Preliminary Seismic Vulnerability Assessment of Existing Reinforced Concrete Buildings in Turkey

Part I: Statistical Model Based on Structural Characteristics

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Abstract: The 1999 earthquakes caused huge damage and economic losses in Turkey. The city of Düzce, hit by the second earthquake of Mw=7.2, suffered widespread damage to many RC buildings. Survey teams conducted post-earthquake evaluations on selected buildings that suffered various degrees of damage. The information collected was analyzed to set up a correlation between the attributes affecting seismic performance and the observed damage. A procedure, developed using a statistical method called discriminant analysis, is presented. The details of the procedure and the content of the database are summarized. The variability of ground motion with respect to the soil properties and the distance to source was incorporated in the improved procedure presented in the companion paper [1].

Key words: Seismic vulnerability, discriminant analysis, reinforced concrete, Düzce earthquake, damage score

1. INTRODUCTION

Up to date procedures on the vulnerability assessment of building structures have primarily focused on the structural system, capacity, layout and response parameters [2-12]. These parameters would provide realistic estimates of the expected performance if the built structural system reflects the prescribed structural and architectural features. In general, the construction practice in Turkey is far beyond reflecting designed structural system, thus violating all assumptions of the usual vulnerability assessment procedures. For this reason, statistical analysis based on the observed damage and significant building attributes would provide more reliable and accurate results for regional assessments. In this context, discriminant analysis technique was used to develop a preliminary evaluation methodology for assessing seismic vulnerability of existing low- to mid-rise reinforced concrete buildings. The main objective is to identify the buildings that are highly vulnerable to damage, that is the seismic performance is inadequate to survive a strong earthquake. Hence, the damage scores obtained from the derived discriminant functions are used to classify existing buildings as “safe”, “unsafe” and “intermediate”. The discriminant functions are generated based on the basic damage inducing parameters, namely number of stories (n), minimum normalized lateral stiffness index (mnlstfi), minimum normalized lateral strength index (mnlsi), normalized redundancy score (nrs), soft story index (ssi) and overhang ratio (or).

The building damage database used in this study contains 484 buildings, which were evaluated by the survey teams after the 1999 Düzce earthquake. The building inventory was formed entirely by low- to mid-rise reinforced concrete buildings. Figure 1 shows the classification of these buildings according to the number of stories and the observed damage. The observed damage states were determined based on the descriptions given in Table 1.

Table 1. Description of damage states

	STRUCTURAL ELEMENTS	NON-STRUCTURAL ELEMENTS
None	No visual sign of damage	No visual sign of damage
Light	Hairline inclined or flexural cracks	Hairline cracks in walls. Flaking of plaster.
Moderate	Concrete spalling	Cracking in walls and joints between panels. Flaking of large pieces of plaster
Severe	Local structural failure	Wide and through cracks in walls
Collapse	Local or total collapse	Crushing of walls or out-of-plane toppling of walls

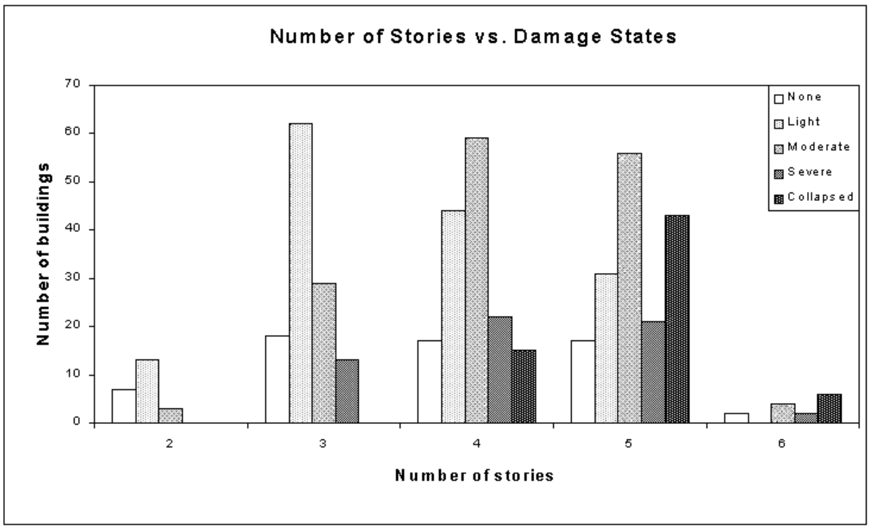

Figure 1. Classification of building data

The description of these parameters and the derivation of discriminant functions are presented in the sections that follow.

This paper serves as the companion paper two the one given in Part II. Therefore, inclusion of the background information on the proposed assessment methodology is given here to provide basis for the improvements that are introduced in the second part.

2. DEFINITION OF THE DAMAGE INDUCING PARAMETERS

In the determination of the estimation variables to be used in the analysis, the basic assumption is that all of the buildings involved in the inventory are exposed to a specific earthquake. In other words, each building stock in itself has faced the same ground motion properties, thus the damage will be evaluated only on the basis of structural responses rather than including the excitation parameters. Considering the characteristics of the damaged structures and the huge size of the existing building stock, the following parameters were chosen as the basic estimation parameters of the proposed method:

i. number of stories (n),
ii. minimum normalized lateral stiffness index (mnlstfi),
iii. minimum normalized lateral strength index (mnlsi),
iv. normalized redundancy score (nrs),
v. soft story index (ssi),
vi. overhang ratio (or).

These parameters are briefly defined in the following paragraphs.

i. Number of stories (**N**): This is the total number of individual floor systems above the ground level.

ii. Minimum normalized lateral stiffness index (**MNLSTFI**): This index is the indication of the lateral rigidity of the ground story, which is usually the most critical story. If the story height, boundary conditions of the individual columns and the properties of the materials used are kept constant, this index would also represent the stiffness of the ground story. This index is calculated by considering the columns and the structural walls at the ground story. While doing this, all vertical reinforced concrete members with "maximum cross-sectional dimension / minimum cross-sectional dimension ratio" less then 7 are considered as columns. All other reinforced concrete structural members are considered as structural walls. The MNLSTFI parameter shall be computed based on the following relationship:

$$\mathrm{MNLSTFI} = \min\left(\mathrm{I_{nx}}, \mathrm{I_{ny}}\right) \tag{1}$$

$\mathrm{I_{nx}}$ and $\mathrm{I_{ny}}$ values in Eq.(1) are to be calculated by using Eq.(2).

$$\begin{aligned} \mathrm{I_{nx}} &= \frac{\sum (\mathrm{I_{col}})_x + \sum (\mathrm{I_{sw}})_x}{\sum \mathrm{A_f}} \times 1000 \\ \mathrm{I_{ny}} &= \frac{\sum (\mathrm{I_{col}})_y + \sum (\mathrm{I_{sw}})_y}{\sum \mathrm{A_f}} \times 1000 \end{aligned} \tag{2}$$

where;

$\Sigma(\mathrm{I_{col}})_x$ and $\Sigma(\mathrm{I_{col}})_y$: summation of the moment of inertias of all columns about their centroidal x and y axes, respectively.

$\Sigma(\mathrm{I_{sw}})_x$ and $\Sigma(\mathrm{I_{sw}})_y$: summation of the moment of inertias of all structural walls about their centroidal x and y axes, respectively.

$\mathrm{I_{nx}}$ and $\mathrm{I_{ny}}$: total normalized moment of inertia of all members about x and y axes, respectively.

$\Sigma\mathrm{A_f}$: total story area above ground level.

iii. Minimum normalized lateral strength index **(MNLSI)**: The minimum normalized lateral strength index is the indication of the base shear capacity of the critical story. In the calculation of this index, in addition to the existing columns and structural walls, the presences of unreinforced masonry filler walls are also considered. While doing this, unreinforced masonry filler walls are assumed to carry 10 percent of the shear force that can be carried by a structural wall having the same cross-sectional area [8, 11, 12]. As in MNLSTFI calculation, the vertical reinforced members with a cross-sectional aspect ratio of 7 or more are classified as structural walls. The MNLSI parameter shall be calculated by using the following equation:

$$\mathrm{MNLSI} = \min\left(\mathrm{A_{nx}}, \mathrm{A_{ny}}\right) \tag{3}$$

where;

$$A_{nx} = \frac{\sum (A_{col})_x + \sum (A_{sw})_x + 0.1 \sum (A_{mw})_x}{\sum A_f} \times 1000$$
$$A_{ny} = \frac{\sum (A_{col})_y + \sum (A_{sw})_y + 0.1 \sum (A_{mw})_y}{\sum A_f} \times 1000 \tag{4}$$

For each column with a cross-sectional area denoted by A_{col}:

$$(A_{col})_x = k_x \cdot A_{col}$$
$$(A_{col})_y = k_y \cdot A_{col} \tag{5}$$

where [11];
k_x=1/2 for square and circular columns;
k_x=2/3 for rectangular columns with $b_x > b_y$;
k_x=1/3 for rectangular columns with $b_x < b_y$; and
$k_y = 1 - k_x$.

For each shear wall with cross-sectional area denoted by A_{sw}:

$$(A_{sw})_x = k_x \cdot A_{sw}$$
$$(A_{sw})_y = k_y \cdot A_{sw} \tag{6}$$

where;
k_x=1 for structural walls in the direction of x-axis;
k_x=0 for structural walls in the direction of y-axis; and
$k_y = 1 - k_x$.

For each unreinforced masonry filler wall with no window or door opening and having a cross-sectional area denoted by A_{mw}:

$$(A_{mw})_x = k_x \cdot A_{mw}$$
$$(A_{mw})_y = k_y \cdot A_{mw} \tag{7}$$

where;
k_x=1.0 for masonry walls in the direction of x-axis;
k_x=0 for masonry walls in the direction of y-axis; and
$k_y = 1 - k_x$.

iv. Normalized redundancy score **(NRS)**: Redundancy is the indication of the degree of the continuity of multiple frame lines to distribute lateral forces throughout the structural system. The normalized redundancy ratio (NRR) of a frame structure is calculated by using the following expression:

$$NRR = \frac{A_{tr}(nf_x - 1)(nf_y - 1)}{A_{gf}} \tag{8}$$

where;
A_{tr} : the tributary area for a typical column. A_{tr} shall be taken as 25 m^2 if nf_x and nf_y are both greater than and equal to 3. In all other cases, A_{tr} shall be taken as 12.5 m^2.
nf_x, nf_y : number of continuous frame lines in the critical story (usually the ground story) in x and y directions, respectively.

A_{gf} : the area of the ground story, i.e. the footprint area of the building.

Depending on the value of NRR computed from Eq. (8), the following discrete values are assigned to the normalized redundancy score (NRS):

$\text{NRS} = 1$ for $0 < \text{NRR} \leq 0.5$
$\text{NRS} = 2$ for $0.5 < \text{NRR} \leq 1.0$
$\text{NRS} = 3$ for $1.0 < \text{NRR}$

v. Soft story index **(SSI)**: On the ground story, there are usually fewer partition walls than in the upper stories. This situation is one of the main reasons for soft story formations. Since the effects of masonry walls are included in the calculation of MNLSI, soft story index is defined as the ratio of the height of first story (i.e. the ground story), H_1, to the height of the second story, H_2.

$$\text{SSI} = \frac{H_1}{H_2} \tag{9}$$

vi. Overhang ratio **(OR)**: In a typical floor plan, the area beyond the outermost frame lines on all sides is defined as the overhang area. The summation of the overhang area of each story, $A_{overhang}$, divided by the area of the ground story, A_{gf}, is defined as the overhang ratio.

$$\text{OR} = \frac{A_{overhang}}{A_{gf}} \tag{10}$$

3. STATISTICAL MODEL

The effects of different parameters on seismic damage vary. In order to make a more rational and systematic evaluation of damage inducing parameters in the prediction of seismic vulnerability of structures, a statistical technique, known as discriminant analysis is adopted.

In the most general sense, earthquake damage to buildings is categorized into five levels, namely: none (N), light (L), moderate (M), severe (S) and collapse (C). Because of the nature of available damage data from the 1999 Düzce earthquake, it was necessary to combine the severe damage and collapse states into one group, denoted by (S+C). Furthermore, if none and light damage states are combined into one group, based on the fact that the distinction between these two damage states is not too crucial for vulnerability analysis, then there will be three different damage states, namely: (N+L), (M) and (S+C).

It is possible to evaluate structures at different performance levels according to different objectives. If the main concern is to identify the buildings that are severely damaged or collapsed, the first three damage states (i.e. N, L and M) can be considered as one group and the severely damaged state and collapsed cases as the other group, reducing the distinct damage states into two. Since the main objective is the identification of severely damaged and collapsed buildings for life safety purposes, this classification can be referred as “Life Safety Performance Classification” (LSPC). Similarly, if the main concern is to identify the structures which suffer no damage or light damage during an earthquake, the first two damage states (N and L) can be considered as one group and remaining damage states (M, S and C) as the other group, reducing the distinct damage states into two. This identification is named as “Immediate Occupancy Performance Classification” (IOPC) since the main concern is to identify the buildings that can be occupied immediately after a strong ground motion.

In the discriminant analysis method, first the set of estimation variables that provides the best discrimination among the groups is identified. These variables are known as the “discriminator variables". Then a "discriminant function", which is a linear combination of the discriminator variables, is derived. The values resulting from the discriminant function are known as "discriminant scores". The final objective of discriminant analysis is to classify future observations into one of the specified groups, based on the values of their discriminant scores.

The unstandardized estimate of discriminant function based on six damage inducing parameters is obtained for life safety performance classification by utilizing the SPSS [13] software and the database constituted after 1999 Düzce earthquake. Here, DILS denotes the damage index or the damage score corresponding to the LSPC and the other parameters are as described. The function given in Eq. (11) is referred to as the unstandardized discriminant function, because the unstandardized (raw) data are used for computing this discriminant function

$$DI_{LS}=0.620n-0.246mnlstfi-0.182mnlsi-0.699nrs+3.269ssi+2.728or-4.905 \tag{11}$$

In the case of immediate occupancy performance classification, the unstandardized discriminant function, where DI_{IO} is the damage score corresponding to IOPC, based on these variables is:

$$DI_{IO}=0.808n-0.334mnlstfi-0.107mnlsi-0.687nrs+0.508ssi+3.884or-2.868 \tag{12}$$

A convenient statistical parameter for interpreting the contribution of each variable to the formation of the discriminant function is the loadings or the structure coefficients [14]. The structure coefficient of a discriminator variable is merely the correlation coefficient between the discriminant score and the discriminator variable and the value will lie between +1 and –1. As the absolute value of the structure coefficient of a variable approaches to 1, the communality between the discriminating variable and the discriminant function increases, or vice versa. The structure coefficients that are obtained as an output from the SPSS software are shown in Table 2. Here the number of stories above the ground level (n) has the highest loading (0.738), indicating that it is the best discriminator variable in LSPC. In the case of IOPC, again the number of stories comes out as the best discriminator variable with the loading of 0.789 and the normalized redundancy score is the second best.

Table 2. Structure matrix for the cases of LSPC and IOPC

Variables	**Structure Coefficients**	
	LSPC	**IOPC**
n	+0.738	+0.789
nrs	-0.555	-0.594
mnlsi	-0.503	-0.481
ssi	+0.418	+0.092
or	+0.167	+0.284
mnlstfi	-0.076	-0.085

4. CLASSIFICATION METHODOLOGY

In the proposed classification methodology, buildings are evaluated according to both performance levels, by using Eqs. (11) and (12), and the final decisions for the damage state of the buildings are achieved by considering the results of the two performance levels simultaneously.

Moreover, the number of stories is the most significant variable in both performance classifications. In order to improve the discriminating contribution of other parameters, new cutoff values are selected depending on the number of stories. For this purpose, a functional relationship is derived between the cutoff values and the number of stories, n, by fitting a least squares curve to the available damage data. In the determination of the cutoff function, two constraints are also imposed at each story level. These constraints are;

(i) the correct classification rate is required to be at least 70 % and,
(ii) the maximum classification error related to damage states leading to life loss (i.e. severe damage and collapse) is restricted to be 5 %.

The resulting cutoff functions based on number of stories, corresponding to the two types of classification, are as follows:

$$\begin{aligned} CF(lspc) &= -0.090 \cdot n^3 + 1.498 \cdot n^2 - 7.518 \cdot n + 11.885 \\ CF(iopc) &= -0.085 \cdot n^3 + 1.416 \cdot n^2 - 6.951 \cdot n + 9.979 \end{aligned} \tag{13}$$

In the proposed classification procedure, firstly the damage scores are obtained by using Eqs.(11) and (12) for the cases of LSPC and IOPC, respectively. Then by comparing these damage scores with the story dependent cutoff values obtained from Eq. (13), the building under evaluation is assigned an indicator variable of “0” or “1”. The indicator variable “0” corresponds to none, light or moderate damage in the case of LSPC and none or light damage in the case of IOPC. Similarly, the indicator variable “1” corresponds to severe damage or collapse in the case of LSPC and moderate or severe damage or collapse in the case of IOPC. In the final stage of the classification procedure, the building is rated as “safe” (i.e. “none or light damage”) or “unsafe” (i.e. “severe damage or collapse”) or “intermediate” depending on the values of the indicator variables obtained from both classification types according to the combinations listed in Table 3.

Table 3. Relationships among different classification criteria according to the proposed classification method

Classification	**Indicator Variable**		**Indicator Variable in Classification**
	LSPC	**IOPC**	
SAFE (None or Light Damage)	0	0	0
UNSAFE (Severe Damage or Collapse)	1	1	1
INTERMEDIATE	1	0	2
INTERMEDIATE	0	1	2

As observed in Table 3, if the indicator variable is consistently “0” or “1” for both LSPC and IOPC cases, the building is rated as “safe” or “unsafe”, respectively. If there is an inconsistency in the classification, in other words if one gives “0” and the other “1” or vice versa, then no final rating is done and the final decision on the seismic safety of the building is left for a more comprehensive detailed seismic evaluation. As the readers may note, in Table 3 all possible ratings are considered, among which the one given in the last row, with an IOPC indicator variable of 0 and LSPC indicator variable of 1, does not have any physical meaning whatsoever. It should be kept in mind that the adopted methodology is a statistical tool and such cases are therefore classified as the cases requiring further study.

Although the decision parameters of the proposed classification method described above are derived from the Düzce damage database, the classification method is applied to the same database in order to check its correct classification efficiency. The resulting output of the proposed classification method is given in Table 4. Out of the 484 buildings forming the seismic damage database, 99 buildings (37+11+51) that correspond to 20.5 % of the entire database, are classified as "intermediate" and left for further detailed evaluation. Among these 99 buildings, only two of them had an IOPC indicator variable of "0" and a LSPC indicator variable of "1". This result actually indicates the success of discriminating ability of the parameters used in the analyses. Out of 122 severely damaged or collapsed buildings, 98 buildings are correctly classified, 13 of them are misclassified and 11 of them are left for further detailed seismic analysis. Thus, the efficiency in identifying the severely damaged or collapsed buildings is increased to 80.3% and among the 484 buildings evaluated only 13 of the severely damaged or collapsed buildings are rated as safe. Thus, the misclassification that may lead to life loss is only 2.7%, i.e. 13/484=0.027.

Table 4. Classification results for the Düzce damage database

			Predicted Group Membership			**Total**
			0	1	2	
Original Group Membership	**Count**	None or Light Damage	130	44	37	211
		Severe Damage or Collapsed	13	98	11	122
		Moderate Damage	37	63	51	151
	Percent (%)	SAFE (None or Light Damage)	61.6	20.8		100.0
		UNSAFE (Severe Damage or Collapsed)	10.7	80.3		100.0
		INTERMEDIATE			20.5	100.0

5. VALIDATION OF THE PROPOSED METHODOLOGY

It is desirable to check the validity of the proposed statistical model by examining the correct classification rates in cases of different databases compiled from different earthquakes. For this purpose, the proposed methodology and the accompanying discriminant functions are applied to damage data assessed from the 1992 Erzincan earthquake and the damage data compiled after 2002 Afyon earthquake.

The classification results according to the proposed classification methodology are presented in Tables 5 and 6 for the Erzincan and Afyon damage databases, respectively.

As it can be observed from these tables, the classification results of the model demonstrate that the correct classification rate for severely damaged and collapsed buildings is quite high. On the other hand, the correct classification rate for none and a light damage state is found to be 96.4 % for the Erzincan database and 75.0 % for the Afyon database. Only 3 buildings forming 9.3 % of the Erzincan database and 22.2 % of the Afyon database cannot be judged. These buildings are identified as "intermediate" and they are the buildings that require further detailed investigations.

Considering the existence of various random factors (such as geotechnical parameters) and sources of uncertainties, these rates are found to be quite satisfactory and support the predictive ability of the proposed statistical model.

Table 5. Classification results for the Erzincan damage database

			Predicted Group Membership			**Total**
			0	1	2	
Original Group Membership	**Count**	None or Light Damage	27	0	1	28
		Severe Damage or Collapsed	0	2	0	2
		Moderate Damage	10	0	3	13
	Percent (%)	SAFE (None or Light Damage)	96.4	0.0		100.0
		UNSAFE (Severe Damage or Collapsed)	0.0	100.0		100.0
		INTERMEDIATE			9.3	100.0

Table 6. Classification results for the Afyon damage database

			Predicted Group Membership			**Total**
			0	1	2	
Original Group Membership	**Count**	None or Light Damage	3	0	1	4
		Severe Damage or Collapsed	1	8	1	10
		Moderate Damage	2	0	2	4
	Percent (%)	SAFE (None or Light Damage)	75.0	0.0		100.0
		UNSAFE (Severe Damage or Collapsed)	10.0	80.0		100.0
		INTERMEDIATE			22.2	100.0

6. CONCLUSIONS

A statistical analysis procedure is used to develop a model proposed for the preliminary assessment of the seismic vulnerability of existing reinforced concrete buildings. The procedure uses discriminant analysis technique that yields discriminant functions in terms of the selected estimation parameters. Six estimation parameters, namely number of stories, existence of soft story, normalized redundancy score, degree of overhang, the minimum normalized lateral stiffness and minimum normalized lateral strength indices, are considered for the assessment of seismic vulnerability. Among these parameters the number of stories is found to be the most discriminating parameter for existing low- to mid-rise reinforced concrete buildings.

The proposed classification methodology improves the correct classification rate especially in the cases where life-safety is involved. For the 1999 Düzce earthquake damage database, the correct classification rate in determining the severely damaged and collapsed structures is increased to 80.3 % whereas total misclassification rate that corresponds to the loss in human lives is only 2.7 percent. Besides the increased efficiency and accuracy of the model, a number of buildings are left for further detailed evaluations instead of evaluating them incorrectly.

The validity of the proposed methodology is checked based on the damage data available for the 1992 Erzincan earthquake and for the 2002 Afyon earthquake. Reasonably high correct classification rates are obtained, demonstrating the predictive ability of the proposed seismic vulnerability estimation methodology.

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