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Response to orthogonal components of ground motion and assessment of percentage combination rules

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SUMMARY

The effects of horizontal components of ground motion on the linear response of torsionally stiff and torsionally flexible systems, on soft and firm soil conditions, are examined. A one-story, two-way asymmetric structural system is used, subjected to uncorrelated ground motion components along their principal directions. Spectral densities for ground accelerations in firm and soft soils are modeled based on recorded data from large intensity Mexican earthquakes. It is shown that for firm soils, in general, these effects are important in the case of torsionally flexible systems that are stiff under translation, or for torsionally stiff systems that are flexible in translation. The percentage combination rules usually specified in seismic design codes are assessed against the dynamic response. Such combination rules can result in overly conservative design forces or underestimated design forces, particularly for torsionally flexible structures. Given the relative magnitude of the response to each ground motion component, it was found that using different percentage values in the combination rules has no significant effect on improving the estimation of the total response. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: multicomponent seismic analysis; combination rules; SRSS rule; 30% rule; 40% rule; orthogonal components

INTRODUCTION

When subjected to earthquakes, buildings are exposed to the simultaneous action of multiple components of ground motion. In seismic design, it is customary to analyze the effects of the translational components independently and then combine them to obtain the design demands. Design codes generally specify the so-called β-percentage combination rules for the effects of the horizontal orthogonal components of ground motion. Let R_X and R_Y denote the response of interest due to the same intensity ground motion acting along the structural axes X and

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Y, respectively. The β-percentage combination rule states that the design response should be taken as the larger of the following: $\beta R_X + R_Y$ or $R_X + \beta R_Y$. The most common rules are the 30% ($\beta = 0.3$) and 40% ($\beta = 0.4$) rules. The 30% rule was developed by Rosenblueth and Contreras [1] and is considered in several codes, see e.g. ICBO [2] and DDF [3]. The 40% rule was proposed by Newmark [4] and is now included in various codes, see e.g. ASCE [5]. The appropriateness of such combination rules to estimate the response obtained under the simultaneous action of two orthogonal components, considering the coupling of the response degrees of freedom of the structure, is yet a subject of examination.

A modal combination rule, denoted CQC3, for linear systems that take into account the correlation between modal responses and the correlation between the horizontal components of ground motion has been proposed by Menun and Der Kiureghian [6] based on the work by Smeby and Der Kiureghian [7]. For the CQC3 rule, horizontal components of ground motion are specified along their principal directions in terms of response spectra. According to Penzien and Watabe [8] there is a set of principal directions for which ground motion components can be considered to be uncorrelated. Specifying ground motion components (or spectral shapes) along any other set of orthogonal axes may lead to unrealistic modeling of the correlation between components. The effect of correlation between ground motion components on the structural linear response is relatively small when the mean-square intensity of the ground motion components is similar to each other or, equivalently, when the ratio of spectral shapes is close to one. The Square Root of the Sum of Squares (SRSS) rule is a particular case of the CQC3 rule. The SRSS rule gives exact results when the structural axes and the ground motion principal directions are aligned; and it is appropriate for combining responses when the ground motion horizontal components are of nearly equal intensity.

The CQC3 rule, as formulated by Menun and Der Kiureghian, is based on the assumption that the ground motion in each principal direction is a wide-band process so that results for white noise response can be used. Also it assumes that both input components have the same spectral shapes. The CQC3 rule has been used to evaluate the appropriateness of the SRSS and the 30% and 40% rules for predicting a critical response, i.e. the largest response over all possible seismic incident angles (Lopez *et al.* [9]). In this paper, the effects of horizontal components are examined for torsionally flexible and torsionally stiff structural systems on soft and firm soil conditions. Input ground motions are characterized by the spectral density functions estimated for each principal component of ground motion based on records for two large-intensity earthquakes in Mexico City. For the soft soil conditions the ground acceleration spectral density is narrow-banded. The dynamic response of a linear, one-story, two-way asymmetric structural model subjected to the principal components of ground motion is analyzed. The effects of horizontal components of ground motion on the response are studied in terms of shear forces in structural elements. The dynamic response to both components of ground motion is then used to assess the β-percentage combination rules.

EQUATIONS OF MOTION

Consider the linear asymmetric structural system shown in Figure 1 consisting of a rigid slab supported by elements with lateral resistance. Let X and Y denote the structural axes. Suppose the system is subjected to a set of horizontal orthogonal components of ground motion along the structural axes. Let $\{X\}^{\mathrm{T}} = \{x_1, x_2, x_3\}$ denote the response displacement vector along the

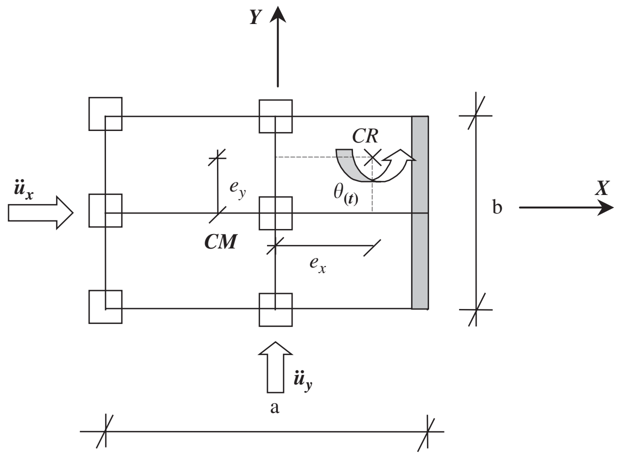

Figure 1. Structural system.

three degrees of freedom of the system and $\{F\}^{\mathrm{T}} = -m\{\ddot{u}_x, \ddot{u}_y, 0\}$ the vector of equivalent forces due to ground motions, where x_1, x_2 are the lateral displacements of the system along the X and Y axes, respectively, x_3 is the rotation of the slab, m is the mass of the system and $\ddot{u}_x$, $\ddot{u}_y$ are the horizontal ground accelerations along the structural axes X and Y, respectively. In matrix form the equations of motion are written as follows:

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & a^2(1+r^2)/12 \end{bmatrix} \{\ddot{X}\} + \omega_y^2 \begin{bmatrix} 1/\lambda & 0 & -\alpha_y ar/\lambda \\ 0 & 1 & \alpha_x a \\ -\alpha_y ar/\lambda & \alpha_x a & \Omega^2 a^2(1+r^2)/12 \end{bmatrix} \{X\} = -\begin{Bmatrix} \ddot{u}_x \\ \ddot{u}_y \\ 0 \end{Bmatrix} \tag{1}$$

where $\Omega = \omega_\theta/\omega_y$ is the ratio of the uncoupled rotation frequency and the translation frequency in the Y direction, $\lambda = k_y/k_x$ is the ratio of lateral stiffness in the Y and X directions, $r = b/a$ is the aspect ratio, and $\alpha_x = e_x/a$, $\alpha_y = e_y/ar$ are the normalized nominal eccentricities. From the free vibration solution one obtains the modal frequencies, ω_i, and the corresponding modal shapes, $\{\phi_i\}^{\mathrm{T}} = \left\{1, -\frac{\alpha_x}{\alpha_y}\frac{\lambda}{r}\frac{(\omega_x^2-\omega_i^2)}{(\omega_x^2\lambda-\omega_i^2)}, \frac{(\omega_x^2-\omega_i^2)}{\omega_x^2\alpha_y ra}\right\}$, $\alpha_y \neq 0$. In the case where the lateral stiffness in both directions is the same, $\lambda = 1$, it can be shown that the second modal frequency is equal to the uncoupled translation frequency, $\omega_2 = \omega_x$, and $\{\phi_2\} = \{1, \frac{\alpha_y}{\alpha_x}r, 0\}$.

Let $\{Z\}$ be the vector of generalized coordinates and $[\Phi]$ the modal matrix, $\{X\} = [\Phi]\{Z\}$. The uncoupled equations of motion for the generalized coordinates are

$$\ddot{z}_i + 2\xi_i\omega_i\dot{z}_i + \omega_i^2 z_i = f_i \quad i = 1, 2, 3 \tag{2}$$

where,

$$f_i = -(\phi_{1i}\ddot{u}_x(t) + \phi_{2i}\ddot{u}_y(t))/\bar{m}_i \tag{3}$$

A modal damping coefficient ξ_i has been incorporated in Equation (2); in (3), $\bar{m}_i$ is the i-th generalized modal mass and ϕ_{1i} and ϕ_{2i} are the components of the modal vector, $\{\phi_i\}$, corresponding to the i-th mode. Suppose now that the horizontal components of earthquake ground acceleration are modeled as zero-mean, stationary random processes. The cross-correlation function between modal response $z_i(t)$ and $z_j(t)$ is equal to

$$R_{ij}(\tau)=E[z_i(t+\tau)z_j(t)]=\int\int h_i(\theta_1)h_j(\theta_2)E[f_i(t+\tau-\theta_1)f_j(t-\theta_2)]\,\mathrm{d}\theta_2\,\mathrm{d}\theta_1 \tag{4}$$

where $h_i(t)$ is a modal unit-impulse response function.

There is a set of principal directions for which the components of ground motion are uncorrelated (Penzien and Watabe [8]). Let $\ddot{u}_x$ and $\ddot{u}_y$ be the principal components of ground acceleration, i.e. the structural axes coincide with the principal axes of ground motion. Given that there is no correlation between the principal components of ground acceleration, $\ddot{u}_x$ and $\ddot{u}_y$, it follows from Equation (3) that the cross-correlation function between $f_i(t)$ and $f_j(t)$ is given by:

$$E[f_i(t+\tau-\theta_1)f_j(t-\theta_2)]=\frac{1}{\bar{m}_i\bar{m}_j}(\phi_{1i}\phi_{1j}R_{xx}(\tau-\theta_1+\theta_2)+\phi_{2i}\phi_{2j}R_{yy}(\tau-\theta_1+\theta_2)) \tag{5}$$

In Equation (5) $R_{xx}(\tau)$ and $R_{yy}(\tau)$ are the auto-correlation functions for the ground acceleration components in the principal directions. Expressing $R_{xx}(\tau)$ and $R_{yy}(\tau)$ in terms of the spectral density functions of the ground acceleration components in both directions, $S_{xx}(\omega)$, $S_{yy}(\omega)$, and substituting Equation (5) in Equation (4), the following expression is found for the cross-correlation function between modal responses,

$$R_{ij}(\tau)=\frac{1}{\bar{m}_i\bar{m}_j}\int(\phi_{1i}\phi_{1j}S_{xx}(\omega)+\phi_{2i}\phi_{2j}S_{yy}(\omega))H_i(\omega)H_j^*(\omega)\mathrm{e}^{i\omega\tau}\,\mathrm{d}\omega \tag{6}$$

where the superscript asterisk indicates complex conjugate and $H_i(\omega)$, $H_j(\omega)$, are the modal transfer functions

$$\int h_i(\theta_1)\mathrm{e}^{-i\omega\theta_1}\,\mathrm{d}\theta_1=H_i(\omega)=1/(\omega_i^2-\omega^2+2i\xi_i\omega\omega_i) \tag{7}$$

Let ψ^2 denote the ratio between the variances of the principal components of ground acceleration, $\sigma_{yy}^2=\psi^2\sigma_{xx}^2$. The ratio ψ^2 is greater (or less) than 1.0 if the component of ground acceleration, $\ddot{u}_y$, is the major (or minor) principal component of motion. The spectral densities in (6) can be normalized as follows:

$$s_{xx}(\omega)=\frac{S_{xx}(\omega)}{\sigma_{xx}^2}\quad s_{yy}(\omega)=\frac{S_{yy}(\omega)}{\psi^2\sigma_{xx}^2} \tag{8}$$

Replacing Equation (8) in Equation (6),

$$\begin{aligned}R_{ij}(\tau)=\;&\frac{1}{\bar{m}_i\bar{m}_j}\sigma_{xx}^2\left(\int\phi_{1i}\phi_{1j}s_{xx}(\omega)\,Re[H_i(\omega)H_j^*(\omega)]\mathrm{e}^{i\omega\tau}\,\mathrm{d}\omega\right.\\&\left.+\int\phi_{2i}\phi_{2j}\psi^2 s_{yy}(\omega)\,Re[H_i(\omega)H_j^*(\omega)]\mathrm{e}^{i\omega\tau}\,\mathrm{d}\omega\right)\end{aligned} \tag{9}$$

The response displacements and rotation of the system can be written as follows:

$$x_k(t)=\sum_{i=1}^{3}\phi_{ki}z_i(t);\quad k=1,2,3 \tag{10}$$

Thus, the cross-correlation function between system responses is given by

$$E[x_m(t+\tau)x_k(t)] = R_{x_k x_m}(\tau) = \sum_{i=1}^{3}\sum_{j=1}^{3} \phi_{ki}\phi_{mj}R_{ij}(\tau) \tag{11}$$

Consider now the structural response along the Y-axis of the stiff edge of the system. i.e. the edge on the side of the location of the center of stiffness. Let k_y denote the lateral stiffness of the structural axis at the stiff edge of the system. The relative displacement of the axis is

$$\delta = x_2 + ax_3/2 \tag{12}$$

and its variance is given by

$$var[\delta] = var[x_2] + a^2 var[x_3]/4 + a\,cov[x_2, x_3] \tag{13}$$

Using Equation (11), then it follows from (13) that

$$var[\delta] = \sum_{i=1}^{3}\sum_{j=1}^{3}\left\{\phi_{2i}\phi_{2j} + \frac{a^2}{4}\phi_{3i}\phi_{3j} + a\phi_{2i}\phi_{3j}\right\}R_{ij}(0) \tag{14}$$

The shear force in the axis is equal to $V = k_y\delta$ with standard deviation given by $\sigma_V = k_y\sigma_\delta$, where $\sigma_\delta = \sqrt{var[\delta]}$. The expected maximum shear force, $V_T = E[V\,max]$, can then be computed by means of a peak factor K_T,

$$V_T = E[V\,max] = K_T\sigma_V \tag{15}$$

where $K_T = \sqrt{2\ln(2vs)} + 0.5772/\sqrt{2\ln(2vs)}$, and v and s are the zero crossing rate and duration of the response window, respectively. Notice that since the structural axes are aligned with the principal axes of ground motion, then Equations (9) and (14) represent the basis for deriving the SRSS combination rule. Assuming that the peak factor for the response V_T is the same as the peak factor for each ground motion response component, then substitution of Equations (9) and (14) into (15) yields the SRSS combination rule for V_T.

CASE STUDY

Ground acceleration records for both horizontal components from the Mexico earthquakes of 25 April 1989, and 9 October 1995, were analyzed to model the spectral density functions. Records were used from stations in the so-called firm soil of Mexico City and from stations in soft soils with ground motion dominant period around 2 seconds. The source characteristics for the two events are listed in Table I; also listed in this table is the number of stations in both types of soil conditions. The recorded motions were decomposed into their principal components along which ground accelerations are uncorrelated (Penzien and Watabe [8]). The spectral density functions were then estimated for these principal directions. Figure 2 shows the average density functions for ground accelerations along the major and minor principal components for the two events and both soil conditions. On soft soil, ground accelerations are a narrow-band process with a dominant period at 2 seconds. On firm soil, ground accelerations are broad-banded with frequencies in the interval [0.2, 1.5] Hz. The spectral density functions for the 9 October 1995 earthquake were selected to model the ground motion.

Table I. Earthquake characteristics and number of recording stations.

Date	Magnitude (Ms)	Longitude	Latitude	Depth (km)	Stations Soft soil	Stations Firm soil
25 April 1989	6.9	99.40	16.60	19	7	9
9 October 1995	7.3	104.67	18.74	20	7	9

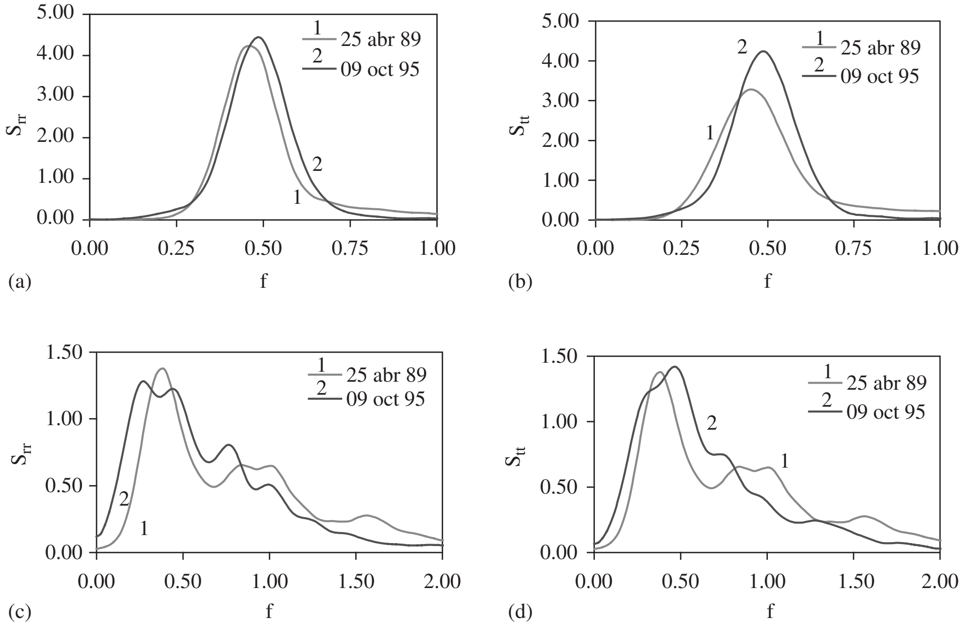

Figure 2. Normalized spectral density functions of ground acceleration along principal directions. (a) Major component, soft soil; (b) minor component, soft soil; (c) major component, firm soil; and (d) minor component, firm soil.

Structural systems such as the one shown in Figure 1 with frequency ratio $\Omega = 0.7, 0.9, 1.2, 1.4$ and aspect ratios $r = 0.5$, 1.0 were analyzed. Nominal eccentricities equal to 5%, 12% and 17% were taken and a critical damping coefficient of 5% was considered for all modes. The minor principal component of ground acceleration was taken acting along the Y structural axes, while the major principal component was taken acting along the X structural axes. For the earthquake motions selected the average ratio of the ground acceleration variance along the minor and major principal components is $\psi = \sigma_{yy}/\sigma_{xx} = 0.78$ for firm soils and $\psi = \sigma_{yy}/\sigma_{xx} = 0.85$ for soft soils. Since the ratio of standard deviations is close to one, it is expected that the effects of correlation between ground motion components will be small for the cases where the structural axes are not aligned with the ground motion principal axes.

We calculated the expected maximum shear force, V_T, in the stiff edge of the system along the Y structural direction due to both principal components of ground acceleration; we also computed the expected maximum shear force, V_Y, due to the principal component in the Y

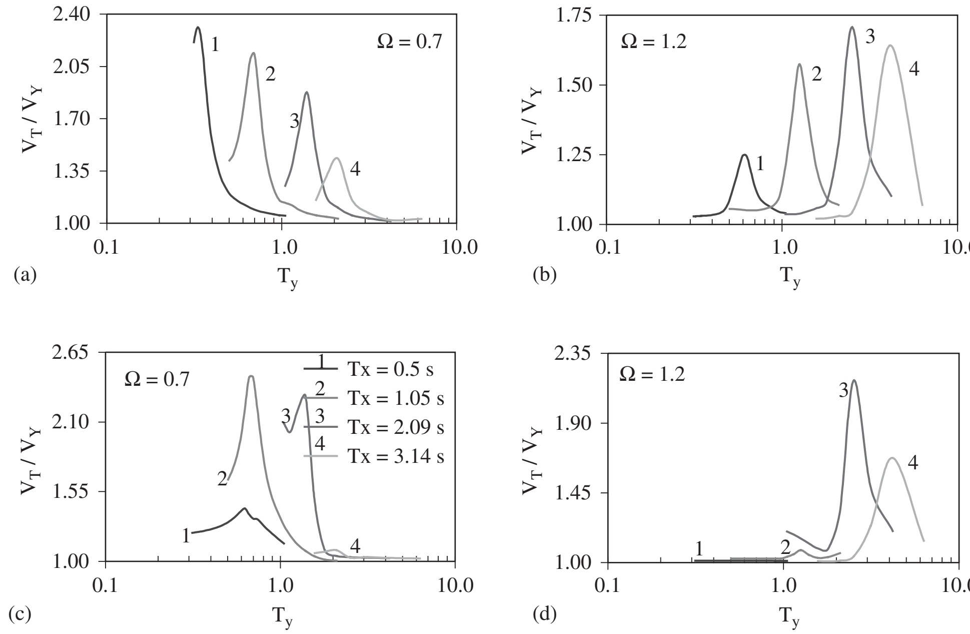

Figure 3. Effects of torsional stiffness on response: (a) Torsionally flexible system, firm soil; (b) torsionally stiff system, firm soil; (c) torsionally flexible system, soft soil; and (d) torsionally stiff system, soft soil.

direction only. Assuming that the ratio of the corresponding peak factors $K_T/K_Y \approx 1.0$, the ratio of the corresponding standard deviations, $\sigma_{V_T}/\sigma_{V_Y}$, can be interpreted as the ratio of the mean maximum shear forces. The figures presented next will show plots of the response ratio V_T/V_Y on the vertical axis versus the uncoupled natural period of translation in the Y direction T_Y on the horizontal axis. In the X direction four values were taken for the uncoupled natural period of the structural system; in the Y direction, the uncoupled natural period was varied considering that the ratio between the lateral stiffness in both directions of the system is at most equal to five.

Figures 3(a) and (b) show the variation of response with uncoupled natural periods for structures on firm soil considering a nominal eccentricity of 5% and on aspect ratio $r = 0.5$. It is clearly seen that the effect of orthogonal components is considerably different for torsionally flexible and torsionally stiff structures. For torsionally flexible systems the significance of orthogonal components is greater when the system is more rigid under translation (Figure 3(a)). As the translation natural periods of the system in both directions become shorter, the effect of orthogonal components becomes greater. The peak increase of response due to orthogonal components varies between 40% and 230%. If $T_Y >> T_X$ the system is much more flexible in the Y direction and the contribution from the major principal component of ground motion ($\ddot{u}_x$) has little effect on the response of interest. For torsionally stiff systems the effect of orthogonal components is greater for systems with long translation natural periods in both directions, i.e. for flexible systems under translation (Figure 3(b)). Peak increases of response due to orthogonal components vary between 25% and 70%. If $T_Y << T_X$ the response is not sensitive to the orthogonal effects since the system is much more flexible in the X direction and therefore the contribution from the major principal component of ground motion ($\ddot{u}_x$) to the response of interest along the Y direction is rather small.

The variation of response with uncoupled natural periods for structures on soft soil is shown in Figures 3(c) and (d). For torsionally flexible structures the effect of orthogonal components is not significant for systems with long natural periods in both directions, i.e. systems that are more flexible under translation ($T_X = 3$ s). On the other hand, for torsionally stiff systems the effect of orthogonal components is negligible for systems with short natural period ($T_X = 0.5$s), i.e. systems that are rigid under translation. Peak increases of response may be as high as 210% and 245% for torsionally stiff and torsionally flexible systems, respectively. The sensitivity of the response to soil conditions can be assessed from Figure 3. For torsionally stiff systems on soft soils the effects of orthogonal components are negligible for $T_X = 0.5$ s; however, on firm soil the peak increase of response due to orthogonal components is 25%. For $T_X = 1$ s the peak response increases from 1.1 for soft soil to 1.6 for firm soil. In the case of torsionally flexible systems, the peak response increases from 2.05 to 2.45 ($T_X = 1$ s) and from 1.9 to 2.3 ($T_X = 2$ s) when the system is founded on soft soil rather than firm soil. However, the peak response is much less for systems on soft soil than for those on firm soil for the short or long periods ($T_X = 0.5$ s and 3 s).

Results are shown in Figure 4 for torsionally stiff systems with nominal eccentricities in the Y direction $\alpha_Y = 5\%, 12\%, 17\%$, aspect ratio $r = 0.5$, $\alpha_X = 17\%$, and natural period $T_X = 1$ s. It is seen that the effects of orthogonal components on the response are sensitive to the

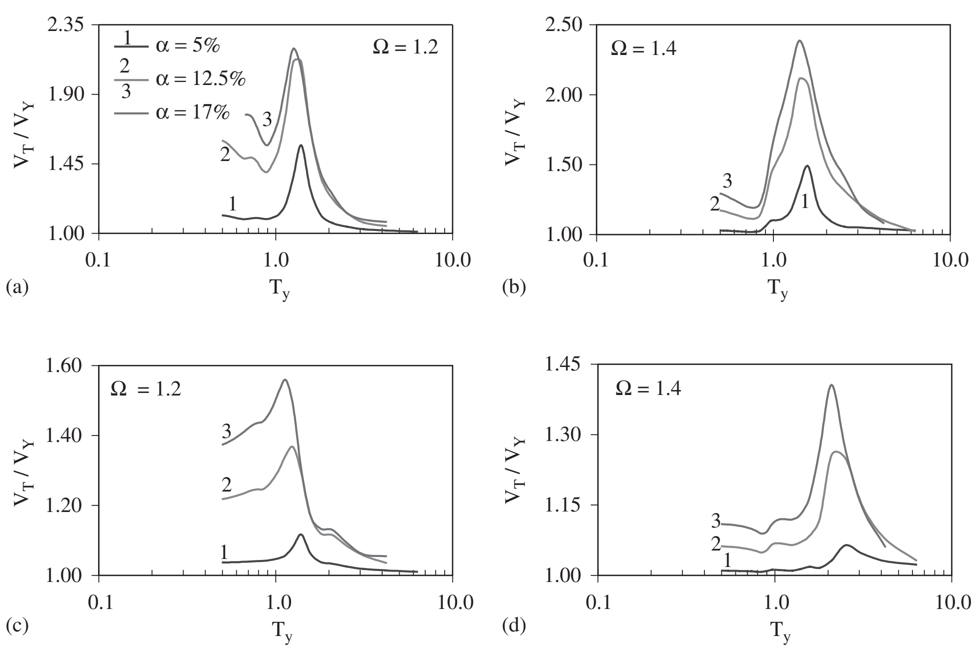

Figure 4. Response of torsionally stiff systems for varying degrees of asymmetry: (a) Firm soil, $\Omega = 1.2$; (b) firm soil, $\Omega = 1.4$; (c) soft soil, $\Omega = 1.2$; and (d) soft soil, $\Omega = 1.4$.

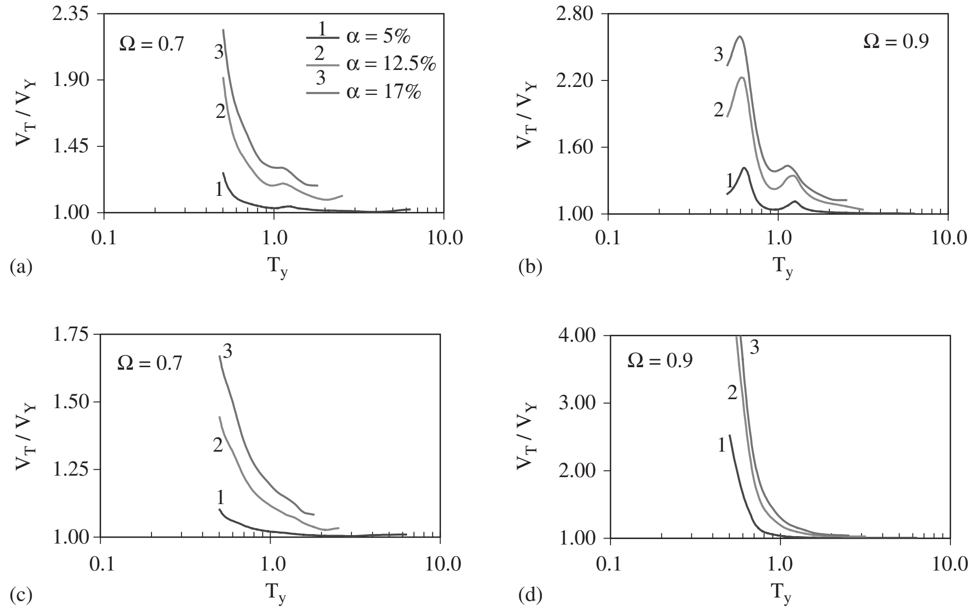

Figure 5. Response of torsionally flexible systems for varying degrees of asymmetry: (a) Firm soil, $\Omega = 0.7$; (b) firm soil, $\Omega = 0.9$; (c) soft soil, $\Omega = 0.7$; and (d) soft soil, $\Omega = 0.9$.

degree of asymmetry of the system and, in general, increase with nominal eccentricity. For instance, the peak response increased from 1.5 to 2.2 ($\Omega = 1.2$) and from 1.5 to 2.40 ($\Omega = 1.4$) when the nominal eccentricity increased from 5% to 17% for systems on firm soil. For small eccentricities (5%), when $T_Y << T_X$the response is not sensitive to the orthogonal effects since the system is much more flexible in the X direction as explained above. Figure 4 shows that for greater degrees of asymmetry, in spite of $T_Y << T_X$ the eccentricity is big enough for the contribution from the major principal component of ground motion ($\ddot{u}_x$) not to be negligible. Consider for example a period $T_Y = 0.5$s and $\Omega = 1.2$: the response increases from 1.1 to 2 (firm soil) and from 1.04 to 1.4 (soft soil) when the nominal eccentricity increased from 5% to 17%. Similar results to those shown were obtained for other values of the natural period in the X direction. Figure 5 shows results for the case of torsionally flexible structures with nominal eccentricities in the Y direction $\alpha_Y = 5\%, 12\%, 17\%$, aspect ratio $r = 0.5, \alpha_X = 17\%$, and natural period $T_X = 1$ s. The effects of orthogonal components on the response are also sensitive to the degree of asymmetry of the system and, in general, increase with nominal eccentricity. Notice that among the torsionally flexible systems analyzed the increase of response due to orthogonal components is the greatest in the case of structures with $\Omega = 0.9$ and on soft soils.

The expected maximum shear force in the stiff edge along the Y-axis of the structural model in Figure 1 was computed using the β-percentage combination rule and then compared to the shear force obtained from the dynamic analysis of response under both horizontal components of ground motion. Let V_{CR} denote the expected maximum shear force in the stiff edge along the Y structure axis when the β-combination rule is applied: $V_{CR} = max(U_Y + \beta U_X, \beta U_Y + U_X)$, where U_X and U_Y denote the shear force of interest due to the same ground

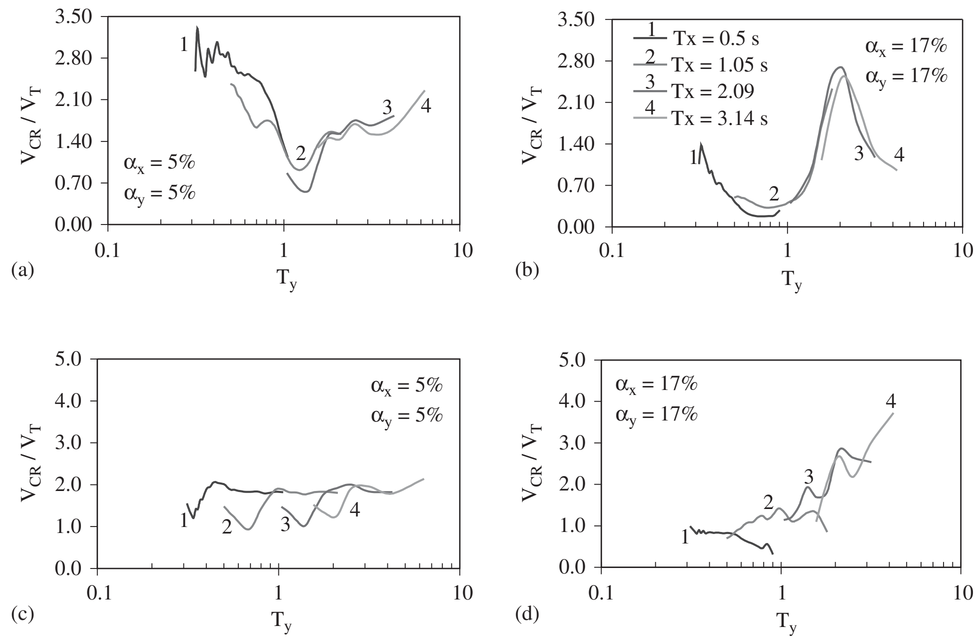

Figure 6. Assessment of the 30% combination rule; torsionally flexible systems: (a) and (b) Soft soil; (c) and (d) firm soil.

motion input acting along each structural axes X and Y, respectively. For computing U_Y and U_X, the spectral density for the major principal component of ground acceleration was used for both structural axes.

First, we analyze results for the 30% combination rule. Figure 6 shows the ratio of the 30% rule shear force, V_{CR}, and the shear force from the dynamic analysis considering both horizontal components of ground motion, V_T, for a torsionally flexible system with $\Omega = 0.7$ and $r = 0.5$. For systems with a low degree of asymmetry ($\alpha = 0.05$) on soft soils the response ratio is in general greater than one. However, there are systems for which the dynamic response can be greater than that computed according to the 30% rule, such as $T_X = 2$ s and T_Y less than 1.3 s. For systems with short period, say T_X and T_Y less than 0.5 s, the 30% rule yields responses that are about three times the dynamic response. In firm soils, the 30% rule response is greater than one for all periods considered. Thus, the 30% combination rule yields conservative responses, which are at most of the order of two times the dynamic response.

For torsionally flexible systems with a higher degree of asymmetry ($\alpha = 0.17$) on soft soils Figure 6 shows that the 30% rule response is greater than the dynamic response for systems with natural period T_Y greater than 1.5 s. For other natural periods it was found that the dynamic response is underestimated by the 30% rule; in the least conservative cases the 30% combination rule response is about 20% the dynamic one. In firm soils, the 30% rule response is greater than the dynamic response for systems with periods T_Y longer than 1 s. For short-period systems ($T_X = 0.5$ s) the combination rule response is 40% to 90% of the dynamic response. On the other hand, for certain long-period systems the 30% rule response can be

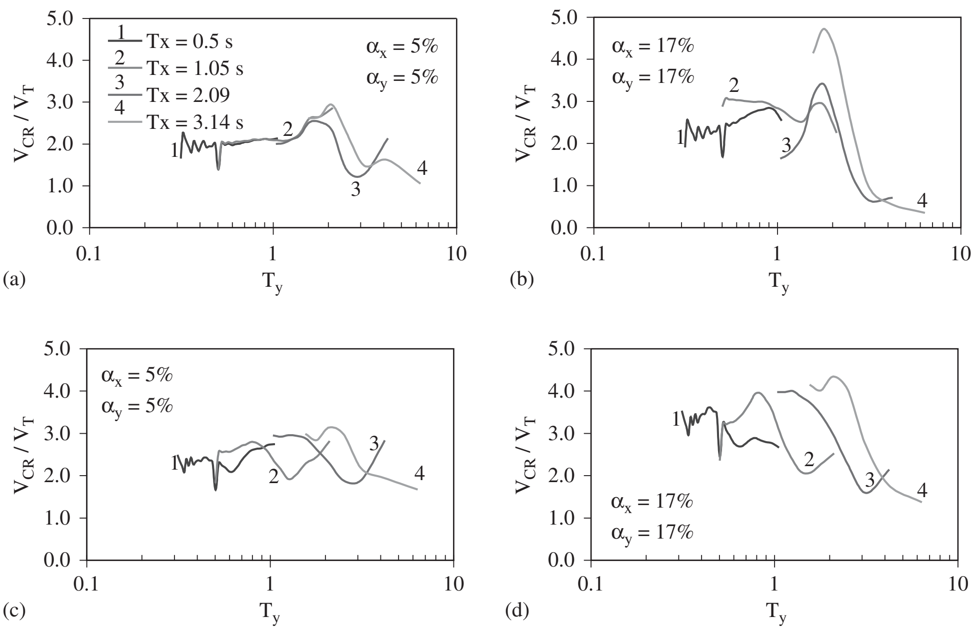

Figure 7. Assessment of the 30% combination rule; torsionally stiff systems: (a) and (b) Soft soil; (c) and (d) firm soil.

considerably higher than the dynamic response. For instance, for systems with T_Y greater than 3 s, the 30% response can be as high as three to four times the dynamic response.

Figure 7 shows the ratio of the 30% rule shear force, V_{CR}, and the shear force from the dynamic analysis considering both horizontal components of ground motion, V_T, for a torsionally stiff system with $\Omega = 1.2$ and $r = 0.5$. On firm soil conditions, the 30% rule response is always greater than the dynamic response thus yielding conservative results. The 30% rule response is more than twice the dynamic response for almost all of the periods considered when $\alpha = 0.05$. For a higher asymmetry ($\alpha = 0.17$) the combination rule procedure can be even more conservative; it yields shear forces that are three to four times the dynamic ones for various period intervals T_Y. The combination rule response is in general at least twice the dynamic one. In the case of soft soils, it was found to produce very variable responses for $\alpha = 0.17$. For instance, for $T_X = 3$ s, the 30% rule response can be more than four times the dynamic response for T_Y less than 2 s, whereas for T_Y greater than 4 s it is less than half the dynamic one. On the other hand, for the other periods T_X considered, the 30% combination procedure yields responses that are 1.5 to 3.5 times the dynamic one for periods T_Y less than 2 s. Finally, it should be pointed out that in almost all of the cases studied above, it was found that $V_{CR} = max(U_Y + \beta U_X, \beta U_Y + U_X) = U_Y + \beta U_X$.

Other percentage combination rules were also compared against the dynamic response. The well-known 40% rule was used for the comparison; in order to examine the variation in the computation of the response with the β-percentage value, a 10% rule was also considered. Figures 8 and 9 show the results for the 10% and 40% combination rules for torsionally flexible and stiff systems on firm and soft soil conditions, $\alpha = 5\%$ and $r = 0.5$. As seen, using

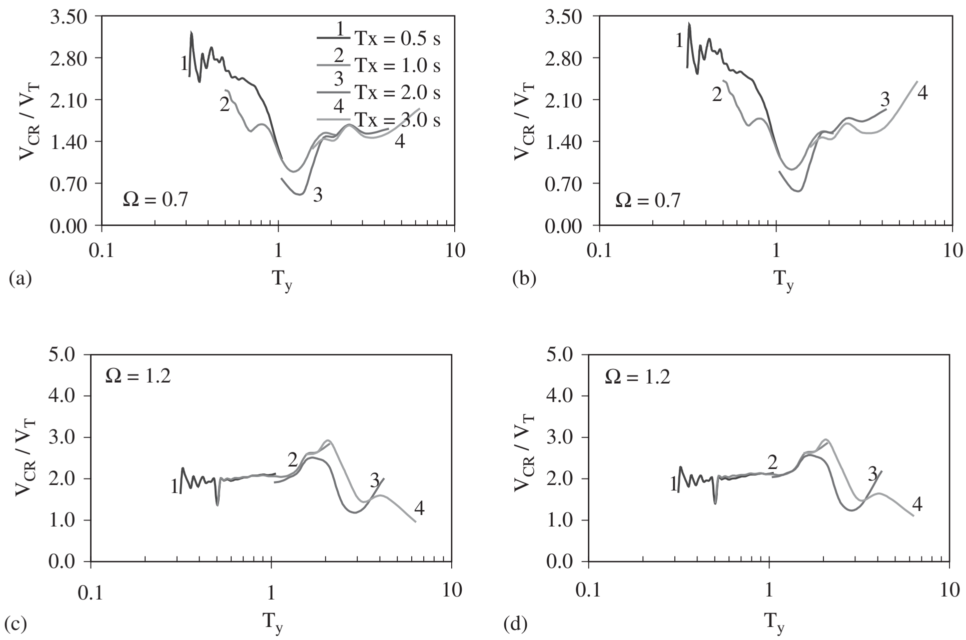

Figure 8. Comparison of results using the 10% and 40% combination rules; soft soil: (a) 10%, $\Omega = 0.7$; (b) 40%, $\Omega = 0.7$; (c) 10%, $\Omega = 1.2$; and (d) 40%, $\Omega = 1.2$.

other β-percentages in the combination rules does not improve significantly the estimation of the dynamic response. When the contribution from the orthogonal component, U_X, is small compared to the response component, U_Y, the weight used in the combination rule, either 10%, 30% or 40%, does not have a significant influence on the results. Figure 10 shows the ratio U_X/U_Y for torsionally flexible and torsionally stiff systems on soft and firm soil conditions. It can be seen that the orthogonal response component in the combination rule, U_X, is very small compared to the response component, U_Y, and thus the weight 'β' has very little influence on the estimation of the dynamic response. It can be shown easily that in order to obtain differences greater than 5% in the computed response using the 40% and 30% rules, the ratio U_X/U_Y should be greater than 0.62; and greater than 0.27 when using the 30% and 10% rules. As seen in Figure 10, the contribution from the orthogonal component is never large enough so as to obtain responses that will differ by more than 5% using the various 'β' values in the combination rule.

CONCLUSIONS

A linear, one-story, asymmetric, structural system was used to examine the effect of orthogonal components of ground motion. The system was subjected to principal components of ground motion acting along the structural axes. Spectral density functions of ground acceleration were modeled based on recorded data for soft and firm soils in Mexico City. The following are the main conclusions of the work.

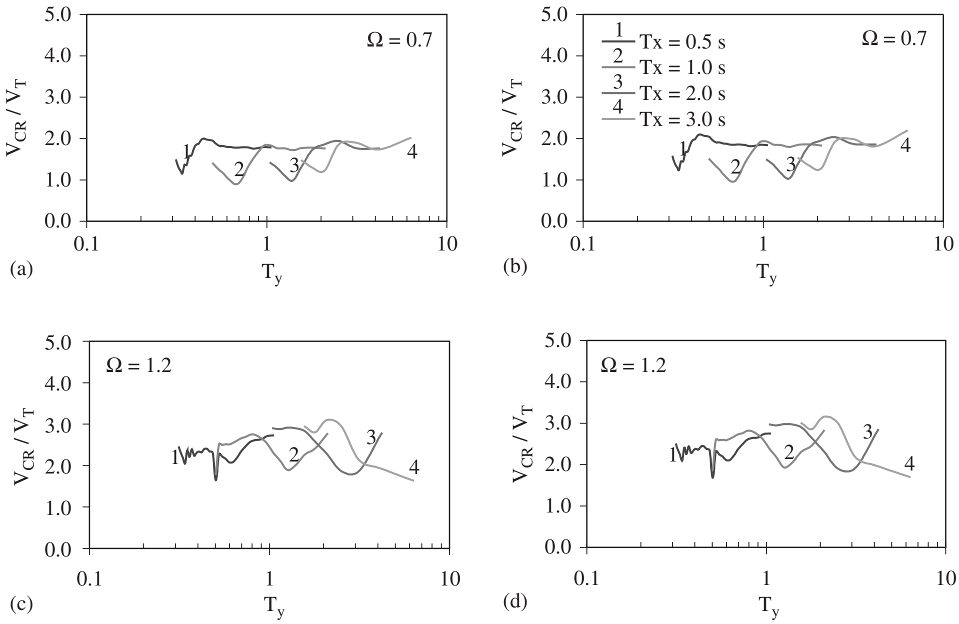

Figure 9. Comparison of results using the 10% and 40% combination rules; firm soil: (a) 10%, $\Omega = 0.7$; (b) 40%, $\Omega = 0.7$; (c) 10%, $\Omega = 1.2$; and (d) 40%, $\Omega = 1.2$.

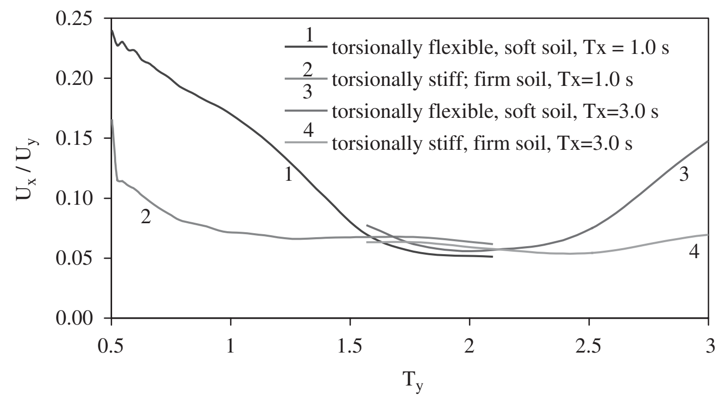

Figure 10. Ratio U_X/U_Y for torsionally flexible and stiff systems.

1. The effect of orthogonal components on structural response varies differently with the natural translation period depending on whether the system is torsionally flexible or torsionally stiff. For torsionally flexible systems on firm soils, the significance of orthogonal components is greater as the natural period becomes shorter; for torsionally stiff systems, the effect of orthogonal components is greater for systems with long translation natural periods.
2. For systems with low asymmetry (5% of nominal eccentricity) overall maximum increases of response due to orthogonal ground motions are about 245% for torsionally

flexible systems and 210% for torsionally stiff systems on soft soils. In the case of firm soils, overall maximum increases of response are about 230% for torsionally flexible systems and 70% for torsionally stiff systems. The increase of response due to orthogonal components can be expected to become larger for higher degrees of asymmetry of the system.
3. The effect of orthogonal components is sensitive to soil conditions. For both torsionally flexible and stiff systems, the increase of response due to orthogonal components may be greater for firm soils than for soft soils, or vice versa, depending on the natural period of the system.
4. The β-percentage combination rules may result in design forces that are less than the dynamic forces due to orthogonal components of earthquake ground motions. On the other hand, they can result in overly conservative design forces as well. Although the combination rules are relatively easy to apply in professional practice, they do not account for the fact that the effects of orthogonal components of ground motion on the response depend significantly on the structural properties and on the soil conditions, as has been shown here.
5. There is not a significant difference in the computation of the dynamic response to orthogonal components when using different percentage combination rules. A comparison of the widely known 30% and 40% rules, and also a 10% rule, shows that because of the relative magnitude of the responses to each ground motion component, the difference in the computed response using such percentage rules is less than 5%.

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