feature	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	Reduction (-) or Increment in the error	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional
Point 1	Base Shear vs. Total Drift	11.92%	7.83%	-34.31%	9.64%	8.46%	-12.24%	6.53%	6.36%	-2.60%	8.62%	6.52%	-24.36%
Point 2	Story 1 - Shear vs. Drift	14.40%	10.40%	-27.78%	11.65%	7.68%	-34.08%	8.13%	4.98%	-38.75%	8.32%	5.92%	-28.85%
Point 5	Story 4 - Shear vs. Drift	4.10%	3.99%	-2.68%	5.41%	5.34%	-1.29%	6.91%	6.91%	0.00%	3.31%	3.64%	9.97%
Point 16	Story 1 - Rightmost column Moment vs. Curvature	11.70%	12.88%	10.09%	9.32%	9.57%	2.68%	16.85%	16.19%	-3.92%	4.50%	4.76%	5.78%

Table 4.2 Capacity curve discrepancy factors (CCDF), frame-wall structures (High and low ductility)

4.2.1. Global level

As in the regular structure, at the global level, adaptive pushover diminishes the difference values, which again are higher for the artificial records than for the natural ones. In this way, for the high ductility structure the difference values for the conventional pushover range from 7.0 % to 11.0 % while the adaptive pushover difference varies from 5.1 % to 6.8 %.

For the low ductility structure, the difference between conventional pushover and dynamic analyses lies between 6.5 % and 11.9 %, while the adaptive pushover from 6.7 % to 8.5 %.

The figures 4.11 and 4.12 show a detailed view of the response at the global level for both structures (high and low ductility).

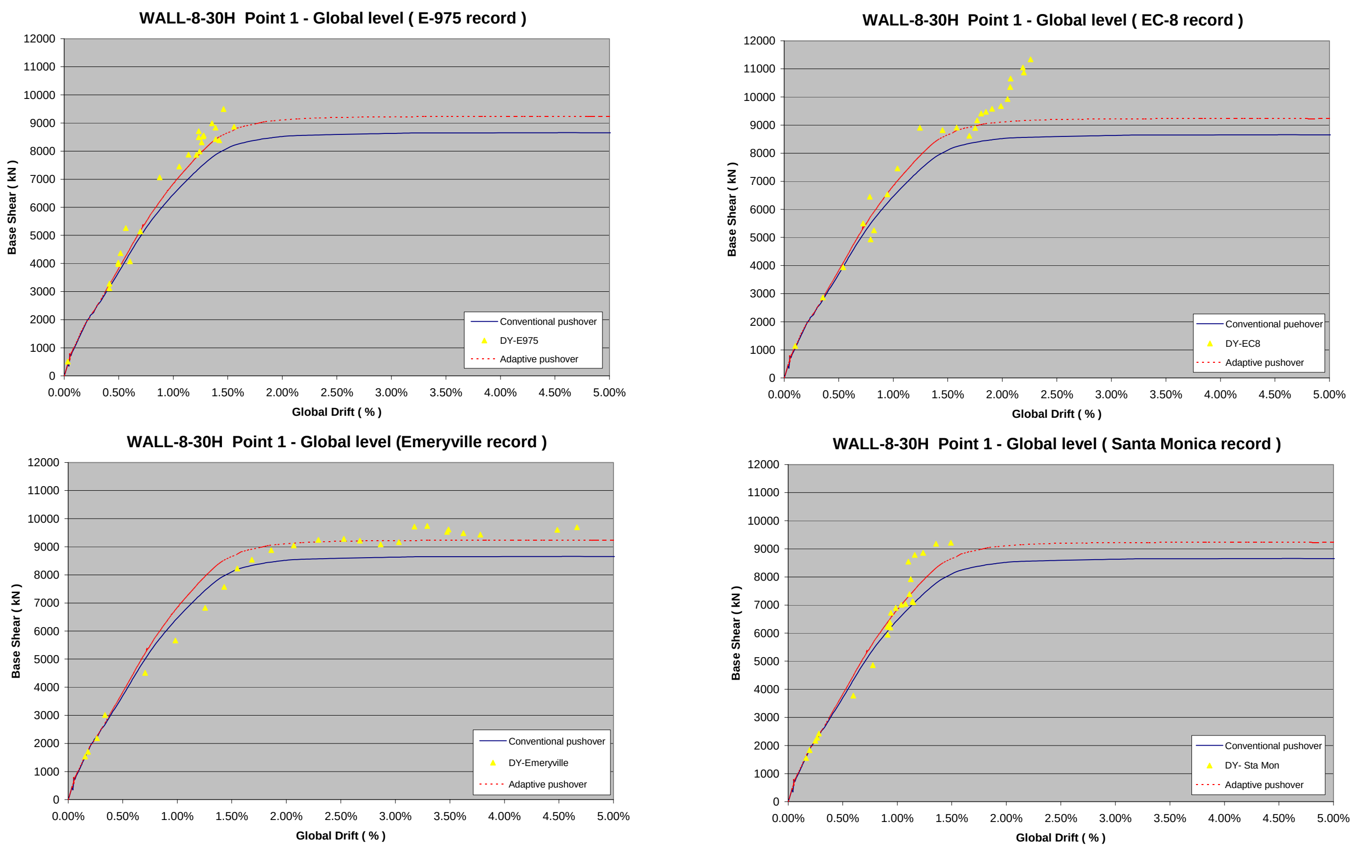


Figure 4.11 Response at the global level, frame-wall structure (high ductility)

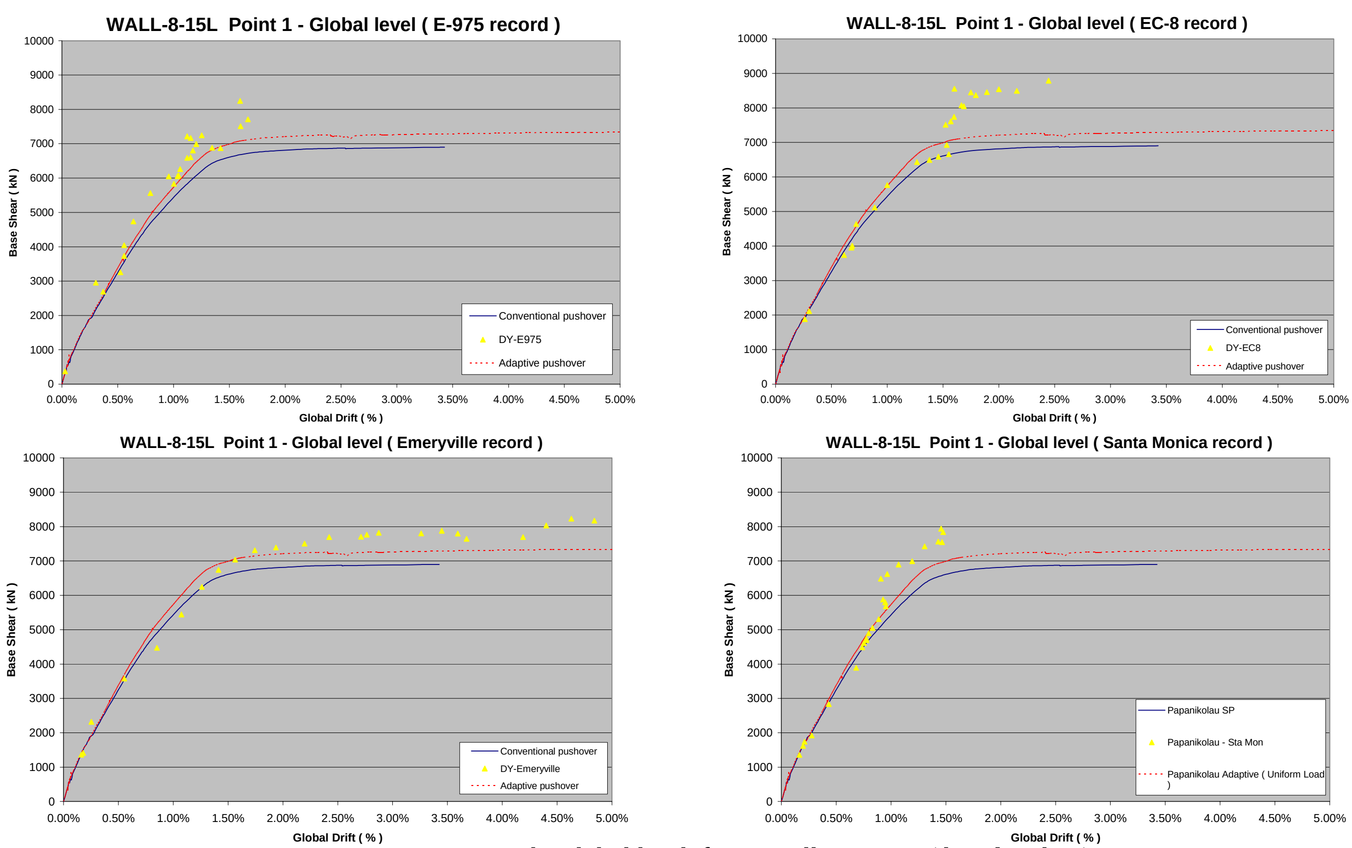


Figure 4.12 Response at the global level, frame-wall structure (low ductility)

From the two set of graphs presented above, several trends, already identified for the regular structures are detected again:

- In the same way that in regular structures, adaptive pushover is less conservative than the conventional one. In capacity terms, it shows a slightly stiffer structure, suggesting a lower application point of the lateral load resultant.

- Again there is a better correspondence between the dynamic analyses performed with the natural records that with the ones executed with the artificial ones.

- In general, in the inelastic range both pushover approaches underestimate the structural response. However, the adaptive pushover shows a slightly better approximation to the dynamic analysis results. It is also clear that the adaptive procedure converges to a fixed load pattern for the high inelastic response, since as the difference between both pushover approaches becomes constant. As has been already pointed out for the regular structures, this is due to the impossibility of the program to find solutions to the eigenvalue problem.

- For the low ductility structure the adaptive pushover allow to the structure to experience a larger global deformation, more similar to displacement levels showed by the dynamic analysis.

4.2.2. First story level

As shown in the figures 4.13 and 4.14, exactly the same comments that apply for the global level, also fit for the response at the first story level.

For comparison purposes, in the high ductility structure the average difference rate of the conventional pushover is between 6.7 % and 12.4 % while for the adaptive technique the difference ranges from 3.9 % to 10.5 %.

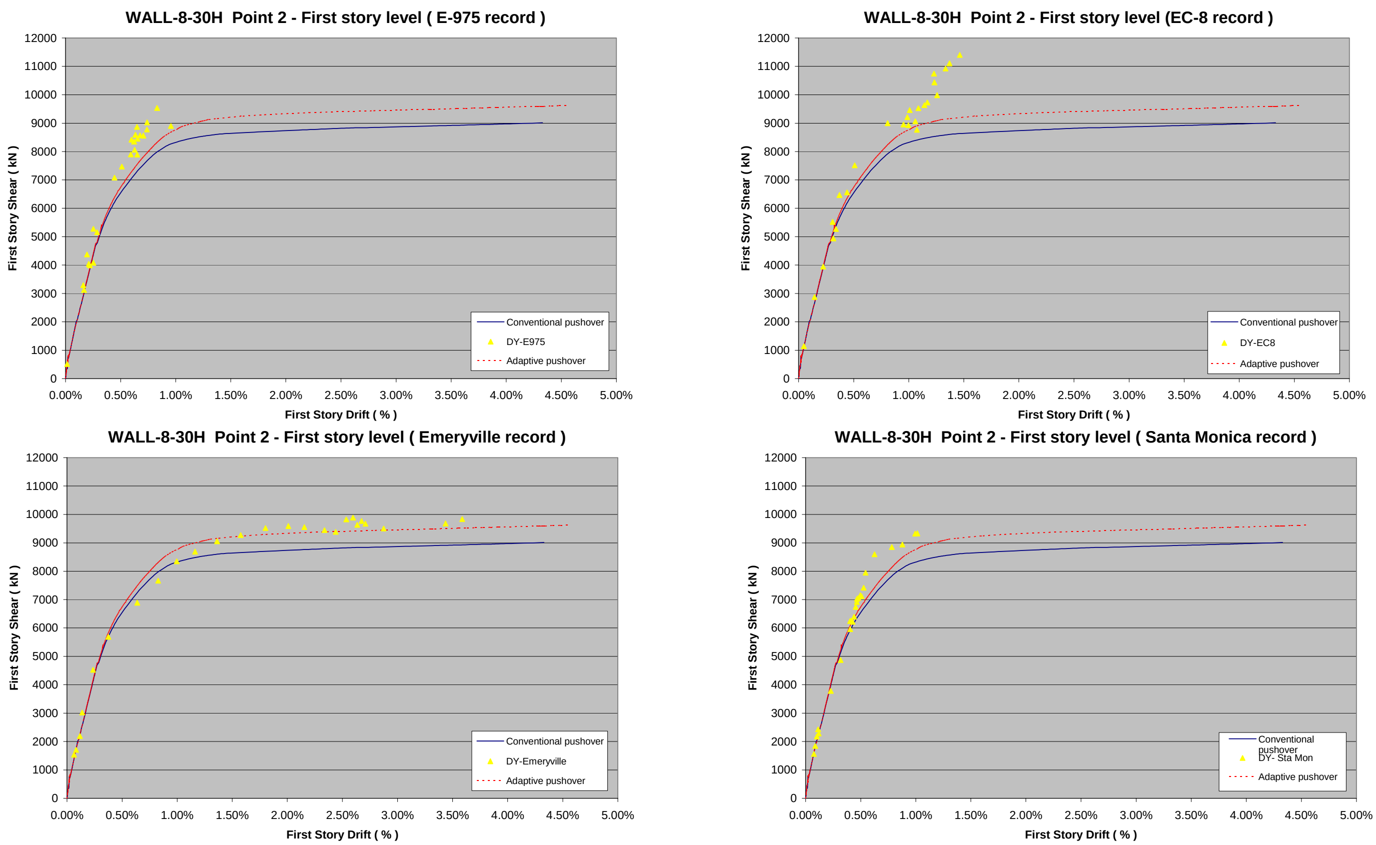


Figure 4.13 Response at the first story level, frame-wall structure (high ductility)

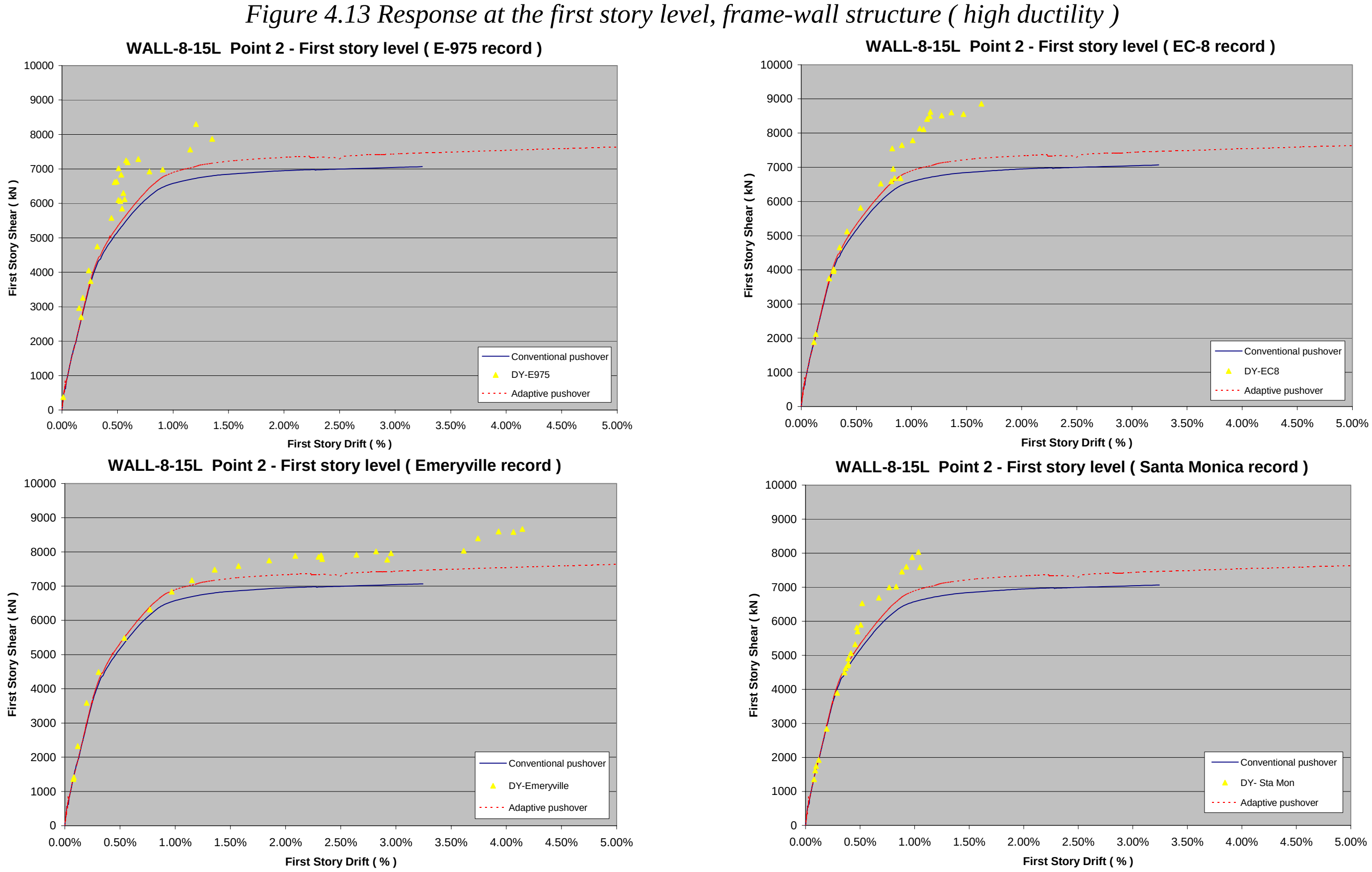


Figure 4.14 Response at the first story level, frame-wall structure (low ductility)

On the other hand, for the low ductility structure the difference rate of the conventional pushover oscillates between 8.1 % and 14.4 % while the adaptive pushover difference varies from 5.0 % to 10.4 %.

4.2.3. Middle story level

The figures 4.15 and 4.16 show that at the mid-story level both pushover analyses converge almost to the same results.

For the high ductility structure the average difference rate of the conventional pushover varies between 2.2 % and 7.7 % while for the adaptive technique the difference ranges from 2.4 % to 7.6 %. On the other hand, for the low ductility structure the difference rate of the conventional pushover oscillates between 3.3 % and 6.9 % while the adaptive pushover difference varies from 3.6 % to 6.9 %.

As can be inferred from the graphs, for the response at this level, it is not apparent that the natural records produce better results. This trend was also observed in the response of the regular frame.

In order to explain the response at this level, it is important to consider the influence of ***the beam to column stiffness ratio (ρ)*** of the structure. As is described in Chopra (1995) this parameter is based on the properties of the beams and columns in the story closest to the middle of the frame, and is an indicator of how much the system is expected to behave as a frame. For an ideal $\rho = 0$ the frame will behave as a flexural beam, while for $\rho = \infty$ the structure behaves as a shear beam with its columns in double curvature in each story.

This structure due to the shear walls is going to have a lower value of ρ than the frame structures, and for this reason its behavior will be more similar to a flexural beam. However, as has been already pointed out by Chopra (1995), the higher mode response is least significant for frames behaving like shear beams, increases its importance as ρ decreases and is largest for buildings deforming like flexural beams.

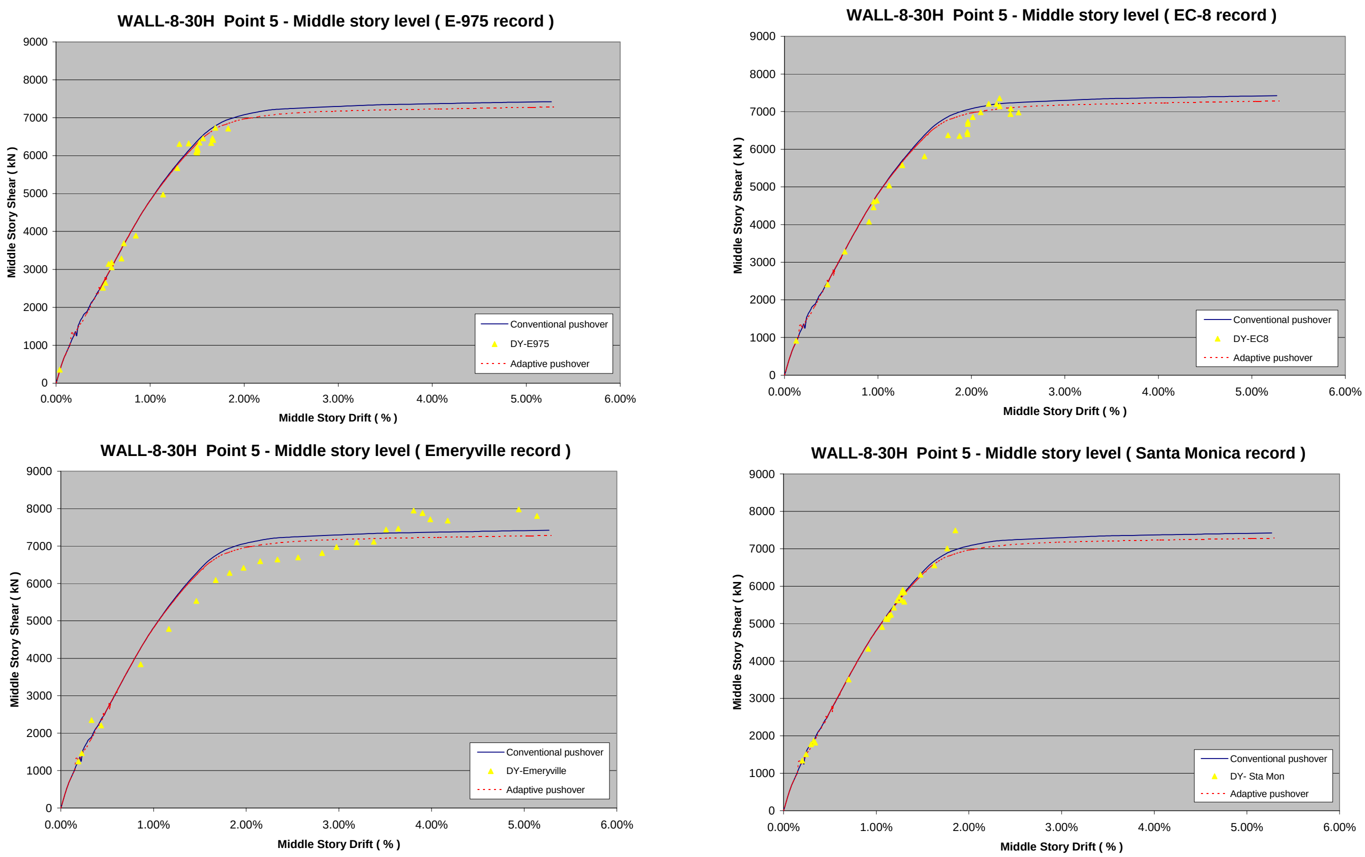


Figure 4.15 Response at the middle story level, frame-wall structure (high ductility)

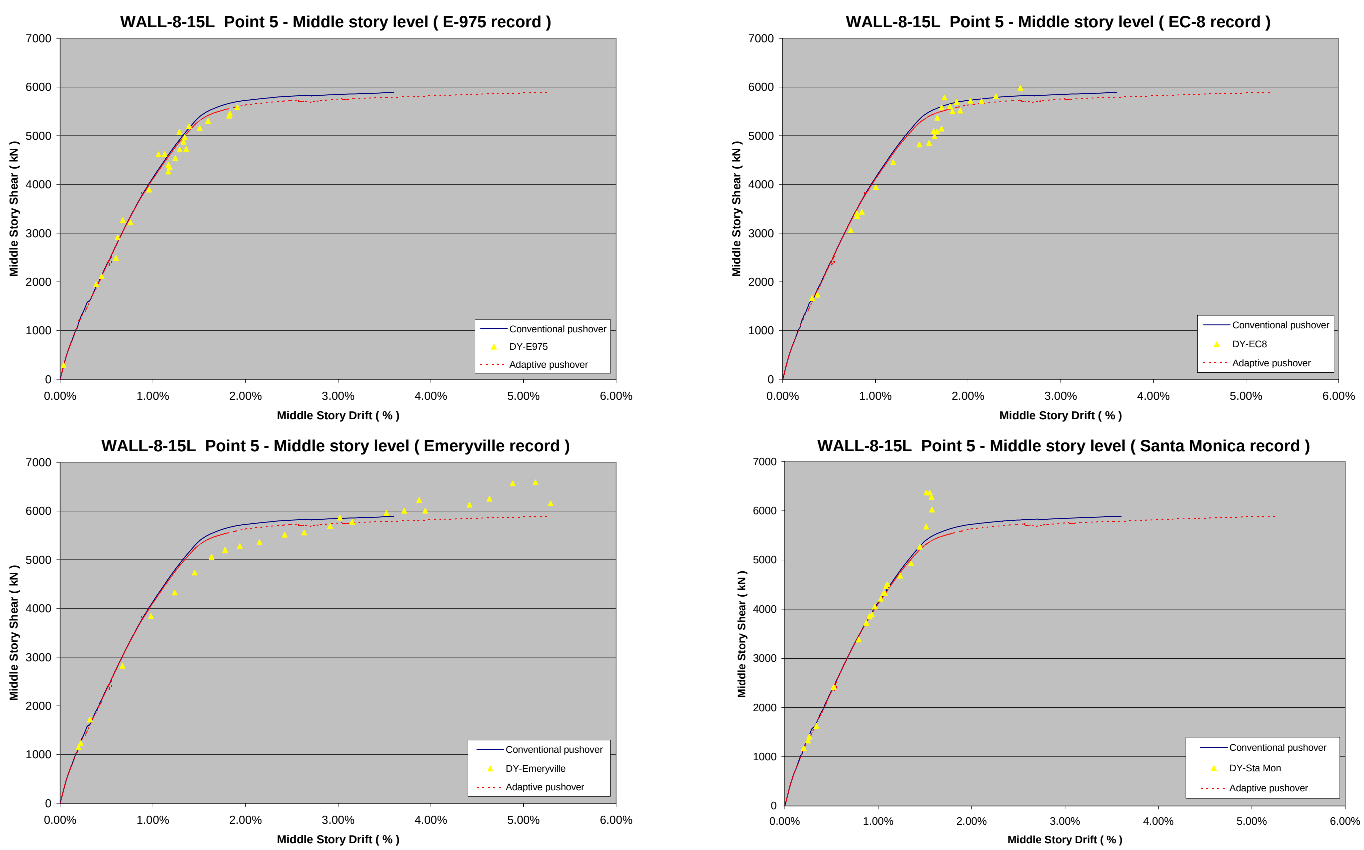


Figure 4.16 Response at the middle story level, frame-wall structure (low ductility).

It seems not very clear if this type of structure is more sensitive to the higher mode response, because there is almost no difference between both pushover approaches at this response level, especially when considerable variations were observed in the response of the regular frame.

Nevertheless, this question can be answered considering the process used to update the lateral load in the adaptive pushover and also the damage distribution along the height of the structure. It can be inferred from the formulations presented in the chapter 2 that the important changes in the lateral load vector come from the alterations in the modal shapes produced by the variation in the structural stiffness, when the damage starts to be concentrated in several localized points of the structure. In the adaptive pushover, the damage concentration is maximized by the method used to combine the modal response, since as was already stated, there is always and increase in the applied load, which is not realistic, because the direction of the modal forces is not considered.

In this case the damage is localized in the base of the structure, and according to these results it seems that there are not big alterations between the modal shapes calculated between the different load steps at the middle story level. The structure goes trough the period elongation process produced by the monotonic loads in the pushover analyses, increasing the damage in the lower level but without experiencing a substantial modification of its stiffness in the upper stories. Hence the lateral load vector shape remains similar to the fundamental mode, since the structure is just suffering a degradation of the stiffness at the lower level and not along its height.

In order to visualize this in a better way, the figure 4.17 presents the variation of the normalized modal scaling vector used to distribute the forces in the adaptive pushover. It is clear that the initial modal vector is similar to the fundamental mode, and how the damage is concentrated at the base of the structure. The graph also confirms that the applied loads remain almost constant at the middle level of the structure.

The adaptive pushover does not show a consistent improvement in the results. It seems that for this level of response, the benefits of the refinement in the pushover analysis technique for this type of structures are marginal.

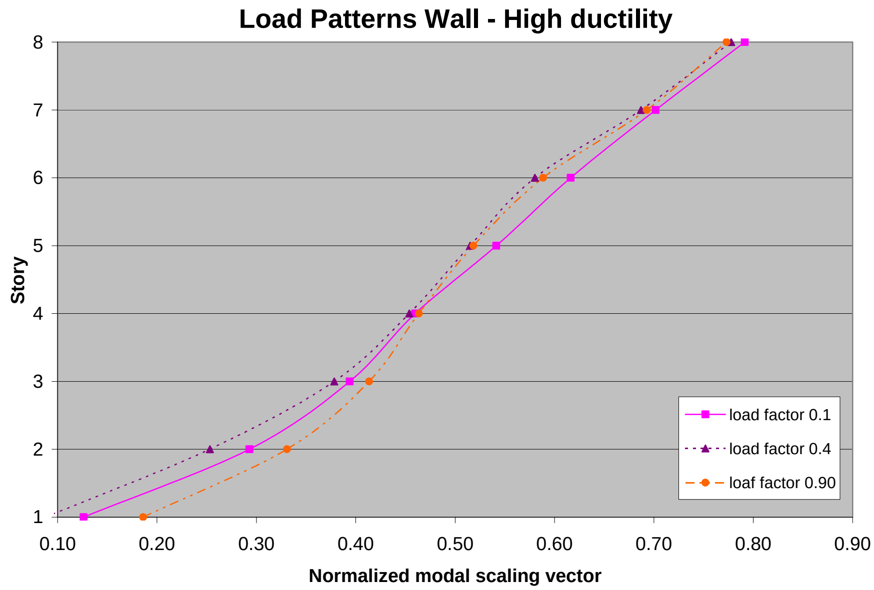


Figure 4.17 Variation on the modal scaling vector, for different load factors.

4.2.4. Section level

Figures 4.18 and 4.19 show a more detailed view of the response at the section level for both structures. Despite that CCDF values show higher differences at this level than the ones obtained from regular structures, this is produced by the method employed to collect the points in the incremental dynamic analysis, using the maximum moment and the maximum curvature at the same time and not by a detriment in the pushover technique results for this type of structures.

For the high ductility structure the average difference of the conventional pushover varies between 5.6 % and 13.4 % while for the adaptive technique the difference ranges from 5.6 % to 14.3 %. On the other hand, for the low ductility structure the difference rate of the conventional pushover oscillates between 4.4 % and 16.8 % while for the adaptive it varies from 4.8 % to 16.2 %.

These results show the same trend detected in the regular frame related with the comparable level of axial forces that is present in both pushover approaches and in the dynamic analysis. This can be inferred if we observe the graphs to verify that the maximum moment in the dynamic analysis coincide with the maximum moment obtained in the pushover analyses.

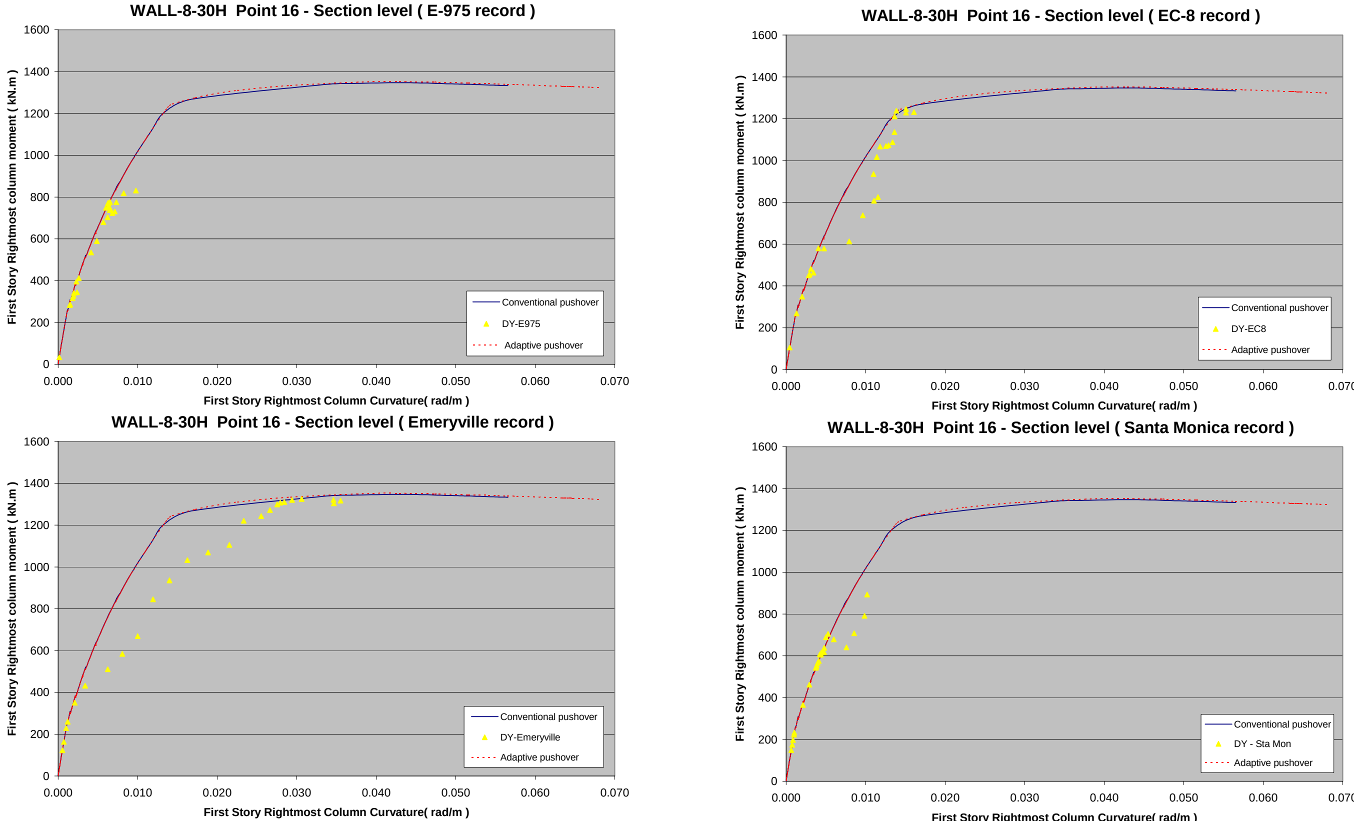


Figure 4.18 Response at the section level, frame-wall structure (high ductility)

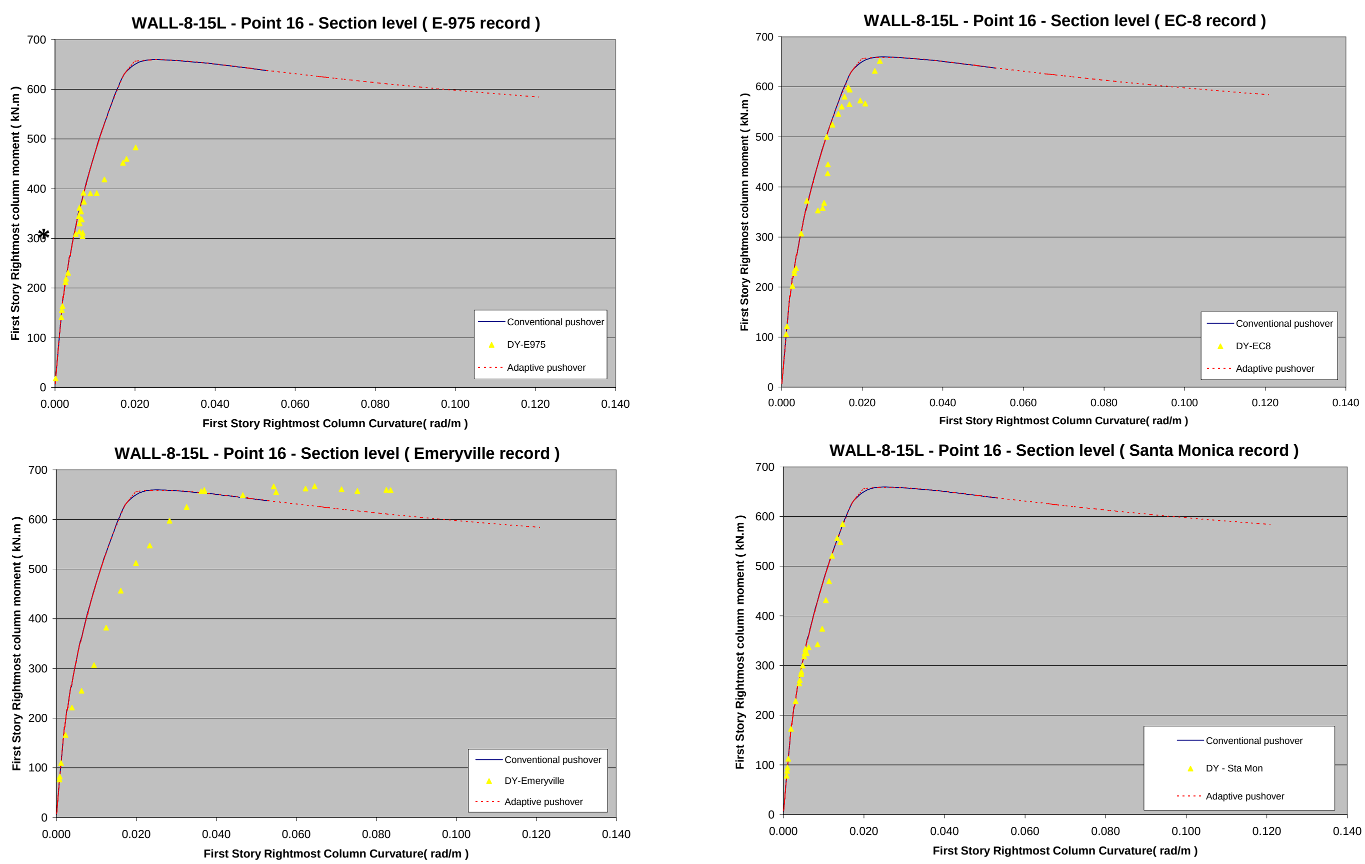


Figure 4.19 Response at the section level,frame- wall structure (low ductility)

Finally, figures 4.20 and 4.21 summarize the difference values presented in this section for regular frame structures.

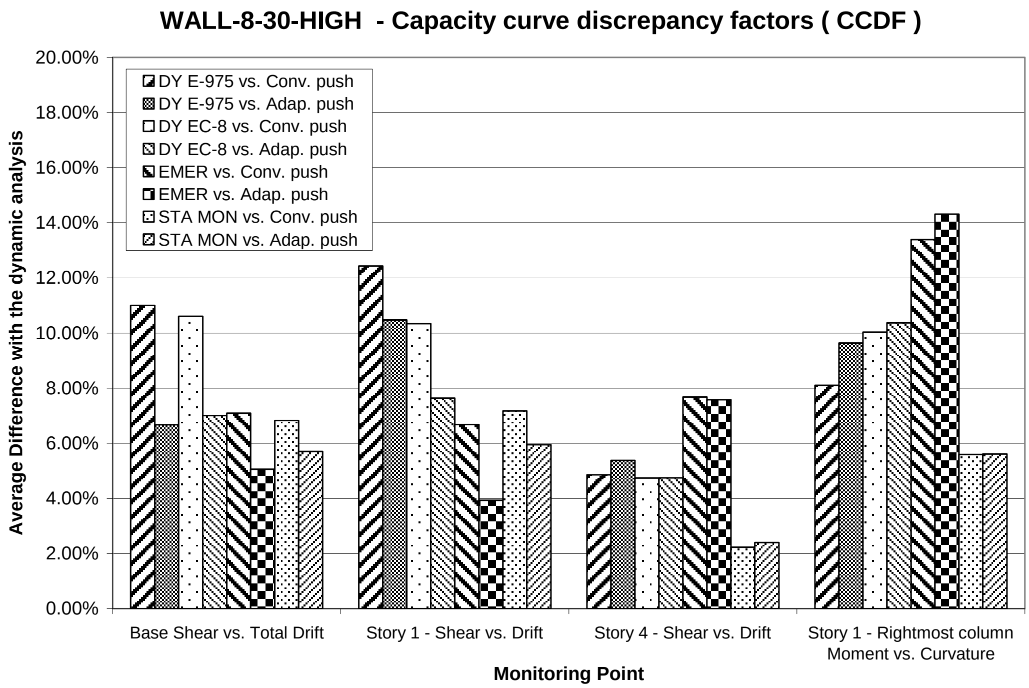


Figure 4.20 Results summary, frame-wall structure (high ductility)

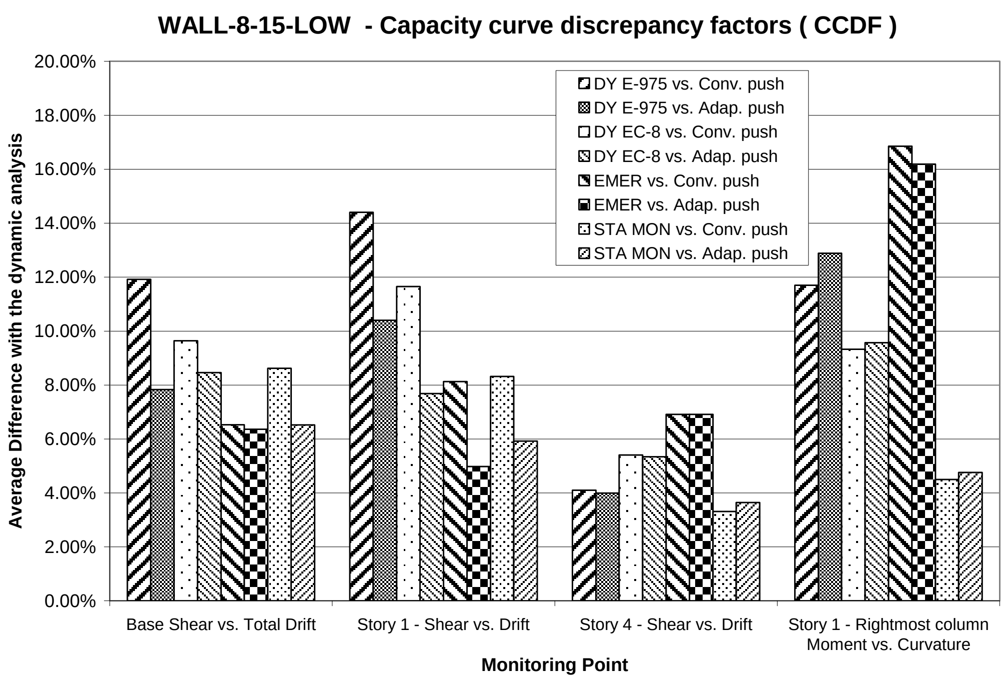


Figure 4.21 Results summary, frame-wall structure (low ductility)

4.3. Irregular structures

Table 4.3 summarizes the difference values between pushover and incremental dynamic analysis for ireggular structures.

HIGH DUCTILITY IRREGULAR STRUCTURE

Monitoring Point	Feature	Dy-1 E-975			Dy-2 EC-8			Dy-3 Emerville			Dy-4 Sta Monica		
		Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	Reduction (-) or Increment in the error	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional
Point 1	Base Shear vs. Total Drift	6.38%	7.85%	23.04%	6.07%	7.29%	20.10%	9.60%	3.20%	-66.67%	5.52%	11.32%	105.07%
Point 2	Story 1 - Shear vs. Drift	5.09%	2.84%	-44.20%	3.97%	2.34%	-41.06%	6.15%	1.93%	-68.62%	2.33%	1.01%	-56.65%
Point 5	Story 4 - Shear vs. Drift	3.93%	5.41%	37.66%	5.52%	9.26%	67.75%	4.01%	6.92%	72.57%	4.93%	6.07%	23.12%
Point 16	Story 1 - Rightmost column Moment vs. Curvature	1.97%	1.65%	-16.24%	1.00%	0.72%	-28.00%	1.85%	1.19%	-35.68%	2.15%	1.72%	-20.00%

LOW DUCTILITY IRREGULAR STRUCTURE

Monitoring Point	Feature	Dy-1 E-975			Dy-2 EC-8			Dy-3 Emerville			Dy-4 Sta Monica		
		Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	Reduction (-) or Increment in the error	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional
Point 1	Base Shear vs. Total Drift	6.02%	4.95%	-17.77%	9.45%	9.59%	1.48%	10.50%	9.60%	-8.57%	7.17%	4.22%	-41.14%
Point 2	Story 1 - Shear vs. Drift	3.92%	2.47%	-36.99%	4.45%	3.62%	-18.65%	2.95%	2.08%	-29.49%	2.62%	1.29%	-50.76%
Point 5	Story 4 - Shear vs. Drift	4.28%	5.17%	20.79%	4.65%	4.52%	-2.80%	3.06%	6.52%	113.07%	3.13%	8.69%	177.64%
Point 16	Story 1 - Rightmost column Moment vs. Curvature	0.84%	0.67%	-20.24%	7.43%	7.04%	-5.25%	8.53%	8.08%	-5.28%	2.07%	1.71%	-17.39%

Table 4.3 Capacity curve discrepancy factors (CCDF), irregular structures (High and low ductility)

4.3.1. Global level

For the high ductility structure the difference values for the conventional pushover vary between 5.5 % and 9.6 % while for the adaptive technique the difference ranges from 3.2 % to 11.3 %. On the other hand, for the low ductility structure the difference for the conventional pushover oscillates between 6.0 % and 10.5 % while the adaptive pushover difference varies from 4.2 % to 9.6 %.

Figures 4.22 and 4.23 show the response of the irregular structures at the global level. It seems that the same trends observed for the regular and shear wall structures are present here. However for the low ductility structure an important stiffness degradation process is observed in the pushover curves, which differs from the results obtained with the incremental dynamic analysis.

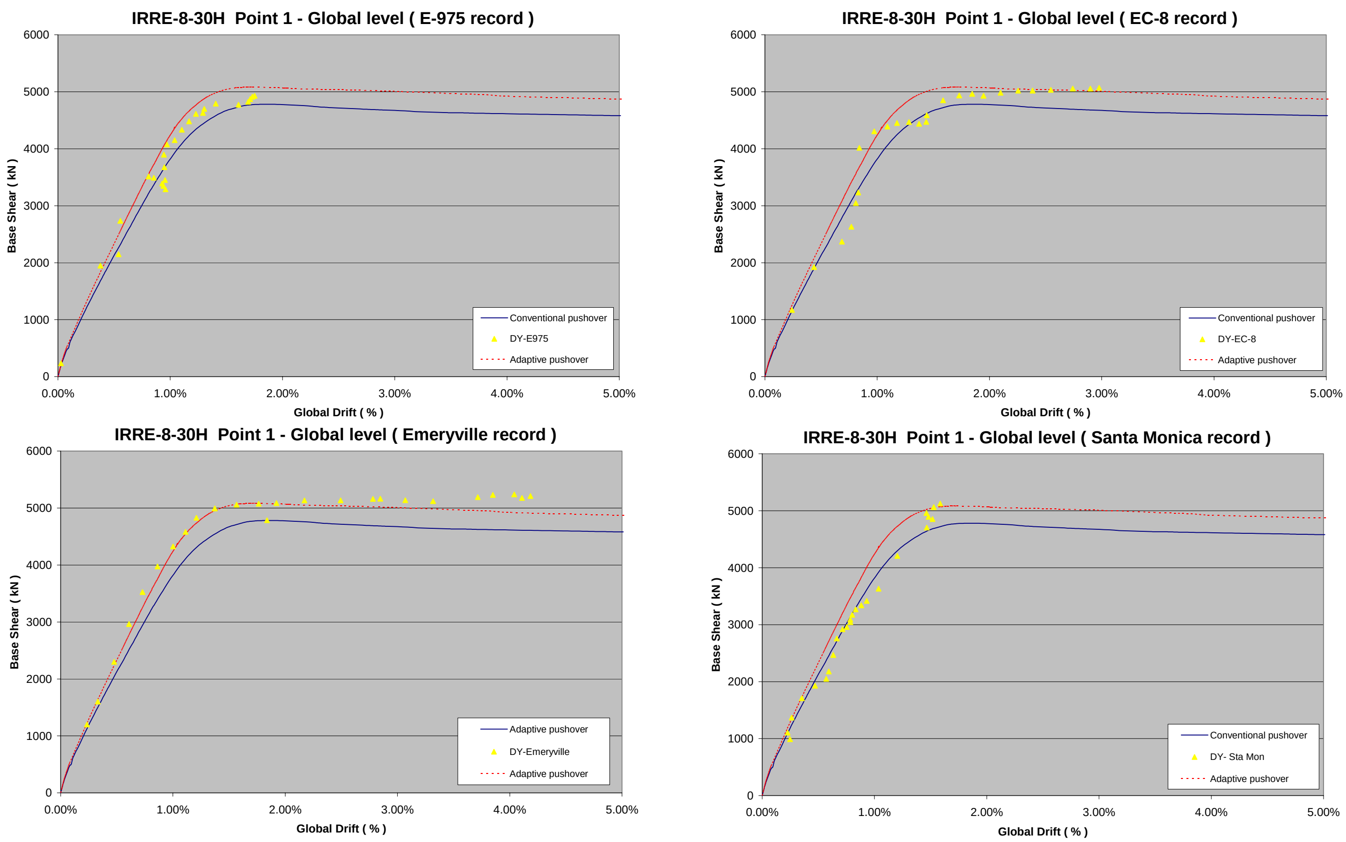


Figure 4.22 Response at the global level, irregular structure (high ductility)

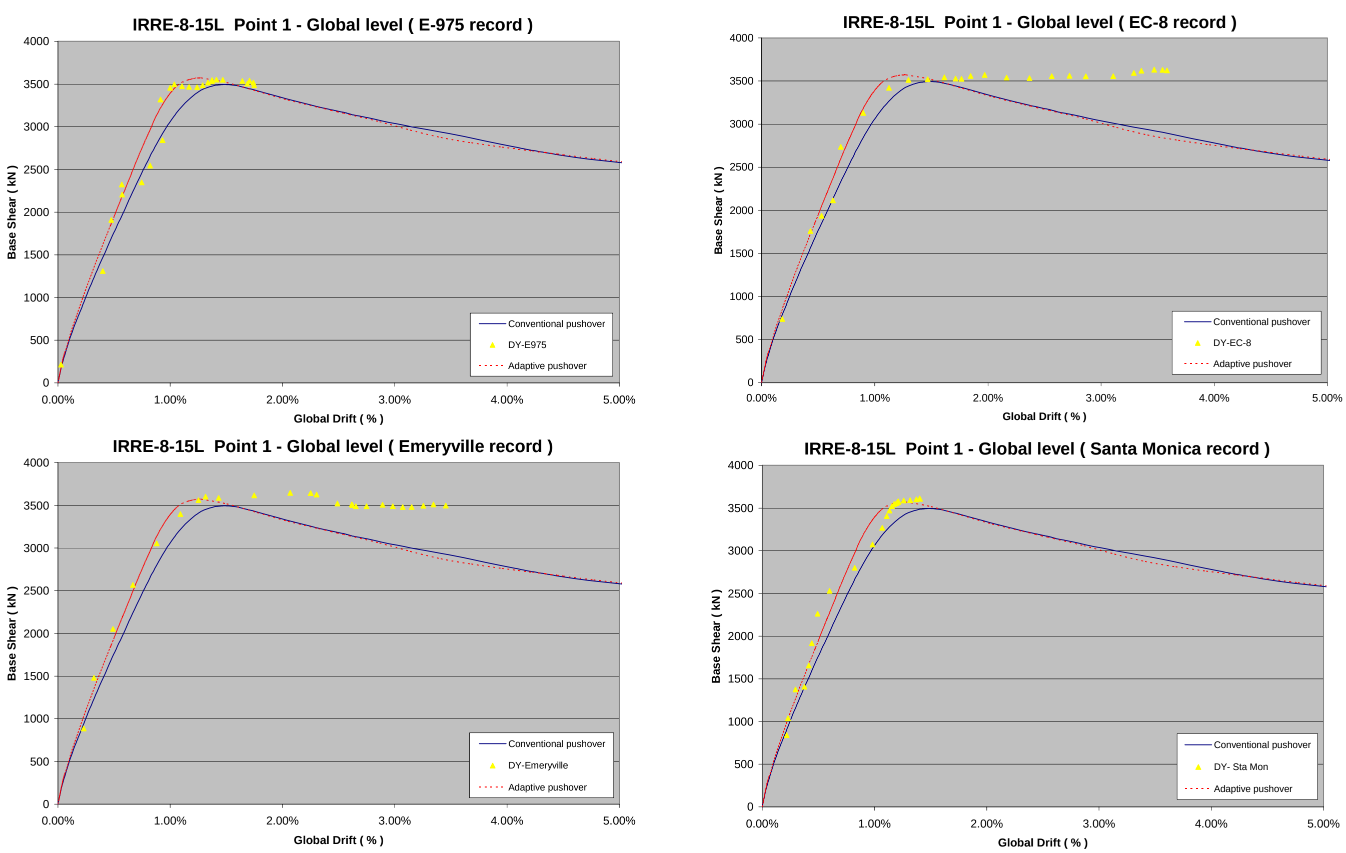


Figure 4.23 Response at the global level, irregular structure (low ductility)

Nevertheless, this strength degradation doesn't imply a bad performance of the pushover approaches, since the maximum value of the base shear is approximately equal in the pushover analyses and in the dynamic calculations. On the other hand, it is clear that for this case, the use of the adaptive pushover approach doesn't introduce any additional benefit in the estimation of the structural response at the global level for the low ductility structure, which presents a brittle behavior.

This non-ductile performance is not weird if the characteristics of the structure are considered : irregular geometry and low ductility design. However, as was previously mentioned, the dynamic analysis doesn't show this decay in the structural strength. Two main factors explain this difference:

- The criteria used to plot the points in the incremental dynamic analysis, using the maximum value of the force and displacement at the same time. As can be observed in the figure 4.23, in the inelastic range the points in the dynamic analysis appear almost in a horizontal line that coincides with the peak base shear in the pushover curves. However since each point is plotted against the maximum displacement, it is very difficult to visualize a process of strength degradation.

- It is important to consider that the monotonic application of the loads produces a conservative estimation of the structural capacity, due to the faster increment in the localized damage and to the constant coincidence of the gravitational effects with the solicitations imposed by the lateral loads at the critical locations. This trend has been observed in the results obtained in this work, since in any case the results obtained with the pushover techniques have been much higher than the peak values determined with the dynamic analysis. In the case of the adaptive pushover, as was stated by Antoniou and Pinho (2004) there is a trend to exaggerate the inelastic deformations in the locations of damage, and in this way sometimes extremely conservative results can be obtained.

Finally it is interesting to consider that in the case of the low ductility structure almost immediately after the structure has reached its peak resistance, both pushover techniques produce almost the same results. This means that this structure exhibits brittle behavior and the formation of the failure mechanism is produced the same time the structure enters into the inelastic range, implying that the eigensolver can no longer find real solutions and hence the lateral load vector remains unchanged.

This observation will be confirmed with the results obtained for the response at the middle story level.

4.3.2. First story level

In the same way that for regular and shear wall structures, at the first story level the adaptive pushover produces slightly better results than the conventional one, especially in the case of the irregular structure with high ductility.

According to table 4.3, for the high ductility structure the difference between conventional pushover and the dynamic analysis (CCDF) varies between 2.3 % and 6.1 % while for the adaptive technique the difference ranges from 1.0 % to around 2.8 %. On the other hand, for the low ductility structure, the CCDF of the conventional pushover oscillates between 2.6 % and 4.4 % while the adaptive pushover difference varies from 1.3 % to 3.6 %.

Figures 4.24 and 4.25 show a detailed view of the response at the first story level for the irregular structures.

From these figures, it is clear that for the high ductility irregular frame, the same trends already discussed in the preceding sections apply to the observed behavior. On the other hand, for the low ductility irregular frame, there is almost no difference in the results provided by the pushover approaches.

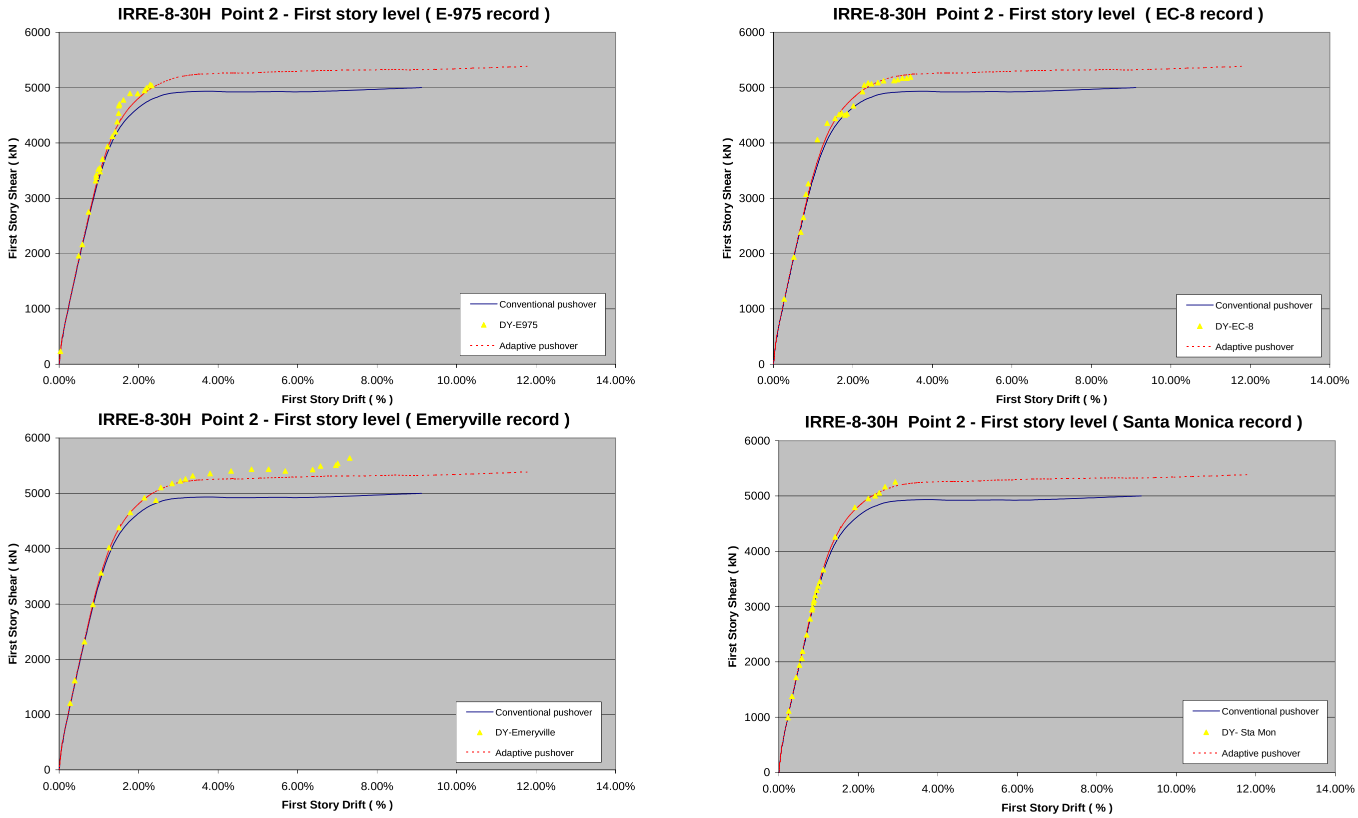


Figure 4.24 Response at the first story level, irregular structure (high ductility)

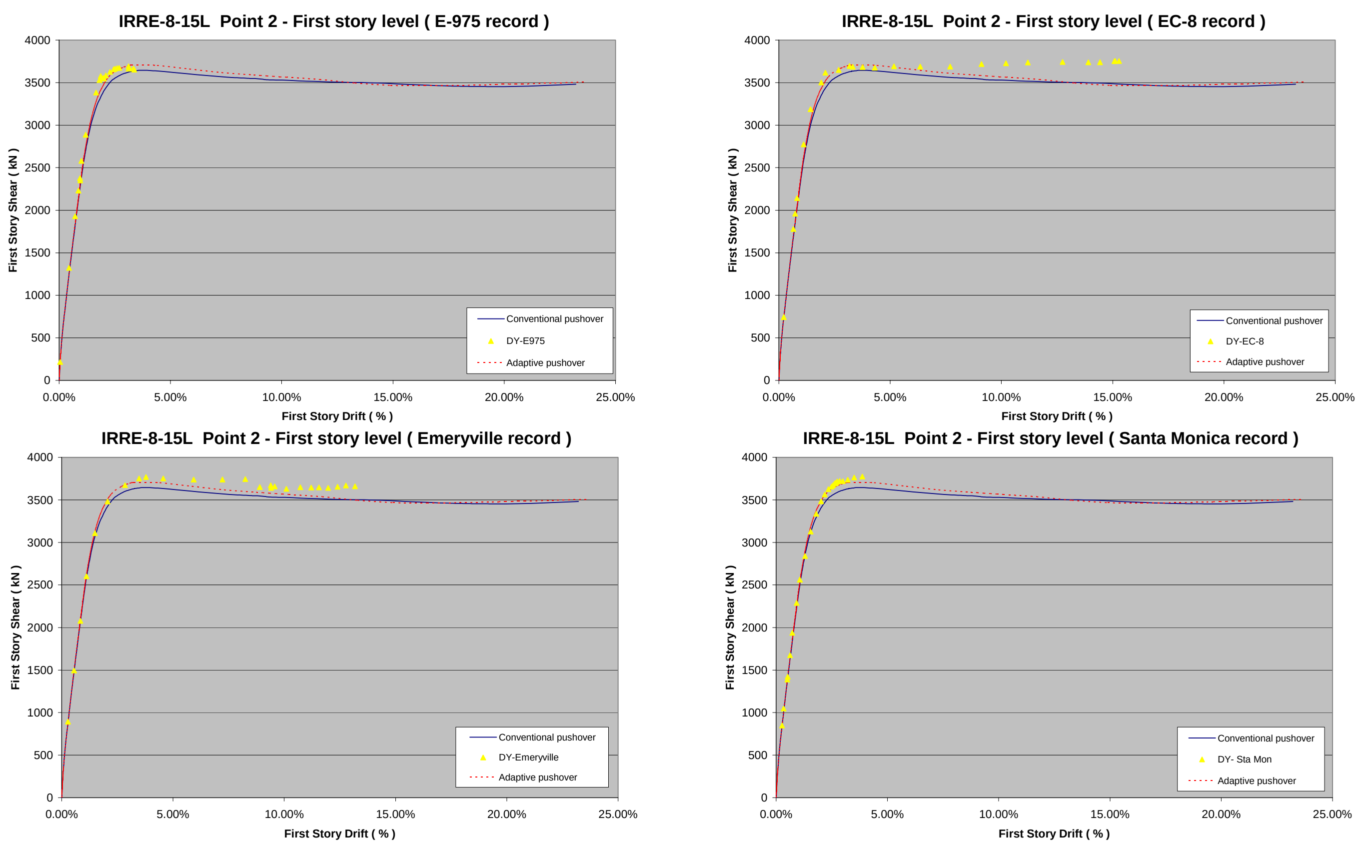


Figure 4.25 Response at the first story level, irregular structure (low ductility)

Since the main differences between conventional and adaptive pushover occur in the post-elastic range, this coincidence in the results suggests a brittle behavior in the structural response. When the structural stiffness is severely deteriorated, the stiffness matrix starts to have negative terms in the main diagonal and the lateral load vector cannot be updated anymore.

It is interesting to observe that according to pushover results the brittle behavior of the structure is not localized at this level which is contrary to the expected effect of the induced soft story. It seems that for this case, the pushover approaches don't supply a correct estimation of the interstory drift and shear distribution in the height of the building. Here is interesting to note that Antoniou and Pinho (2004) have reported cases where the totally inadequate predictions of these parameters were made by the pushover techniques with adaptive and fixed load distributions.

4.3.3. Middle story level

At the middle story level, in the same way that in the regular structures, the adaptive pushover loses its superiority over the conventional one.

For the high ductility structure the difference between conventional pushover and dynamic analysis (CCDF) varies between 3.9 % and 5.5 % while for the adaptive technique the CCDF ranges from 5.4 % to 9.3 %. On the other hand, for the low ductility structure the conventional pushover difference oscillates between 3.1 % and 4.6 % while the adaptive pushover difference varies from 4.5 % to 8.7 %.

Figures 4.26 and 4.27 show in detail the structural response at the middle story level.

These graphs present the same trend observed at the middle story in the regular structure : Both pushover approaches are too conservative in the estimation of the structural response at this level, predicting a very low structural capacity, especially in the low ductility irregular frame. In fact, figure 4.27 explains why at the global and first story levels, both pushover approaches show the same results in the inelastic range for the low ductility frame : there is a brittle failure mechanism,

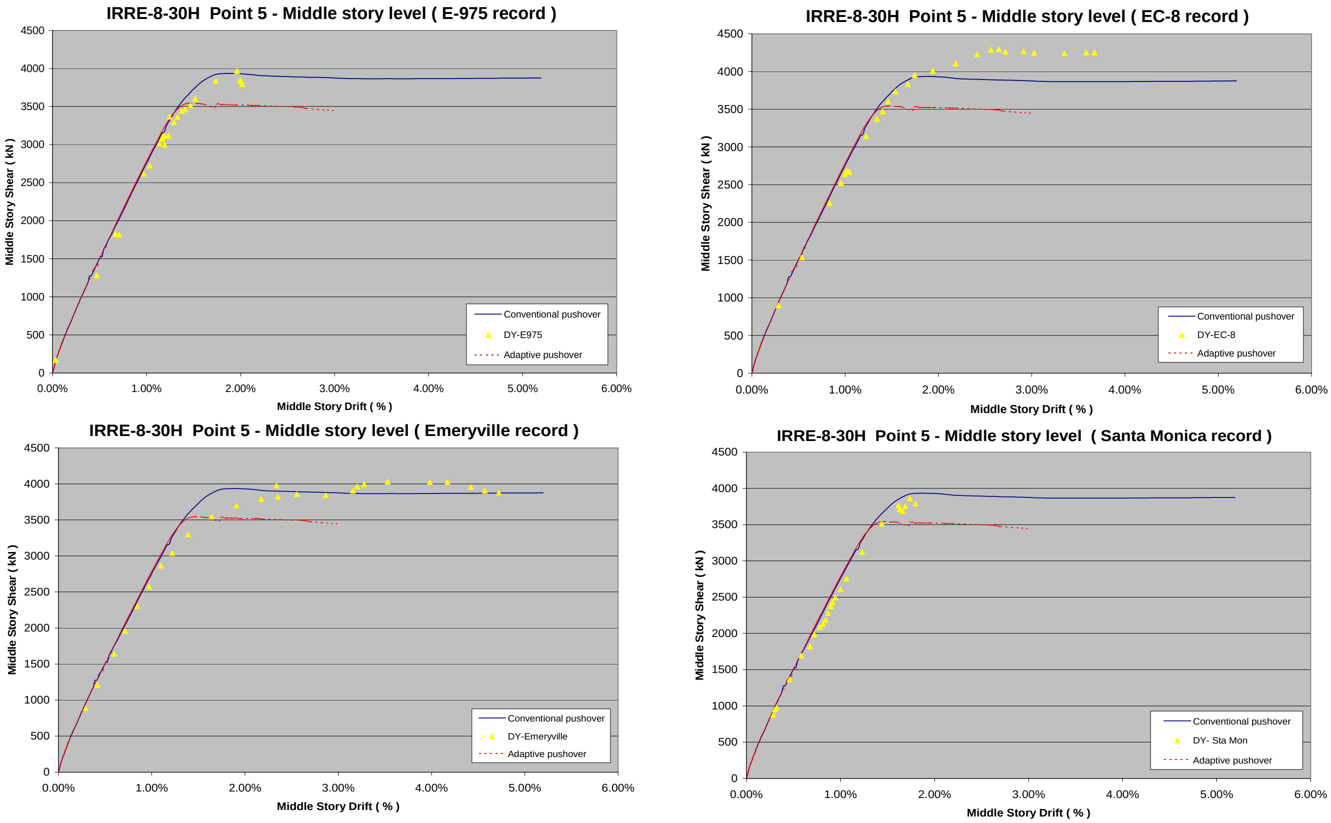


Figure 4.26 Response at the middle story level, irregular structure (high ductility)

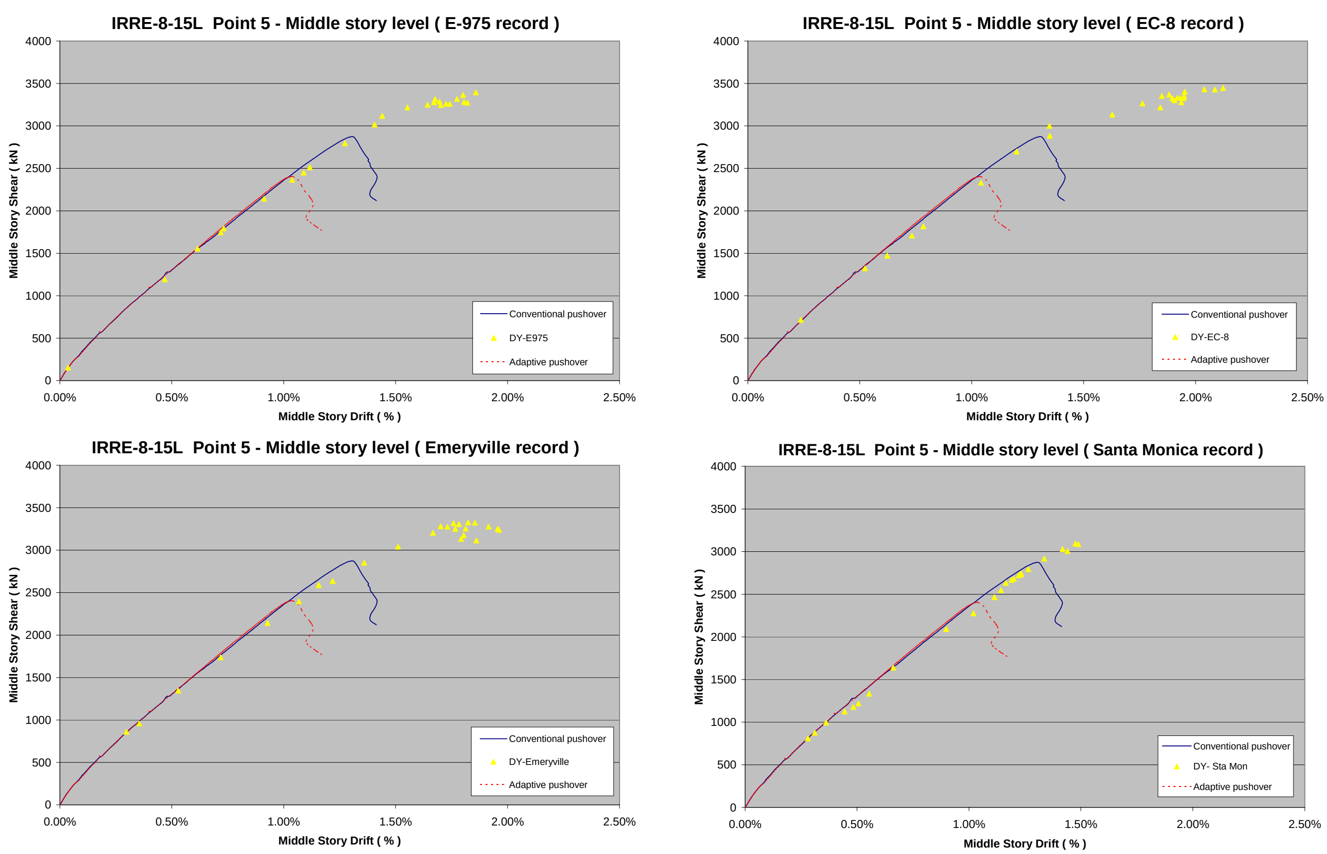


Figure 4.27 Response at the middle story level, irregular structure (low ductility)

that generates a fast degradation in the stiffness of the structure, creating problems to update the lateral load vector in the adaptive procedure.

Again in order to explain this behavior the following factors should be considered:

- As was mentioned before, the combination of the load forces using the SRSS or the CQC method seems to produce extremely conservative results, especially at this level, since the sign reversal in the different modal loads is not taken into account.

- Since in an irregular frame structure the damage tends to be distributed in a less localized way than, for instance, a shear wall structure, the alterations in the stiffness matrix of the structure will generate higher changes in the modal shapes and hence this can lead to wrong estimations of the structural capacity with the adaptive technique, due to the excessive increasing in the lateral forces applied in the damaged locations.

Another interesting observation is that the conventional pushover seems to be a better estimator of the response at the middle story level. As is shown in the figure 4.28, it is clear how the adaptive pushover concentrates the forces increment in the lower middle of the structure and applies higher forces than the fixed pattern in the stories where the damage is concentrated.

This inferior level of forces in the lower stories, allows the conventional pushover to reach higher level of deformations that supply slightly better estimations of the interstory drift and shear. Similar observations were made by Antoniou and Pinho (2004).

Finally, it should be pointed out that according to these results adaptive pushover doesn't introduce any advantage in the estimation of the response at this level for this type of structures, but rather generates a very conservative estimation of the structural capacity exaggerating the applied forces in the locations of the structure where the damage is concentrated.

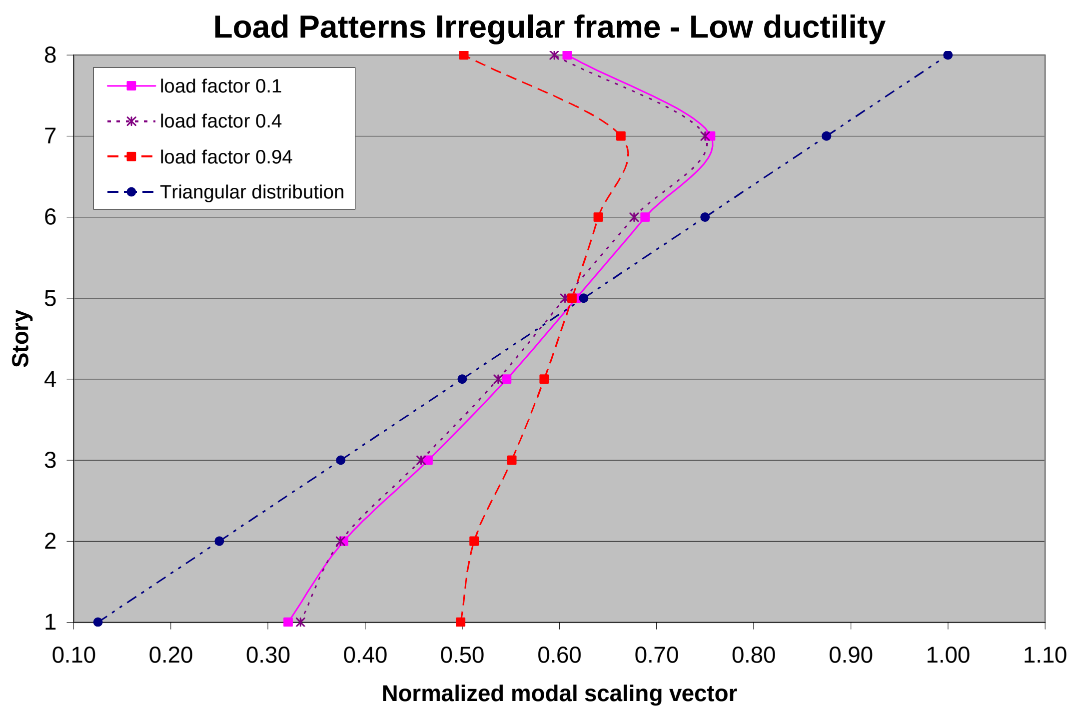


Figure 4.28 Variation on the modal scaling vector, for different load factors.

4.3.4. Section level

In the same way that for regular structures, at the section level both pushover analyses produce basically the same results, and no substantial improvement is produced applying the adaptive pushover technique.

For the high ductility structure the CCDF of the conventional pushover varies between 1.0 % and 2.1 % while for the adaptive technique the difference with the dynamic analyses ranges from 0.7 % to 1.7 %. On the other hand, for the low ductility structure the difference rate of the conventional pushover oscillates between 0.8 % and 8.5 % while the adaptive pushover difference varies from 0.7 % to 8.1%.

The figures 4.29 and 4.30 present a detailed view of the response at the section level for this type of structures. Again it is clear that the use of the adaptive pushover doesn't introduce any significant improvement in the results compared to dynamic analysis.

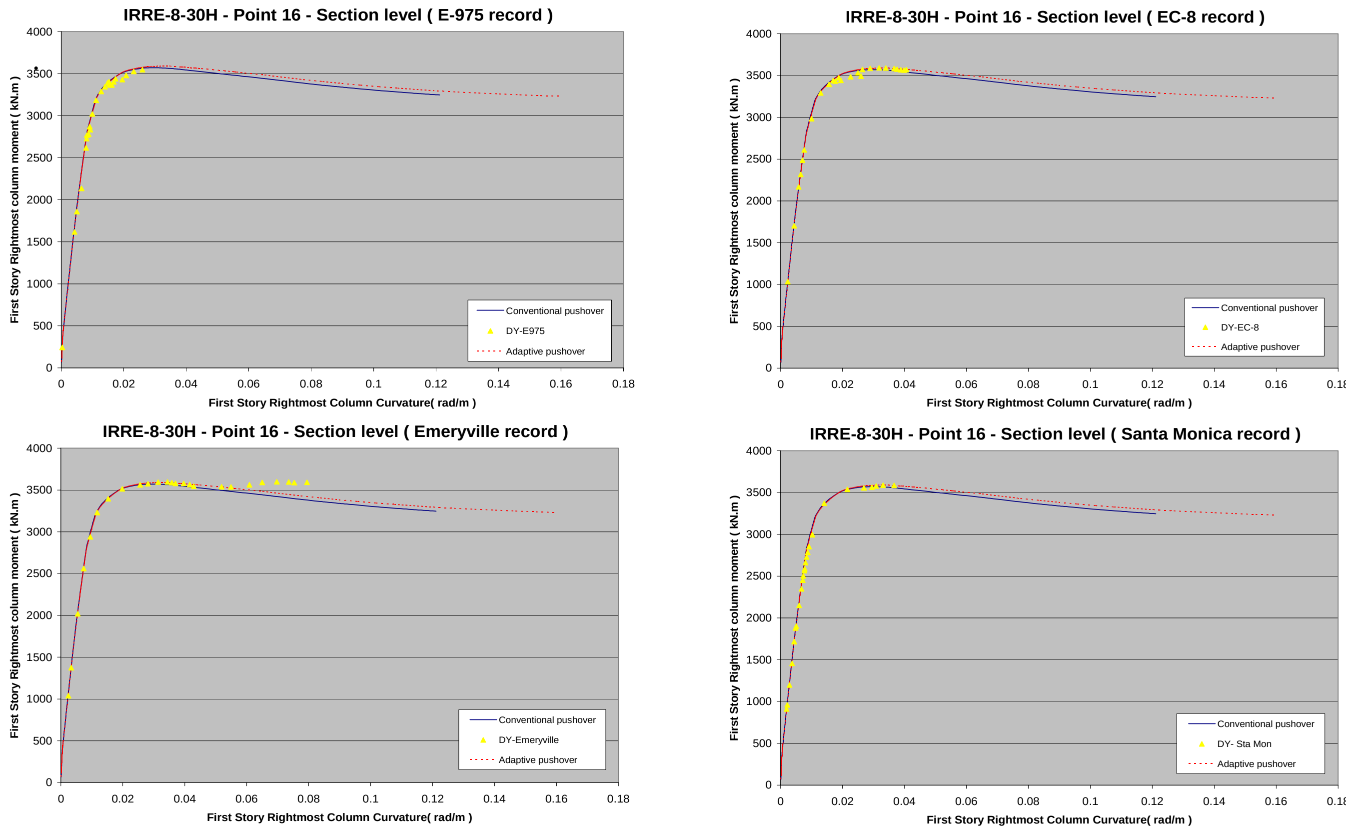


Figure 4.29 Response at the section level, irregular structure (high ductility)

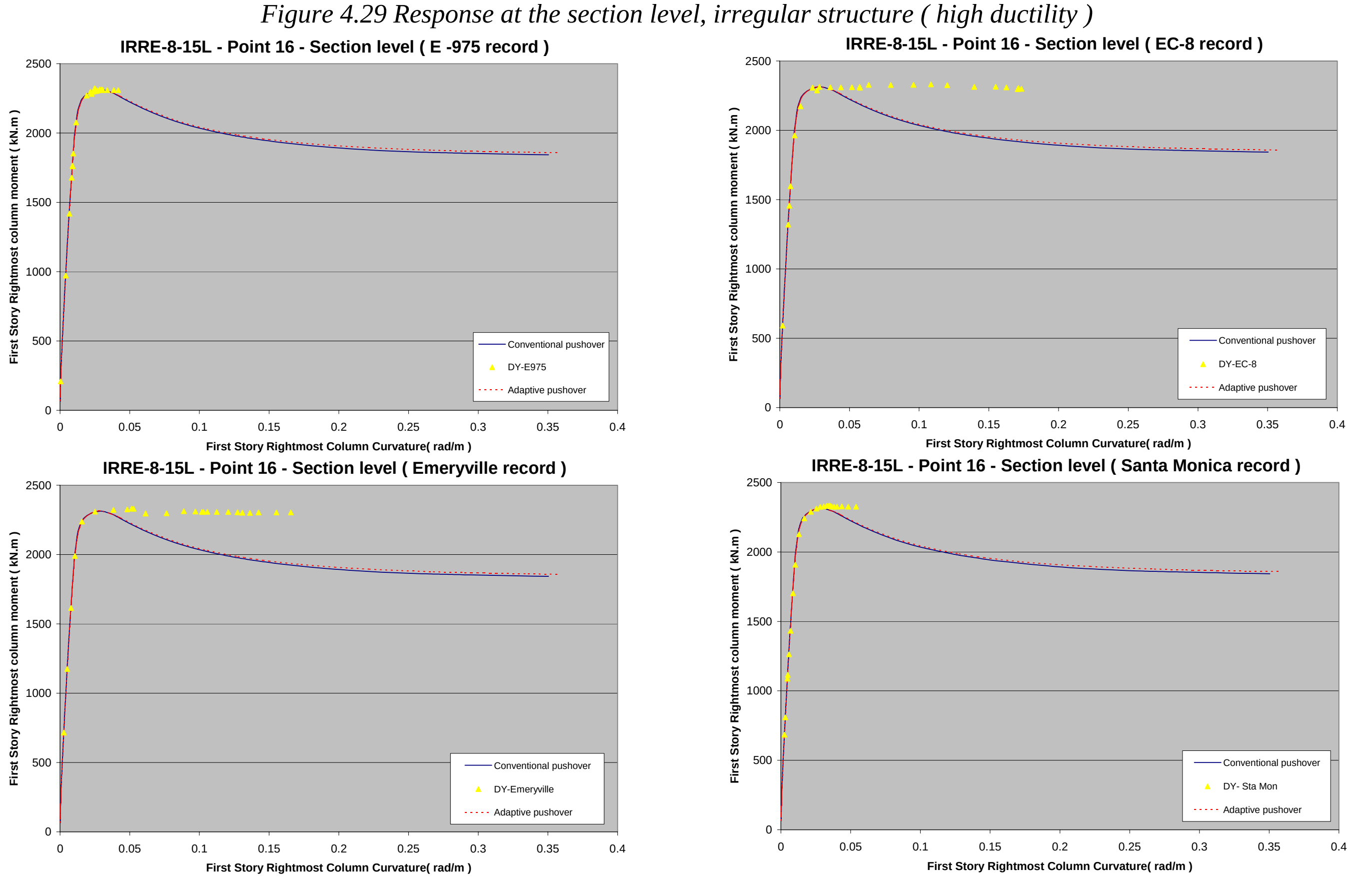


Figure 4.30 Response at the section level, irregular structure (low ductility)

However, the maximum moment estimated by dynamic analysis is very close to the maximum moment predicted by the pushover methods and hence it can be inferred that the level of axial loads are comparable between pushover and dynamic analysis.

As has been already pointed out, any differences observed in the figure 4.30 are due more to the method employed to plot the points in the incremental dynamic analysis than to the inaccuracy of the static pushover techniques

Finally, figures 4.31 and 4.32 summarize the difference values presented in this section for irregular frame structures.

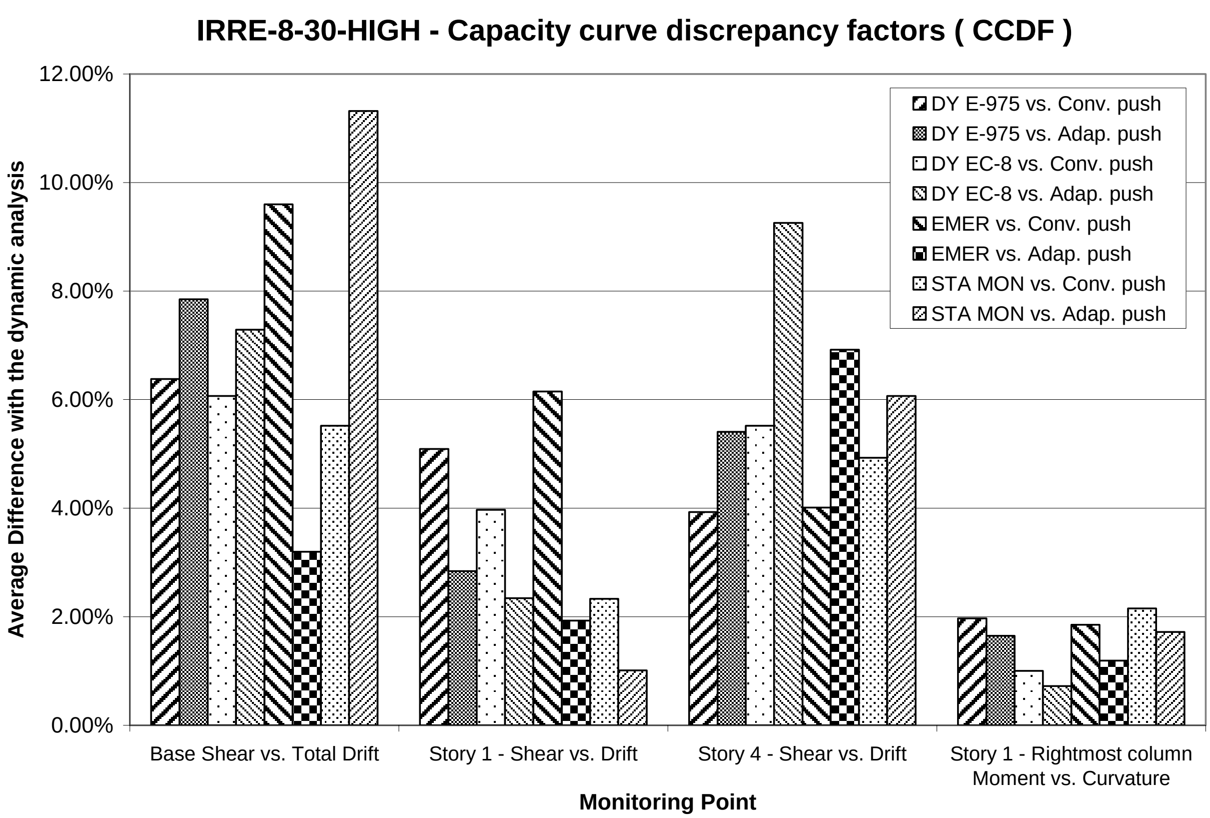


Figure 4.31 Results summary, irregular structure (high ductility)

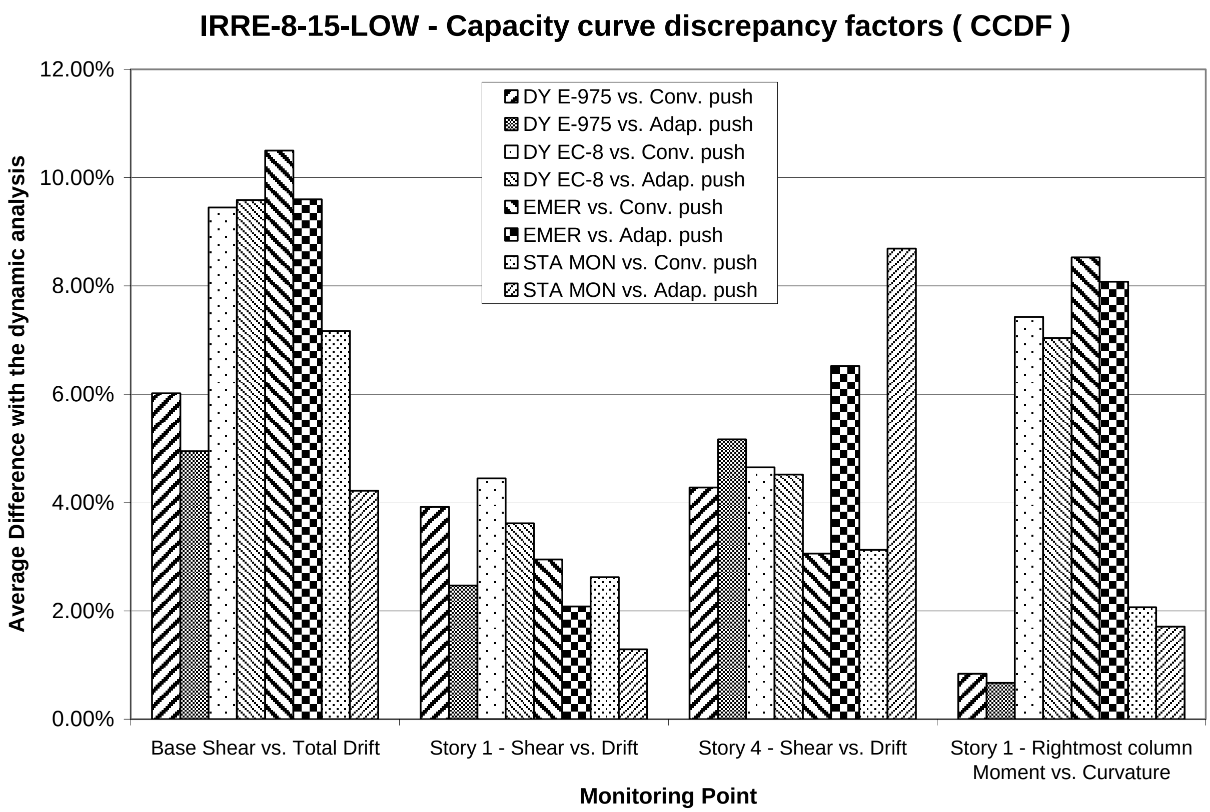


Figure 4.32 Results summary, irregular structure (low ductility)

4.4. ICONS bare frame

Table 4.4 summarizes the difference values between pushover and incremental dynamic analysis for the ICONS bare frame.

ICONS FRAME - BARE

Monitoring Point	Feature	Dy-1 E-975			Dy-2 EC-8			Dy-3 Emerville			Dy-4 Sta Monica		
		Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	Reduction (-) or Increment in the error	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional
Point 1	Base Shear vs. Total Drift	35.62%	34.02%	-4.49%	35.58%	44.04%	23.78%	36.27%	32.87%	-9.37%	34.60%	32.71%	-5.46%
Point 2	Story 1 - Shear vs. Drift	25.98%	26.27%	1.12%	29.16%	24.30%	-16.67%	28.13%	24.66%	-12.34%	29.77%	25.38%	-14.75%
Point 4	Story 3 - Shear vs. Drift	23.90%	27.32%	14.31%	21.24%	23.68%	11.49%	16.90%	25.28%	49.59%	17.85%	19.45%	8.96%
Point 18	Story 1 - Rightmost column Moment vs. Curvature	9.73%	9.66%	-0.72%	8.97%	8.88%	-1.00%	12.53%	12.47%	-0.48%	7.08%	6.99%	-1.27%

Table 4.4 Capacity curve discrepancy factors (CCDF), ICONS frame (Bare)

4.4.1. Global level

At the global level, difference values between conventional pushover and dynamic analysis range from 34.6 % to 36.2 % and for adaptive pushover difference values range between 32.7 % and 44.1 %.

The same trends observed in the response at the global level for the structures analyzed before are also present in this case. Differences for adaptive pushover are higher than the conventional case. However, observing the graphs it is easy to understand the reason : Since the adaptive pushover allows to the structure to reach higher deformation levels at the global level, the range of comparison for the CCDF is wider for the adaptive technique and hence the difference compared to dynamic analysis is increased. This observation reminds the importance to understand the meaning of the CCDF with very good criteria considering that it only applies to cases whit the same range of deformations.

The figure 4.33 presents in detail the bare frame response at the global level.

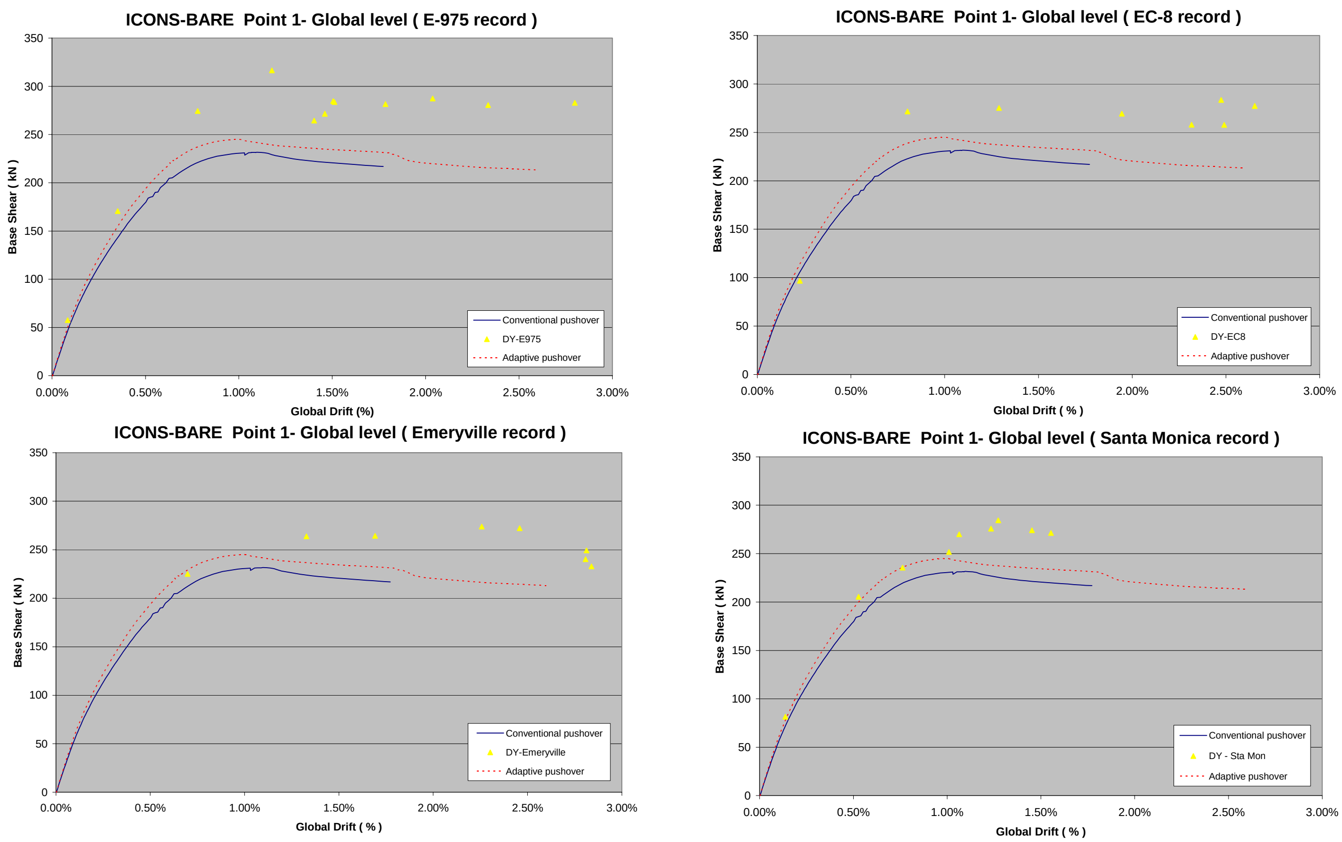


Figure 4.33 Response at the global level, ICONS bare frame

As was previously mentioned, the same trends already discussed for the response at the global level are observed here. Nevertheless it is interesting to point out that the adaptive pushover seems to be more stable up to higher levels of inelastic deformations showing a better correspondence with the maximum deformation levels observed in the dynamic analyses.

4.4.2. First story level

For this bare frame the differences with the dynamic analyses at this level, vary from 26.0 % to 29.7 % for the conventional pushover and from 24.3 % to 26.3 % for the adaptive technique.

These percentages, and the detailed comparison presented in the figure 4.34 indicate that at the first story level the main improvement of the adaptive pushover is to enhance the calculations stability more than to increase the results accuracy. The increase in stability can be clearly appreciated observing that the adaptive pushover reaches higher level of inelastic deformations.

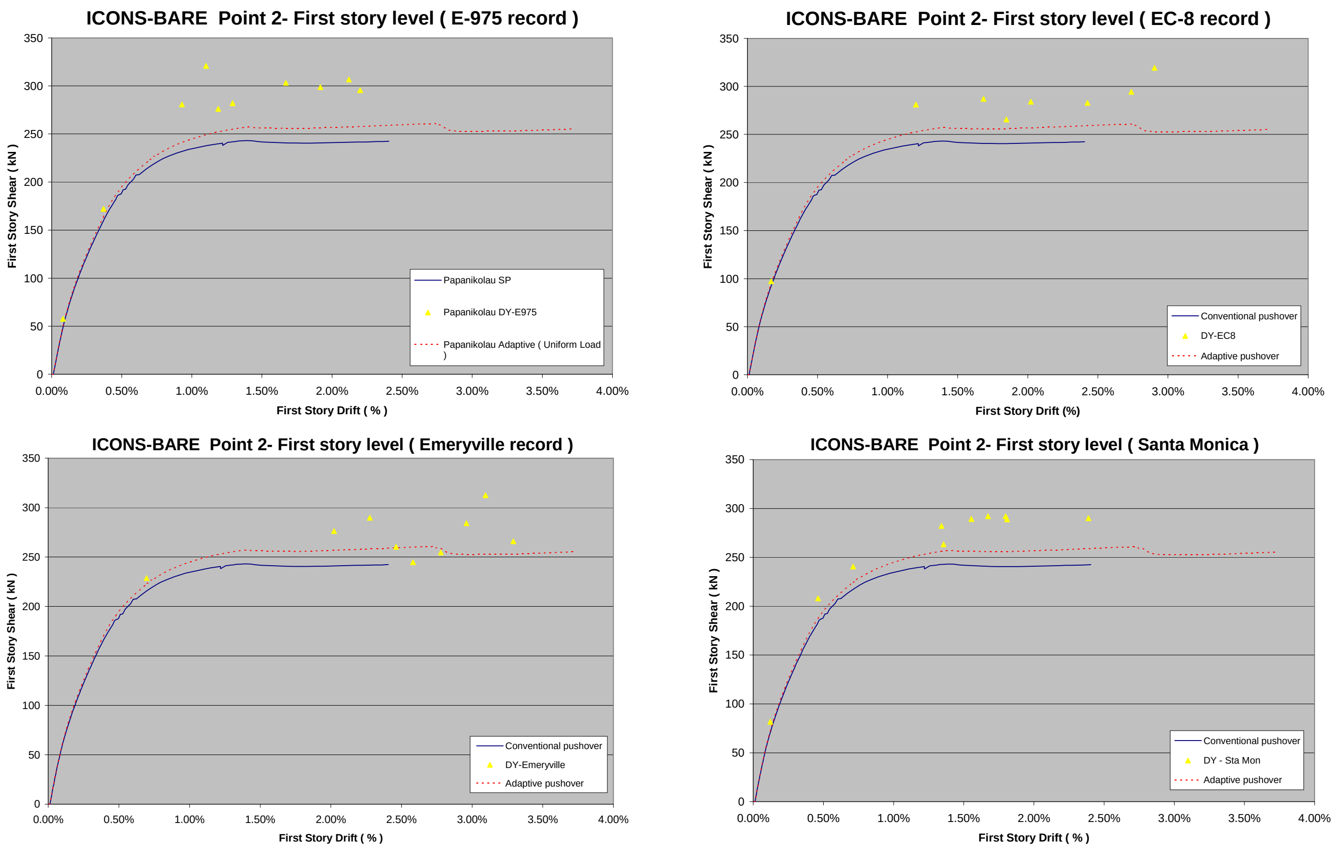


Figure 4.34 Response at the first story level, ICONS bare frame

The same trends observed at the global level are present here. There is almost no difference in the results of the pushover techniques in the elastic range and in the high inelastic range the differences between the two approaches remains constant implying that the lateral load vector is not updated anymore.

4.4.3. Middle story level

In the same trend observed for the regular and the irregular structures, the adaptive pushover loses its superiority over the conventional at this level.

For this irregular bare frame the differences between conventional and dynamic analysis fluctuate between 16.9 % and 23.9 %, and for the adaptive pushover between 19.4 % and 27.3 %.

The figure 4.35 shows a detailed view of the response at the middle story level.

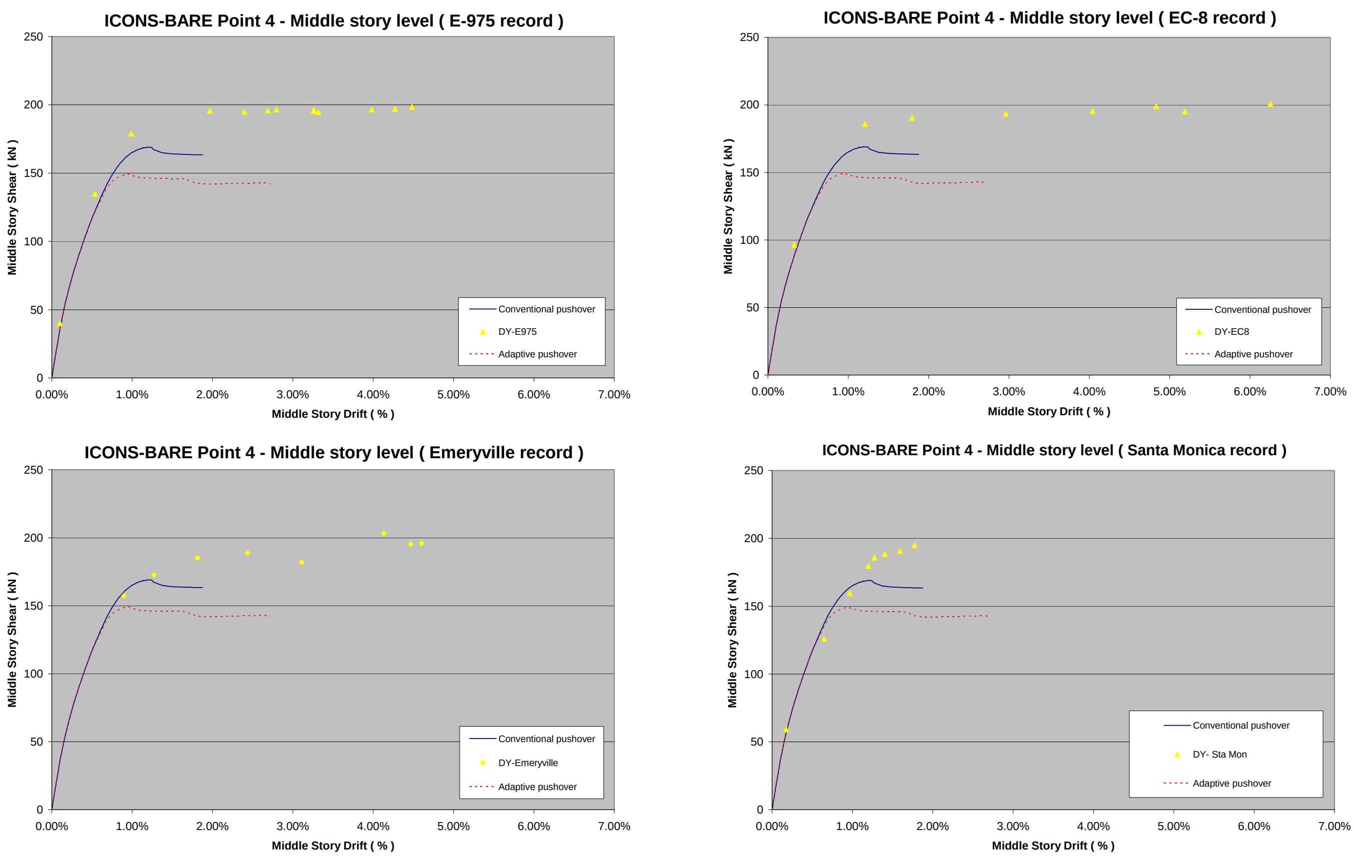


Figure 4.35 Response at the middle story level, ICONS bare frame

It is clear that at the middle story level both pushover approaches are very conservative in the prediction of the structural capacity. As in the case of the framed structures analyzed before, the fixed triangular lateral force distribution used in the conventional pushover gives better results than the adaptive technique. As was stated before and confirmed by Antoniou and Pinho (2004), this behavior is explained by the lower level of forces applied by the conventional pushover in the lower floor levels of the structure where the damage is concentrated.

Hence, according to these results the adaptive pushover methodology doesn't improve the accuracy in the response assessment of the middle story level for this type of structures. In fact, both pushover techniques seem to overestimate the damage at this location.

4.4.4. Section level

The same trend observed in the previous analysis is detected here. There is no significant improvement using the adaptive pushover technique, and basically both pushover approaches produce the same results.

For this irregular frame, the differences with the dynamic analyses fluctuate from 7.1 % to 12.5 % for the conventional pushover, and from 7.0 % to 12.5 % for the adaptive.

The figure 4.36 presents in detail the obtained response at the section level.

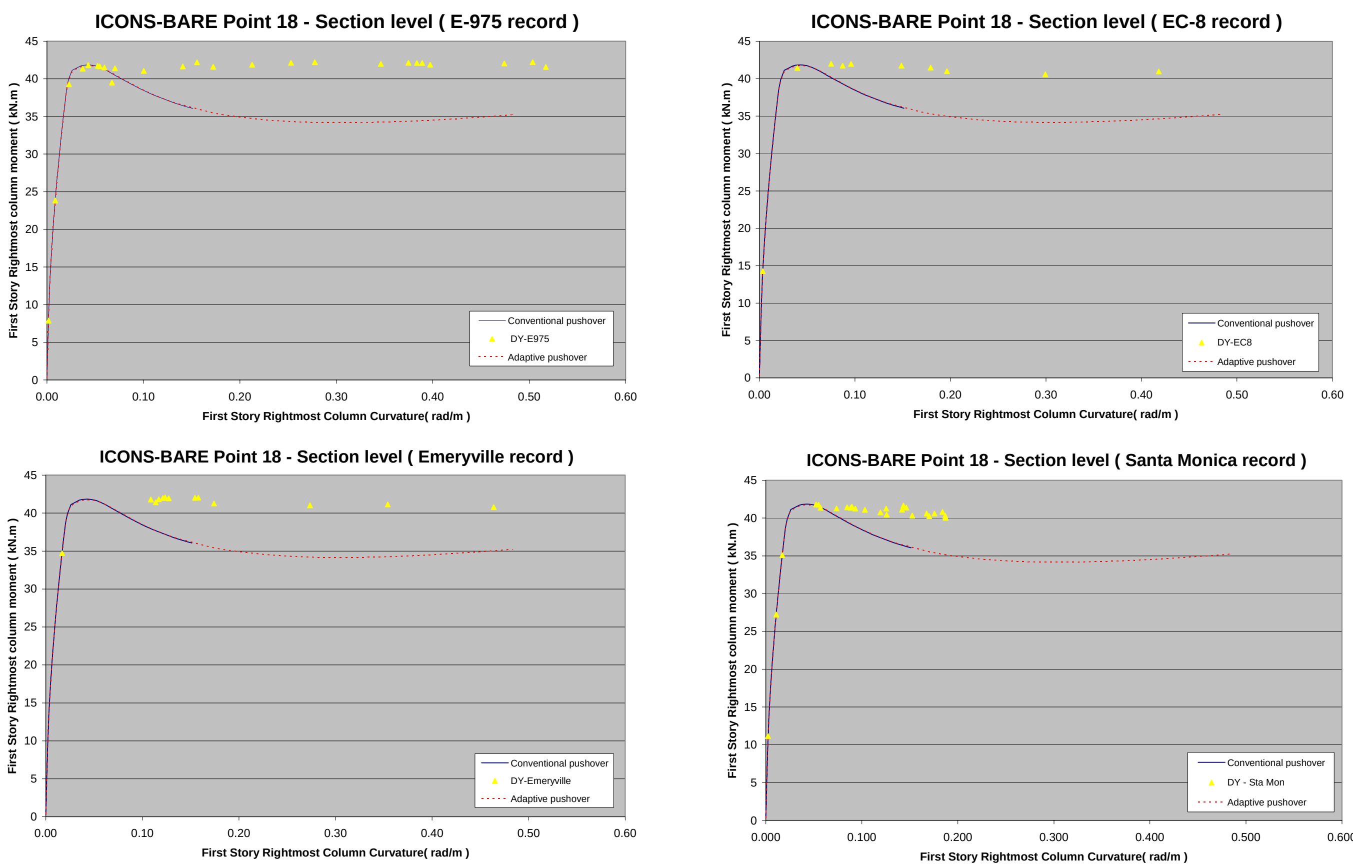


Figure 4.36 Response at the section level, ICONS bare frame

These percentages clearly indicate that no benefit is achieved using the adaptive pushover. Again both inelastic static analysis approaches show the same results. Nevertheless, the maximum levels of moment and curvature reached by the section are very similar between dynamic analysis and pushover procedures, indicating that at the section level inelastic static analysis is an adequate indicator of structural capacity.

Finally, figure 4.37 summarizes the difference values presented in this section for the ICONS bare frame.

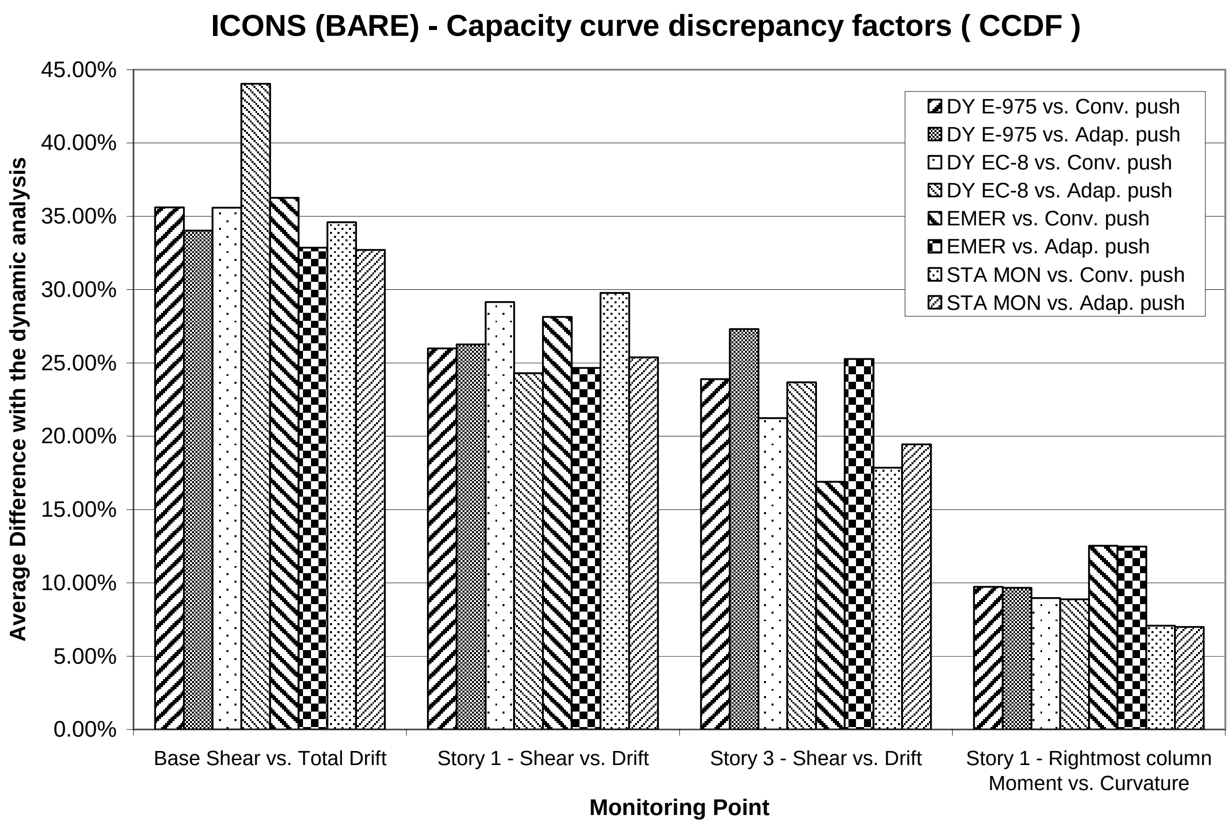


Figure 4.37 Results summary, ICONS bare frame

4.5. SPEAR 3D frame

Table 4.5 summarizes the difference values between pushover and incremental dynamic analysis for the SPEAR frame.

SPEAR 3D-FRAME

Monitoring Point	Feature	Dy-1 E-975			Dy-2 EC-8			Dy-3 Emerville			Dy-4 Sta Monica		
		Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	Reduction (-) or Increment in the error	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional	Avg Difference Conventional Pushover	Avg Difference Adaptive Pushover (Uniform Load)	% Error above or below the Conventional
Point 1	Base Shear vs. Total Drift	29.48%	41.23%	39.86%	38.79%	50.19%	29.39%	49.80%	61.15%	22.79%	28.76%	39.89%	38.70%
Point 2	Story 1 - Shear vs. Drift	12.31%	14.16%	15.03%	10.86%	13.12%	20.81%	13.37%	15.23%	13.91%	11.11%	12.58%	13.23%
Point 3	Story 2 - Shear vs. Drift	10.95%	6.96%	-36.44%	9.46%	3.84%	-59.41%	12.00%	1.62%	-86.50%	4.28%	13.64%	218.69%
Point 11	Story 1 - Rightmost column Moment vs. Curvature	13.20%	15.16%	14.85%	9.33%	13.43%	43.94%	13.20%	19.31%	46.29%	11.46%	13.45%	17.36%

Table 4.5 Capacity curve discrepancy factor (CCDF), SPEAR-3D frame

4.5.1. Global level

Contrary to the trends already observed, for this structure the conventional pushover analysis performs consistently better than its adaptive counterpart, although there are large differences between both pushover analyses and results obtained with dynamic analysis.

For the conventional pushover the differences with the dynamic analyses vary from 29.5 % to almost 49.8 %, while the adaptive pushover produces differences fluctuating between 39.9 % and 61.1 %.

Figure 4.38 shows a detailed view of the response at the global level. For first time, the conventional pushover performs consistently better than the adaptive technique. However, the maximum base shear predicted by the dynamic analyses is very similar to the peak base shear observed in the pushover curves. The main difference is the fast strength degradation observed in the pushover curves in the inelastic range which is not suggested by the obtained incremental dynamic analysis points.

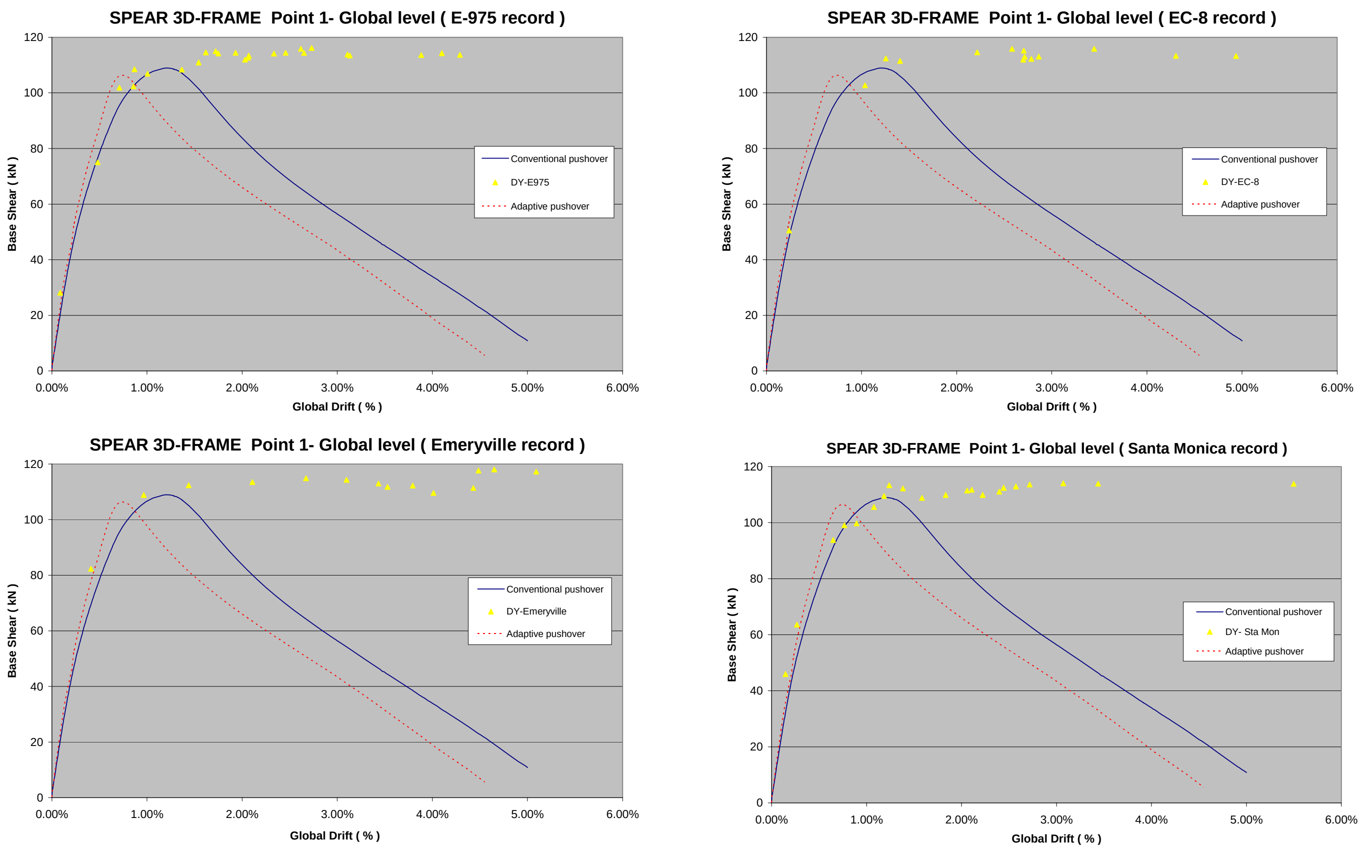


Figure 4.38 Response at the global level, SPEAR-3D frame

According to the trends observed in the 2D structures, when the structure starts to develop considerable damage which is not concentrated at its base, the pushover analyses have a tendency to underestimate the structural capacity and this observation is even more notorious in the adaptive pushover.

According to its characteristics, this structure is susceptible to experiment this behavior since it was designed according to outdated design codes, and with no seismic design provisions. Weak concrete and smooth bars were used, and additionally it had heavily imbalanced stiffness in the two orthogonal directions as well as large eccentricities in plan and irregularity in elevation that coupled produce high sensitivity to the torsional effects.

Keeping in mind these considerations the following observations about the performance of the pushover techniques for this irregular 3D structure can be made :

- Despite both pushover approaches show a strong process of structural stiffness degradation in the inelastic range, it is interesting to observe that with the adaptive procedure, the structural peak resistance is reached for a lower level of deformation at the global level. Additionally the graphs illustrate that with conventional pushover the transition between the elastic and the inelastic range is slightly more gradual. These observations imply that a higher concentration of forces is present at the damage locations when the adaptive procedure is employed, confirming the fact that when the structural damage is spread irregularly in the structure, the performance of the pushover techniques is diminished.

- As was mentioned for the irregular 2D structures, the methodology used to plot the points that represent the results of the dynamic analyses makes difficult to visualize a process of stiffness degradation since the maximum force is plotted against the maximum displacement despite they don't occur at the same time. However according to the results of the dynamic analyses the pushover approaches seems to predict in a good way the maximum capacity of the structure at the global level.

- The inclusion of torsional effects in the analysis increment the structural damage, generating higher alterations in the stiffness matrix and hence producing problems of damage concentration and for the lateral load vector updating in the adaptive procedure. Fajfar (2002) has reported the problems in the application of the pushover technique to asymmetric and torsionally flexible structures. The use of effective eccentricities has been suggested in order to improve the results of the pushover analysis. In this work, the lateral loads were applied in a conventional way since the main objective is to measure the improvement in the results produced by the adaptive technique. However, this is an issue that requires further analysis.

Finally, it should be pointed out that the behavior observed at the global level for this 3D highly irregular structure, indicates that the adaptive pushover doesn't improve the accuracy in the estimation of the structural response.

4.5.2. First story level

At the first story level, differences between the conventional pushover and the dynamic analyses are between 10.8 % and 13.4 %, and for the adaptive pushover differences oscillate from 12.6 % to 15.2 %.

As can be inferred from these comparisons, and confirmed with figure 4.39, the adaptive pushover does not introduce any significant improvement in the estimation of the response at the first story level.

There is almost no difference between the results obtained with conventional pushover approach and the ones calculated with the adaptive technique. It is important to observe that contrary to the response at the global level, at the first story level the structure doesn't show any brittle behavior according to the pushover results.

However, the closest coincidence in the results of the pushover approaches suggest that the lateral load vector is not updated anymore just after the initial stages of the inelastic response and this is produced by the presence of a brittle failure mechanism which generates negative terms in the stiffness matrix.

The graphs presented in the figure 4.39 illustrate how in this case the adaptive pushover produces slightly more conservative results than the conventional one. On the other hand, despite that there is not a good correspondence between the dynamic analyses and pushover approaches in the inelastic range, the peak resistance predicted with the pushover techniques is similar to the maximum solicitations calculated with the dynamic analyses.

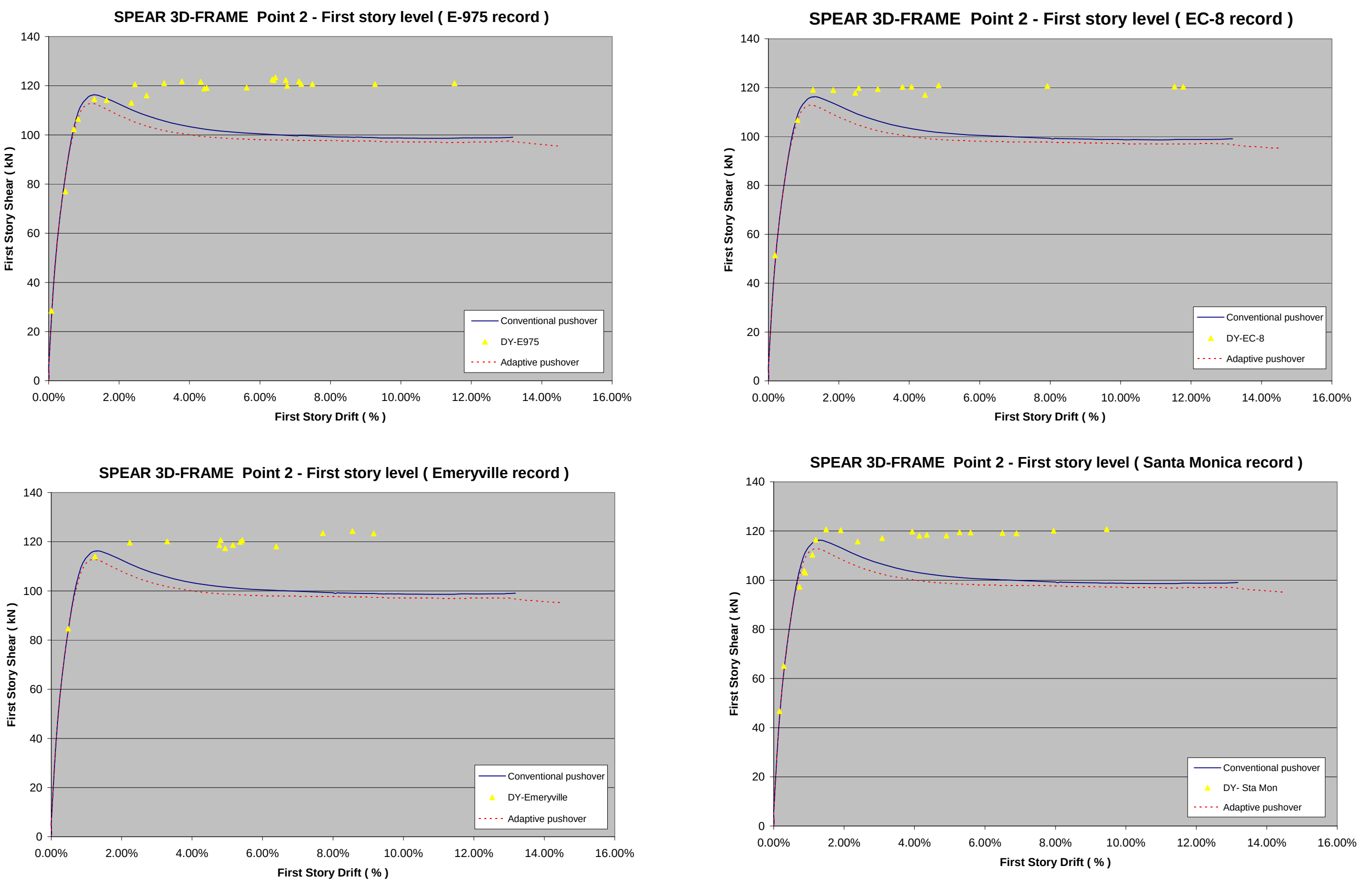


Figure 4.39 Response at the first story level, SPEAR-3D frame

4.5.3. Middle story level

It is clear form the figure 4.40 than at the middle story the adaptive pushover totally fails to estimate the dynamic structural response, and in fact the conventional pushover provides a more acceptable estimation in terms of strength and ductility for all the considered ground motions.

The differences between the dynamic analysis and the conventional pushover oscillate from 4.3 % to 12.0 %, while for the adaptive technique the differences are between 1.6 % and 13.6 %.

Nevertheless, in this case the lower differences in the adaptive procedure are produced by the lack of comparison points, and for this reason they don’t represent an improvement in the results accuracy.

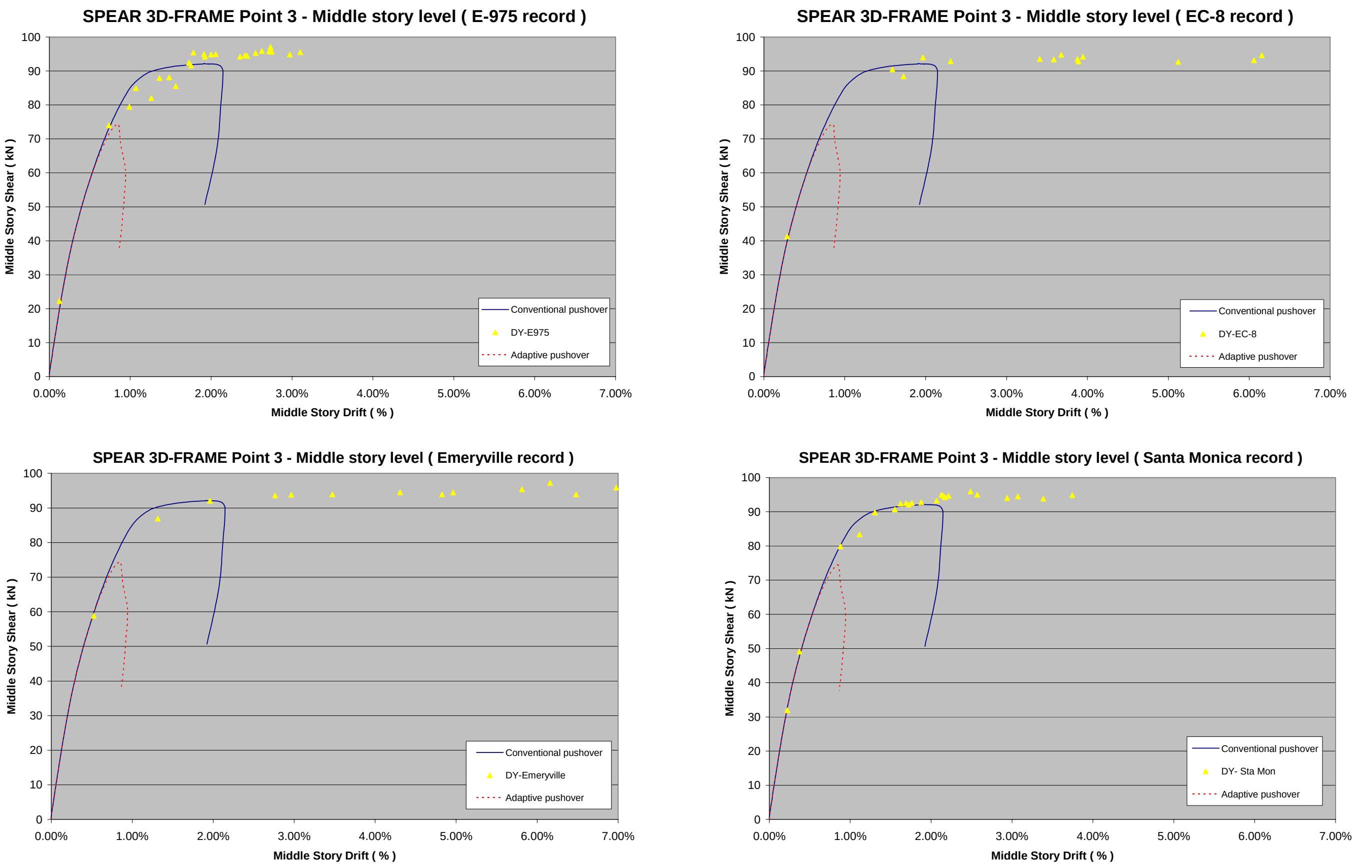


Figure 4.40 Response at the middle story level, SPEAR-3D frame

In this structure the bad performance of the adaptive procedure is obvious becausen it completely underestimates the structural capacity at this level. Additionally, this behavior explains why the conventional pushover is always showing better results than the adaptive one for this structure, since as can clearly be observed, a brittle failure is produced for very low deformation levels. This explains why the adaptive technique consistently shows more conservative results and why the lateral load vector is not updated anymore after the very early stages of the inelastic range as has been already mentioned.

Here it is important to say that the results presented in the figure 4.40 correspond to the second story in the middle frame of the building. Considering that this structure is subjected to torsion solicitations due to its stiffness imbalance, even worst results should be obtained in the frames located in the perimeter of the building. In order to verify the concentration of forces at the middle story level that seems to occur in the adaptive procedure and the variation of the lateral loads in the frames of the building the figure 4.41 was produced.

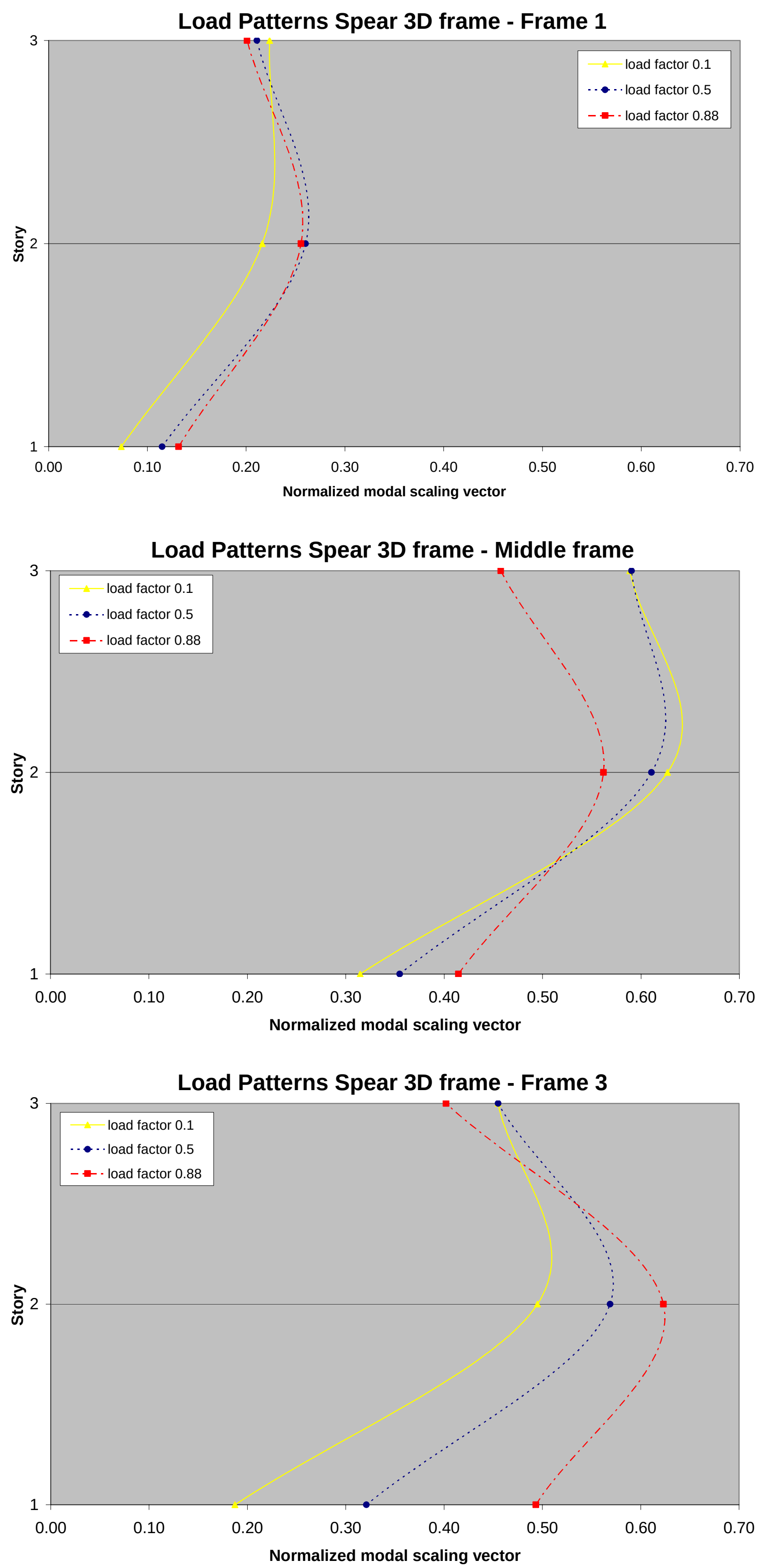


Figure 4.41 Variation on the modal scaling vector, for different load factors in the SPEAR building frames.

In this figure the variation in the normalized modal scaling vector for several load factors in each one of the frames of the structure is presented. The frame 1 corresponds to the one at the front of the building.

It is obvious that the influence of torsion is evident when the load distributions in the frames 1 and 3 are compared. However, it seems that the adaptive pushover procedure overestimates the forces applied to the middle frame, since they are even higher at the initial load stages than the ones applied in the frame 3, which should have higher solicitations produced by torsion effects. This observation confirms why further research is required in order to determine the optimum point to apply the lateral loads when asymmetric 3D structures are evaluated using simplified static procedures

4.5.4. Section level

Observing the graph 4.42, it is clear that the conventional pushover produce better results than the conventional one, especially in the estimation of the structural peak response and in the initial stages of the inelastic range.

For the conventional pushover the differences compared to dynamic analysis vary from around 9.3 % to 13.2 % while for the adaptive pushover, differences oscillate between 13.4 % and 19.3 %.

Observing these results it is clear that for this structure the adaptive procedure underestimates the structural capacity. As has been already pointed out, since the moment–curvature relation is an intrinsic property of the section (under the same axial load) , these results suggest that in this case higher level of axial loads are present in the adaptive procedure.

This is the first time that the conventional pushover produced consistently better results than the conventional one at the section level.

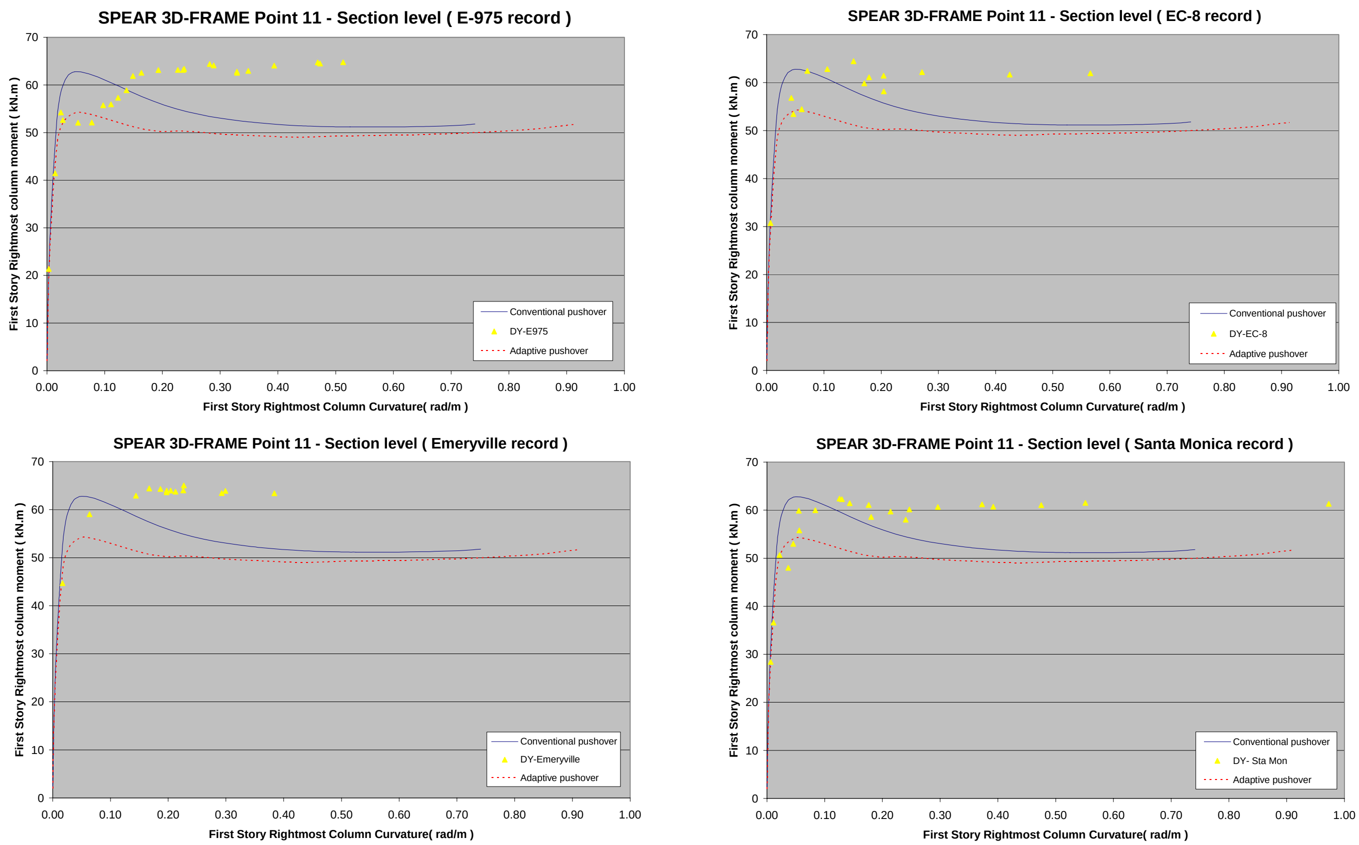


Figure 4.42 Response at the section level, SPEAR-3D frame

Finally it is important to note that despite the adaptive pushover analysis doesn't improve the quality of the results, the conventional pushover still gives a good estimation of the section maximum capacity.

Finally, figure 4.43 summarizes the difference values presented in this section for the SPEAR frame.

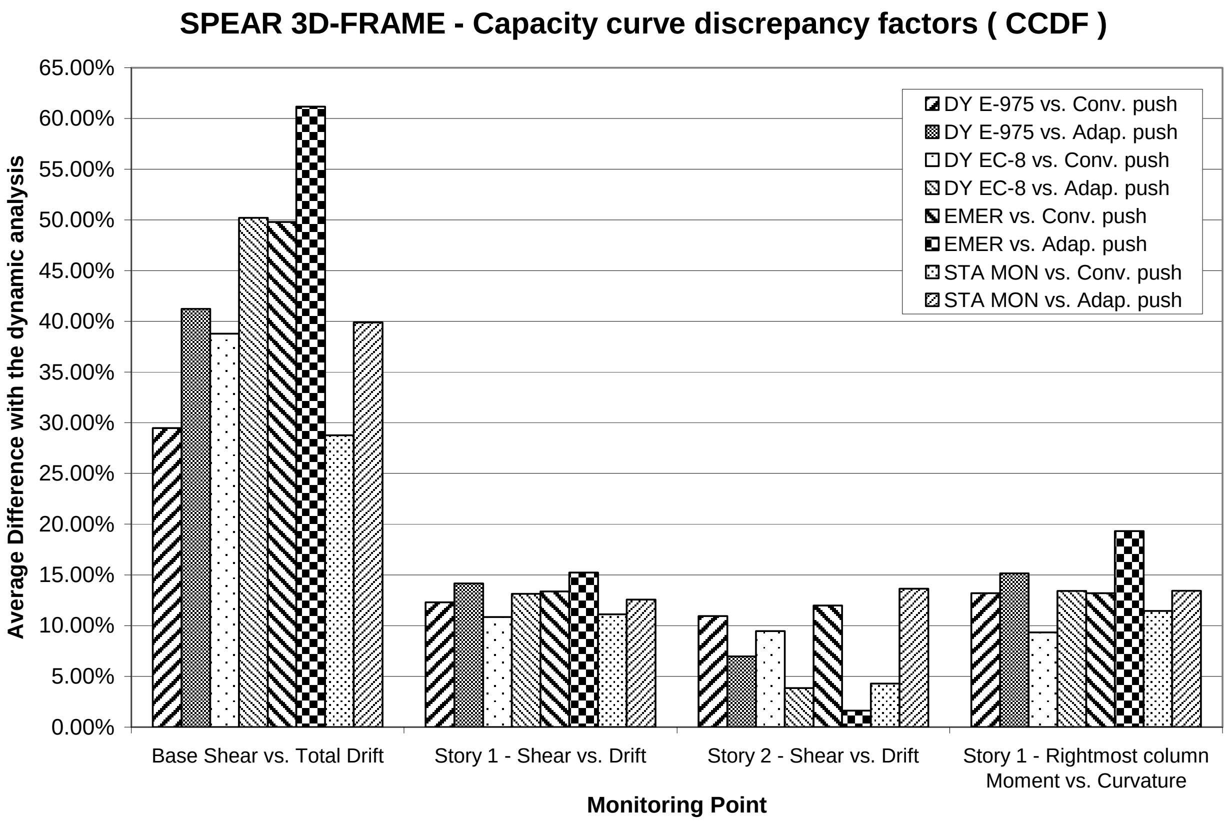


Figure 4.43 Results summary, SPEAR-3D frame structure

5. EVALUATION OF RESULTS

In this chapter, the already presented analysis results are evaluated in a summarized manner and the most important trends are outlined. Specifically, an overall comparison among the complete set of analyzed structures is included for each one of the monitored response parameters.

5.1 Response at the global level

Figure 5.1 shows an overall comparison of the difference (CCDF) values between pushover and dynamic analysis at the global level, for each one of the considered structures.

It is obvious that for bi-dimensional structures, adaptive pushover produces in general slightly better results than the conventional one (lower CCDF values), especially for the frame-wall structures.

It is also interesting to observe that when the structure is highly irregular (ICONS frame), the improvement in the results offered by the adaptive approach is minimal, or completely inexistent. This reduction in the performance of adaptive pushover in high irregular structures is attributed to the brittle failure mechanisms generated at the middle story level by the adaptive procedure.

For the 3D SPEAR structure, both pushover techniques are unable to predict the response at the global level with reasonable accuracy. In fact the adaptive pushover shows a worse performance than the conventional one. The effects of excessive force concentration at the middle level in the adaptive pushover approach seem to be the reason of this behavior.

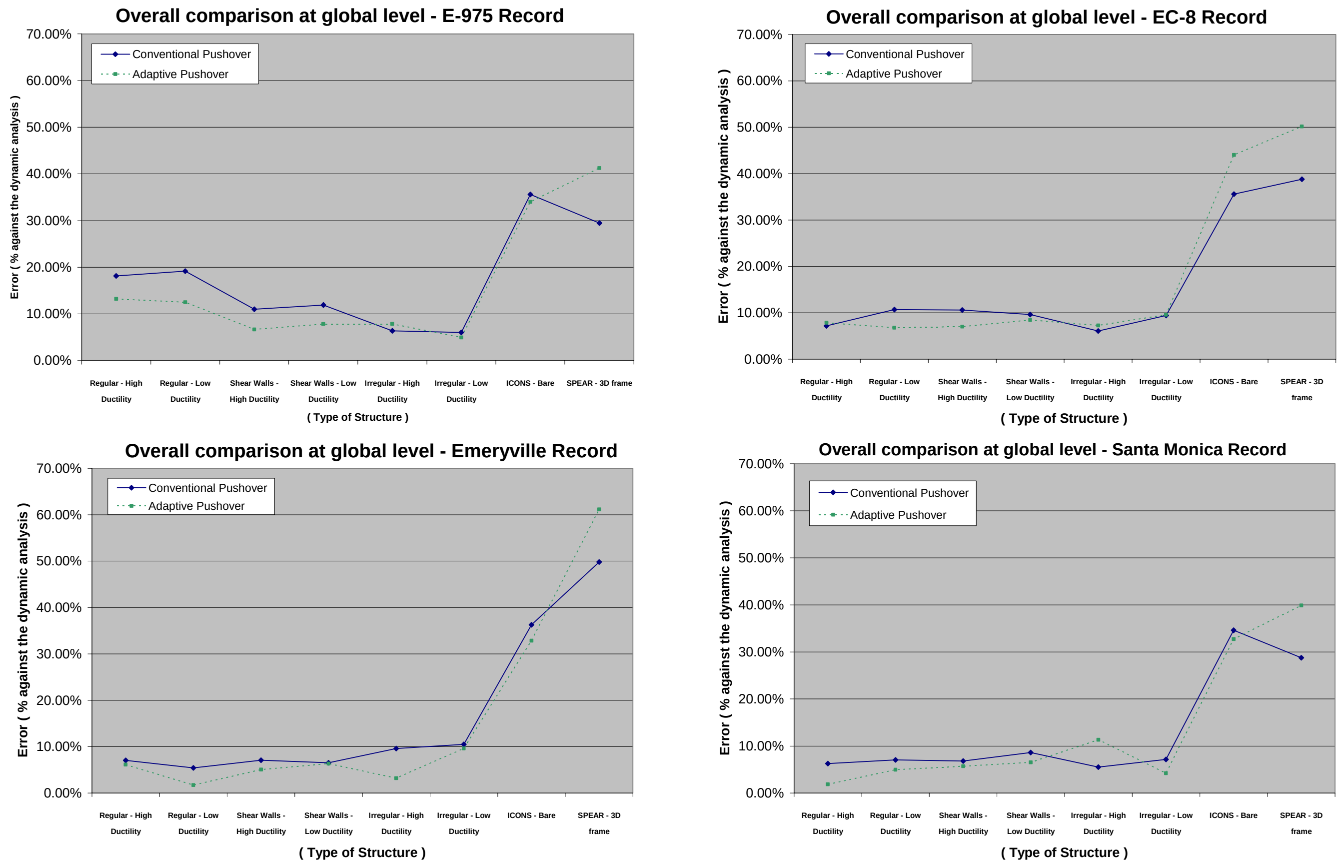


Figure 5.1 Overall comparison of the response at the global level

Additionally, as was already pointed out in section 4.5, it is necessary to improve the results of pushover analysis in asymmetrical 3D structures, by using eccentricities in the application of the lateral forces in order to represent the torsional effects in a more realistic way.

Finally, it is important to highlight the fact that the adaptive procedure improved the results at the initial stages of the inelastic response. However, for higher inelastic deformations, the adaptive pushover becomes similar to a conventional one, since the program is not able to find real solutions for the eigenvalue problem due to changes in the structural stiffness.

5.2 Response at the first story level

In the assessment of the structural response at the first story level, figure 5.2 confirms that the same observations and trends that were drawn for the response at the global level also apply in this case.

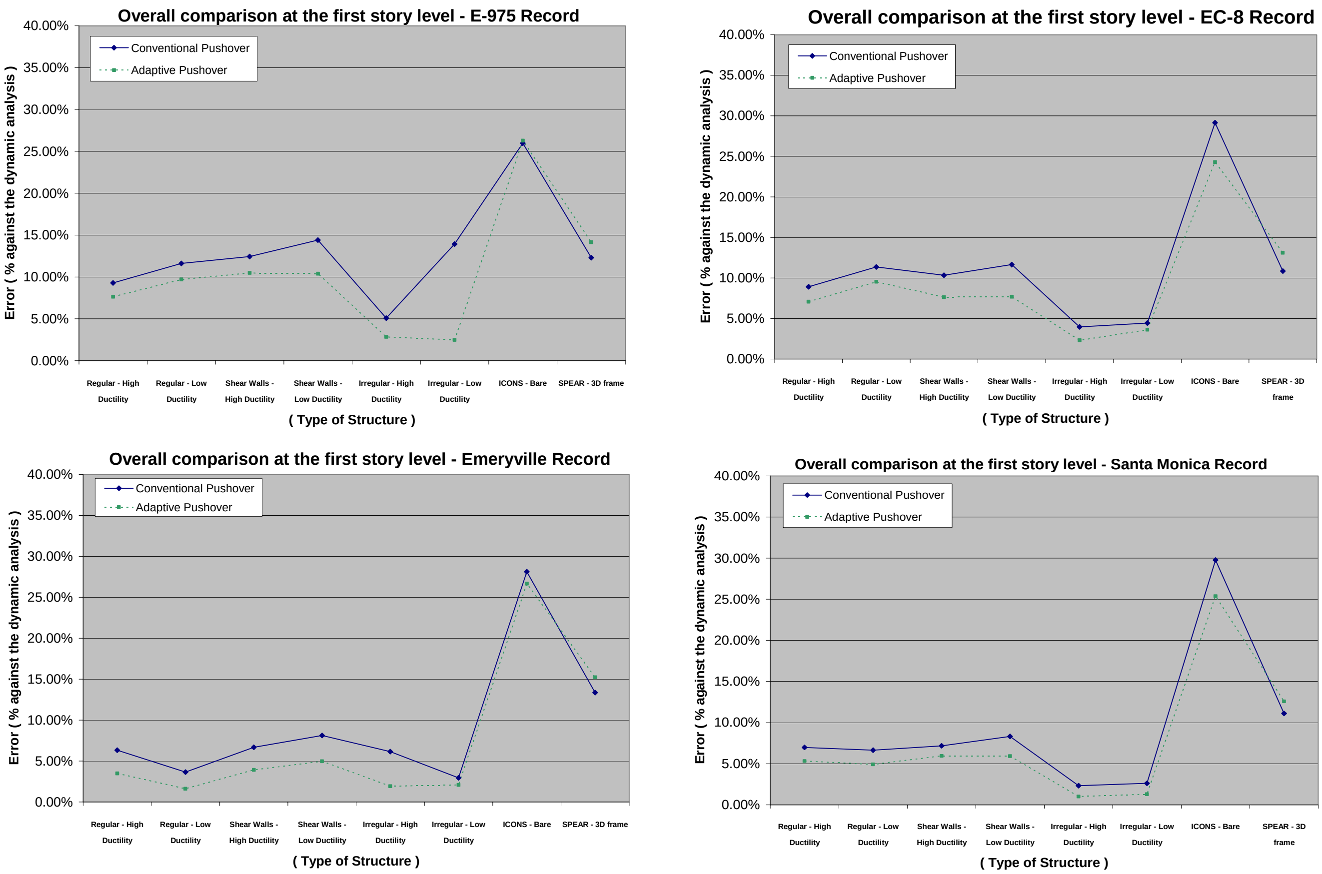


Figure 5.2 Overall comparison of the response at the first story level

It is obvious again, that for regular structures, the adaptive pushover produces slightly better results. However, for the highly irregular structures (ICONS and SPEAR), any benefits from applying the adaptive pushover approach disappear. This implies that the updating of the lateral load vector in the adaptive process stops almost immediately after the structure reaches its peak resistance, due to the presence of a brittle failure mechanism that degenerates the stiffness matrix introducing problems in the solution of the eigenvalue problem.

5.3 Response at the middle story level

The main drawback in the performance of adaptive pushover was detected at the middle story level. With the exception of the frame-wall structures, for all the other analyzed cases the adaptive pushover was extremely conservative in the estimation of the structural response producing an exaggerate concentration of damage at this level.

The figure 5.3 shows a summary of the obtained results in the assessment of the response at the middle level.

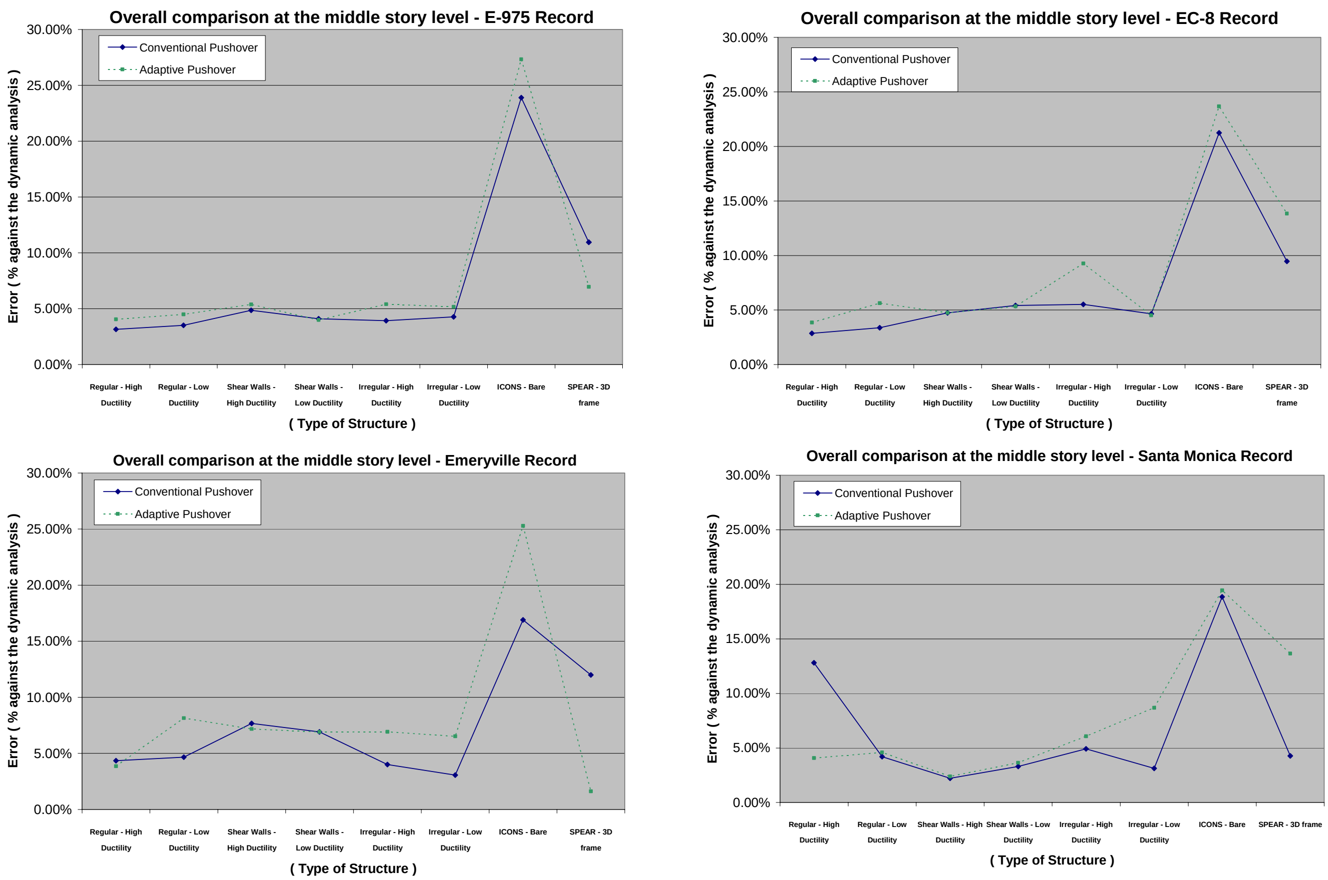


Figure 5.3 Overall comparison of the response at the middle story level

This problem has also been reported by Antoniou and Pinho (2004) and is mainly produced by the difficulties faced by the adaptive procedure to model the sign reversal in the modal load vectors. For this reason, more refined methods to combine the modal forces are required in order to improve the performance of adaptive pushover analysis.

5.4 Response at the section level

Figure 5.4 shows a summary of the response at the section level for all the considered structures.

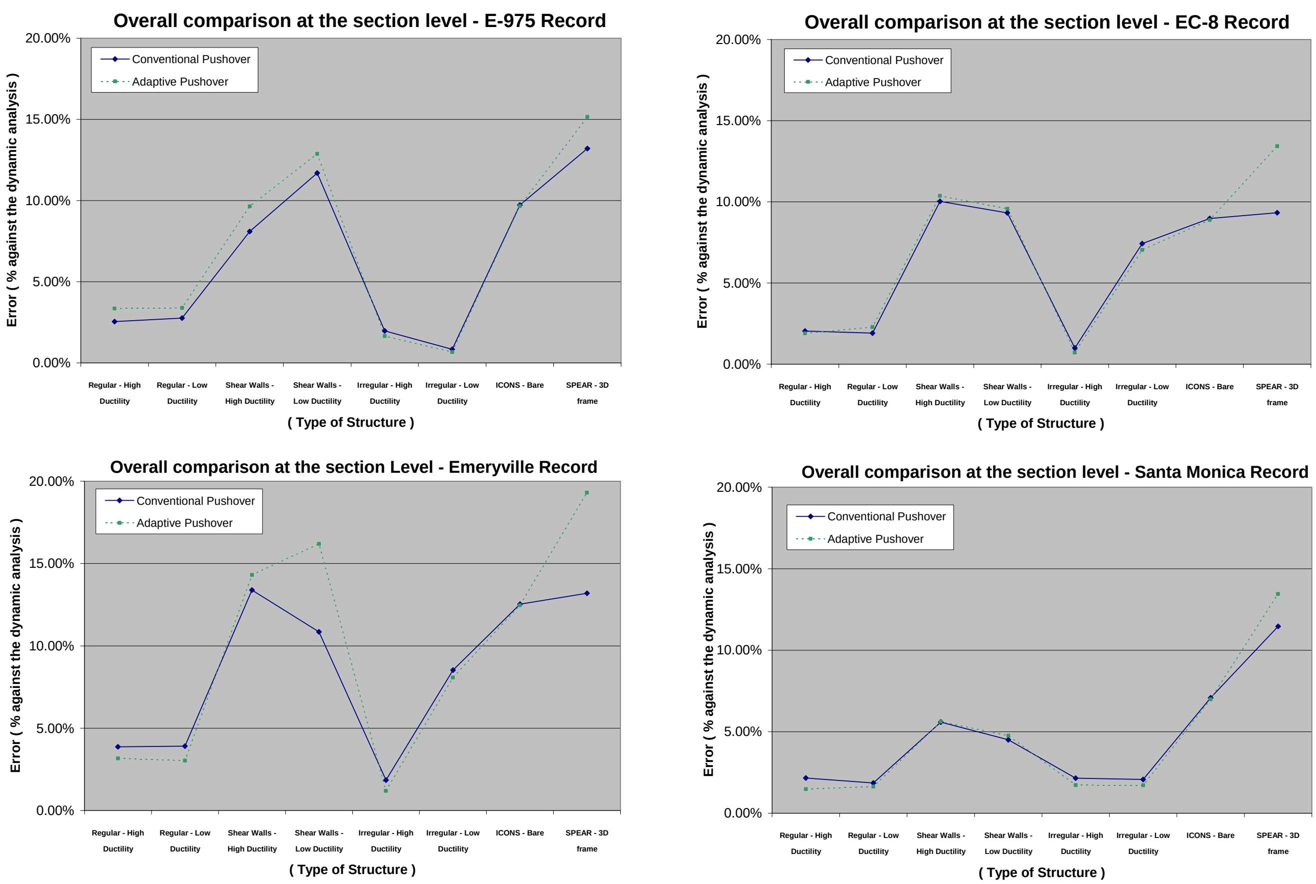


Figure 5.4 Overall comparison of the response at the section level

From the above plots it is observed that there is practically no gain from applying the adaptive pushover procedure when the section behavior is evaluated. Again, the difference between pushover and dynamic analysis is lower for regular structures with the exception of the frame-wall structures where the dynamic response underestimates the section capacity.

Nevertheless, it is important to point out that at the section level, the differences between the dynamic analyses and the pushover curves are expected to be minimal, due to the fact that the moment-curvature relationship is an intrinsic characteristic of a reinforced concrete section under the same magnitude of axial load. Hence, since both pushover approaches produce basically the same results for each one of the considered structures and predict almost the same peak section capacity compared to the dynamic analysis (confirming that the static procedures are an adequate tool to estimate the structural capacity) the differences quantified through the CCDF are generated basically by the methodology used to collect the points that represent the results of the dynamic analyses, using the maximum moment against the maximum curvature. In order to clarify this concept, in figure 5.5 the moment-curvature curve for the first story rightmost column of the ICONS bare frame has been complemented with results of incremental dynamic analysis using the maximum moment versus its corresponding curvature and the maximum curvature versus its corresponding moment.

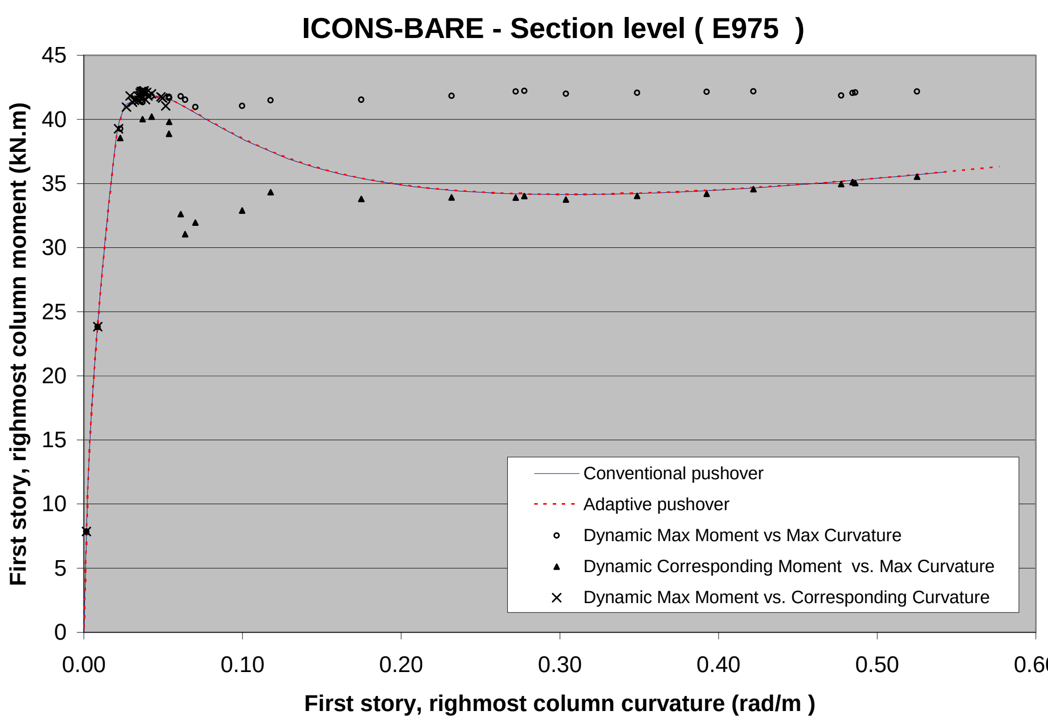


Figure 5.5 Comparison between several methodologies to plot the dynamic analyses results

From this graph it is clear that the absolute maxima approach is not always adequate, and hence it is an issue of further discussion and research.

6. CONCLUSIONS

The main conclusions obtained from the present study are presented below :

- The overall assessment of the extensive results from this study indicates that the adaptive pushover in general does not provide major advantages over the traditional methodology.

- Despite the apparent enhancement in the assessment of the response at the global level from adaptive pushover results, it is also clear that it presents serious deficiencies in the estimation of the structural response at the local level, especially for the intermediate stories. Careful assessment of the local response is necessary before an assessment method is used with confidence; global results comparisons may be misleading.

- It is clear that the adaptive procedure presents a problem of excessive force concentration at the locations of the structure where the damage is concentrated. This is a consequence of the approach used to combine the modal forces using the SRSS or CQC methodologies, since the sign reversals in the load vectors are not included. Hence, more refined methodologies are required to compute the normalized scaling load vector.

- In the adaptive pushover procedure updating of the lateral load vector is directly related to the calculation of the vibration modes for each of the load steps. Therefore, when the structure reaches very high inelastic deformations or presents a brittle failure the benefits generated with the application of the adaptive methodology are eliminated because the load vector is no longer updated, since the mode shapes include imaginary components.

- The results obtained for the 3D SPEAR frame show that both pushover techniques still require further refinement in order to provide reliable estimates of the dynamic response of 3D asymmetrical structures. Torsional effects are not adequately represented in pushover analysis.

- The methodology used to plot the dynamic (pushover) analysis results using temporally coincident maxima of force/moment and deformation/curvature yields in general a conservative estimate of the capacity. It has, however, drawbacks that generate difficulties in the comparisons with the results of the pushover curves. The option of comparing the results of the pushover curves with those of the dynamic analyses plotted using the maximum force/moment versus the corresponding displacement/curvature and with the force/moment corresponding to the maximum displacement/curvature (as suggested in Section 5.4) provides a better understanding of the differences between the static and the dynamic procedures.

- The Capacity Curve Discrepancy Factor (CCDF) used in this work should be considered as a first approach to systematically quantify the differences between the results of dynamic and pushover analyses, for the reasons explained in the body of the report.

- Further research is required in order to validate the adaptive and other forms of advanced pushover techniques as a viable (or even enhanced) tool to replace the estimation of the dynamic response through inelastic time-history analyses.

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7. REFERENCES

Antoniou, S., Pinho, R. (2004). Advantages and limitations of adaptive and non adaptive force-based pushover procedures. *Journal of Earthquake Engineering* Vol 8, No. 4 (2004), pp. 497 – 522.

Antoniou, S. (2003) Pushover Analysis for Seismic Design and Assessment of RC Structures, *PhD Thesis*, Engineering Seismology and Earthquake Engineering Section, Imperial College, London, UK.

Bathe, K.J. (1982), "Finite Element Procedures in Engineering Analysis", Prentice-Hall Inc.

Bracci, J.M, Reinhorn A.M. and Mander J,B (1995a) Seismic resistance of reinforced concrete frame structures designed for gravity loads : Performance of structural system, *ACI Structural Journal*, Vol. 92, No. 5, 597-609.

Bracci, J.M, Reinhorn A.M. and Mander J,B (1995b) Seismic retrofit of reinforced concrete buildings designed for gravity loads : Performance of structural model, *ACI Structural Journal*, Vol. 92, No. 6, 711-723.

Bracci, J.M., Kunnath, S.K. and Reinhorn, A.M. (1997) Seismic Performance and Retrofit Evaluation of RC Structures, *Journal of Structural Engineering*, 123, 3-10.

Chopra, A.K. (1995) Dynamics of Structures; Theory an applications to Earthquake Engineering, Prentice Hall, New Jersey 1995.

Elnashai A.S, Papanikolaou V. and Lee D.H (2002-2004) Zeus-NL - A Program for Inelastic Dynamic Analysis of Structures – User Manual, Mid-America Earthquake Center, University of Illinois at Urbana-Champaign.

Elnashai, A.S. (2000) Advanced Inelastic Static (Pushover) Analysis for Seismic Design and Assessment, *G.Penelis International Symposium on Concrete and Masonry Structures* (Thessaloniki, 2000).

Elnashai, A.S. (2002) Do we really need inelastic dynamic analysis ?, *Journal of Earthquake Engineering*, Vol. 6, Special Issue 1, pp. 123-130.

Elnashai, A.S., Pinho, R. and Antoniou, S. (2000) INDYAS - A Program for Inelastic Dynamic Analysis of Structures, *ESEE Research Report*, Imperial College, London.

Faella, G. (1996) Evaluation of RC structures seismic response by means of nonlinear static pushover analyses, *11th World Conference on Earthquake Engineering*, Acapulco, Mexico, Paper no. 1146, 8 pp.

Fajfar, P. (2002) Structural analysis in earthquake engineering – a breakthrough of simplified non linear methods, *12th European Conference on Earthquake Engineering*, Paper reference 843.

Fajfar, P. and Fischinger, M. (1988) N2 - Method for Nonlinear Seismic Analysis of Regular Structures, *Proceedings of the Ninth World Conference on Earthquake Engineering* (Tokyo-Kyoto, Japan, 1988), Vol. 5, pp. 111-116.

Fardis, M.N. (1994) Analysis and Design of Reinforced Concrete Buildings According to Eurocodes 2 and 8. Configuration 3, 5 and 6, *Reports on Prenormative Research in Support of Eurocode 8*.

Gulkan, P. and Sozen, M.A. (1974) Inelastic response of reinforced concrete structures to earthquake motions, *ACI Journal*, Vol. 71, pp. 604–610.

Izzudin, B.A. and Elnashai, A.S. (1989) ADAPTIC - A Program for Adaptive Large Displacement Elastoplastic Dynamic Analysis of Steel, Concrete and Composite Frames , *ESEE Research Report*, No. 89-7, Imperial College, London.

Kim, S. and D'Amore, E. (1999) Push-over analysis procedures in earthquake engineering, *Earthquake Spectra*, Vol. 15, No. 3, August, pp. 417-434.

Krawinkler H. (1995) New trends in seismic design methodology *Proceedings 10th ECEE*, The Netherlands, Rotterdam, pp. 821–830.

Krawinkler, H. and Seneviratna, G.D. (1998) Pros and cons of pushover analysis of seismic performance evaluation, *Engineering Structures*, Vol. 20, No.4-6, pp. 452-464.

Kunnath, S.K., Reinhorn, A.M. and Lobo, R.F. (1992) IDARC Version 3.0 – a program for inelastic damage analysis of reinforced concrete structures, National Centre for Earthquake Engineering Research Technical Report no. NCEER-92-0022, State University of New York at Buffalo.

Lawson, R.S., Vance, V. and Krawinkler, H. (1994) Nonlinear static pushover analysis, why, when and how ?, *Proceedings of 5th US National Conference on Earthquake Engineering*, Chicago, vol. 1, pp. 283-92.

Lefort, T. (2000), Advanced Pushover Analysis of RC Multi-story Buildings, MSc dissertation, Engineering Seismology and Earthquake Engineering Section, Imperial College, London, UK.

Mwafy, A.M. and Elnashai, A.S. (2000) Static Pushover versus Dynamic Collapse Analysis of RC Buildings, *Journal of Engineering Structures*, 23, pp. 407-424, 2001.

Papanikolaou, V. (2000) Development and Verification of Adaptive Pushover Analysis Procedures, MSc dissertation, Engineering Seismology and Earthquake Engineering Section, Impeial College, London, UK.

Pinho, R. and Elnashai, A.S. (2001) Dynamic Collapse Testing of a Full-Scale Four Story RC frame, *ISET Journal of Earthquake Technology*, Vol. 37, No. 4, Special Issue, pp. 143-164.

Pinho, R. (1999) Selective Repair and Retrofitting of RC Structures using Selective Techniques, Ph.D. Thesis, Imperial College, University of London, London.

Sasaki, K.K., Freeman, S.A. and Paret, T.F (1998) Multimodal Pushover Procedure (MMP) - A Method to Identify the Effects of Higher Modes in a Pushover Analysis, *Proceedings of the Sixth US National Conference on Earthquake Engineering* (Oakland, California, 1998) [computer file], Earthquake Engineering Research Institute, 12 pages.

Saiidi, M and Sozen, M.A. (1981) Simple nonlinear analysis of RC structures, *ASCE, ST Division*, Vol. 107, No. ST5, pp. 937-951.

Tso W.K., Moghadam A.S. (1998) Pushover procedure for seismic analysis of buildings, *Progress in Structural Engineering and Materials*, Vol. 1, No.3, pp.337-344.