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Model of damage for RC elements subjected to biaxial bending

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Abstract

This paper describes the modeling of the biaxial bending of RC frames within a new framework called lumped damage mechanics (LDM). In this alternative approach, the models are based on the methods of fracture mechanics and the concept of plastic hinge. LDM can be considered as a branch of fracture mechanics for framed structures. LDM integrates concepts such as plastic hinge, damage variable, energy release rate, deformation equivalence hypothesis and RC standard theory. In order to validate the model, some numerical simulations of tests reported in the literature are presented. A good agreement between tests and model can be observed.

Keywords: Fracture mechanics; Damage mechanics; Structural analysis; Biaxial bending; Reinforced concrete

1. Introduction

Very often, the structures are subjected to outstanding loads, such as earthquakes and impacts. Evaluation or prediction of structural damage in such cases is very important for the structural engineer.

The linear elastic fracture mechanics theory accepts the existence of cracks that can grow even in elastic media. In this theory, the crack propagation is described through the Griffith criterion. The theory validity relies on small area crack over structure size rate.

On the other hand, the damage introduced by continuum damage theory characterizes the microcracks and microvoids surface density. However, it is not evident how fracture mechanics or continuum damage theories can be used in many real engineering applications. The structure representation as a three-dimensional solid and the estimation of the damage in it, is a very expensive and cumbersome procedure.

This paper describes the modeling of the biaxial bending of RC frames within a new framework called lumped damage mechanics (LDM). In this alternative approach, the models are based on the methods of fracture and damage mechanics and the concept of plastic hinge. LDM can be considered as a branch of fracture mechanics for framed structures. This general approach has been described in some previously published papers [1–5], two of them appeared in this journal [6,7]. Other works, using the same general framework, have also been independently reported by researchers from two European Universities [8,9].

The use of LDM for RC members allows for the representation of a very complex behavior that includes stiffness and strength degradation, variable axial loads, crack closure effects and pinching. This framework permits the analysis of complicated frames using few elements. The structural and damage analysis are coupled and the modeling is carried out in a more rational and physically sound way.

However, all the aforementioned references considered only the case of planar frames. It is clear that any realistic representation of the behavior of RC structures should include three-dimensional aspects. This paper deals with the modeling of RC frame members under compression and biaxial bending. It is shown that the same basic concepts of the planar frames can be straightforwardly extended to the biaxial case. The result is a conceptually simple model and nevertheless accurate enough for engineering purposes.

So far, it seems that the main direct approach to the problem is the use of finite elements, specially those models that discretize the member as set of fibers charac-

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terized by some kind of uniaxial constitutive law. Another strategy, a simplification of the FE approach, has been proposed by Lai and Will [10]. This approach, called 'triaxial spring model', consists of a linear elastic element between two inelastic connections. Each connection comprises a certain number of effective concrete and steel springs. Uniaxial constitutive laws, made of a series of straight lines, represent the behavior of the springs. The model is able to represent biaxial bending with stiffness degradation and variable axial load. Takisawa and Aoyama [11] proposed the use of a biaxial trilinear model. In the biaxial moment space the model consists of two elliptical surfaces that represent a cracking surface and a yielding surface. Both surfaces can exhibit isotropic hardening as in the conventional plasticity theory. This model can also represent biaxial bending and stiffness degradation.

The model proposed in this paper is radically different since none of the above includes concepts of fracture mechanics. Taking into account that stiffness degradation is mainly due to concrete cracking, it seems useful to explore this alternative.

2. Variables definition

2.1. Generalized stresses and deformations

Let us consider a three-dimensional RC frame. A member of the structure, between the nodes i and j, is isolated. A set of coordinate axes is chosen so that the directions Y and Z are the principal axes of the cross-section and X coincides with the neutral line of the member.

The matrix $\mathbf{\Phi}^{\mathrm{T}} = (\phi_{iy}, \phi_{jy}, \delta, \phi_{iz}, \phi_{jz}, \phi_x)$ contains the generalized deformations of the member. The meaning of the elements of $\mathbf{\Phi}$ is indicated in Fig. 1.

It can be noticed that ϕ_{iy} and ϕ_{jy} are flexural rotations of the tangents to the member with respect to the chord i–j, in the XZ plane; while ϕ_{iz} and ϕ_{jz} are flexural rotations in the XY plane; δ represents the elongation of the chord and ϕ_x is a torsional rotation

If $\mathbf{\Phi} = 0$, there is no change of shape of the frame member and vice versa. In general, in a rigid body rotation or displacement of the member the generalized deformation matrix is nil.

Another matrix, denoted by $\mathbf{M}^{\mathrm{T}} = (m_{iy}, m_{jy}, n, m_{iz}, m_{jz}, m_x)$, represents the generalized stresses of the member. The meaning of the elements of $\mathbf{M}$ is indicated in Fig. 2.

It can be noticed that m_{iy} and m_{jy} are flexural moments in the XZ plane; the moments m_{iz} and m_{jz} act in the XY plane; n is the axial force and m_x the torsional moment.

The matrices $\mathbf{M}$ and $\mathbf{\Phi}$ are conjugated with respect to the mechanical work on the frame member.

2.2. Plastic rotations

The inelastic behavior of the member is described with the help of the conventional lumped inelasticity representation that assumes all inelastic phenomena are concentrated in hinges. These hinges are located at the ends i and j of the member (Fig. 3). It is assumed in this paper that the permanent elongation of the chord and the plastic torsional rotation are negligible. Therefore, a generalized plastic deformations matrix is defined as follows: $\mathbf{\Phi}_{\mathrm{p}}^{\mathrm{T}} = (\phi_{iy}^{\mathrm{p}}, \phi_{jy}^{\mathrm{p}}, 0, \phi_{iz}^{\mathrm{p}}, \phi_{jz}^{\mathrm{p}}, 0)$, where the non-nil terms in the matrix $\mathbf{\Phi}_{\mathrm{p}}$ are the plastic rotations of both inelastic hinges, in the XZ and XY planes respectively.

In this work, it is assumed that plastic rotations in RC frame members are mainly the consequence of reinforcement yielding, while the inelastic phenomena associated with concrete cracking may be represented by the damage variables that are introduced in the next section.

2.3. Damage in a member under biaxial bending

Two additional sets of internal variables, called damage, are introduced. They represent a generalization to

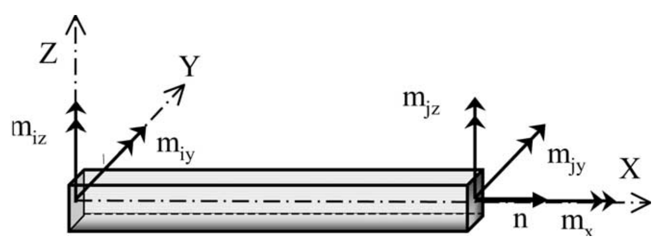

Fig. 2. Generalized stresses in a three-dimensional frame member.

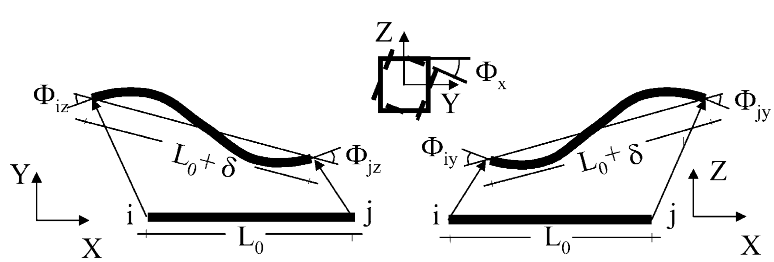

Fig. 1. Generalized deformations in a three-dimensional frame member.

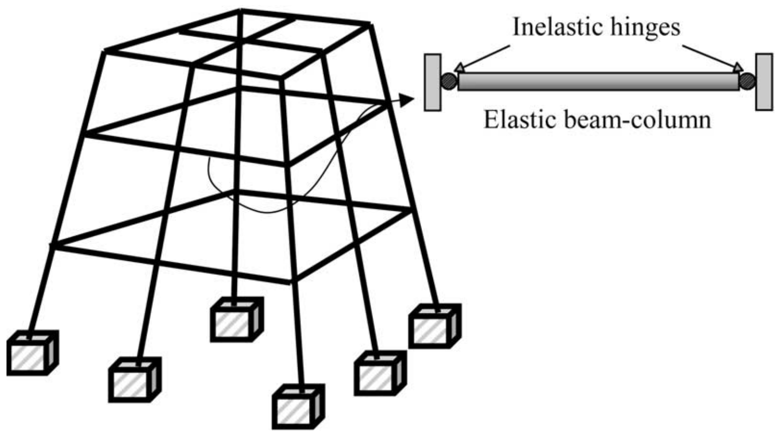

Fig. 3. Inelastic hinges in a frame member.

the biaxial bending case of a similar concept introduced in the case of planar frames ([2,3]): $\mathbf{D}^+ = (d_{iy}^+, d_{jy}^+, d_{iz}^+, d_{jz}^+)$ and $\mathbf{D}^- = (d_{iy}^-, d_{jy}^-, d_{iz}^-, d_{jz}^-)$. All those damage parameters can take values between zero and one, where zero represents a non-damaged hinge and one a totally damaged hinge with no stiffness at all, i.e. a totally damaged hinge behaves as internal hinges in elastic frames.

The damage parameters with the superscript + (respectively −) represent damage, i.e. concrete cracking, due to positive (negative) moments as indicated in Fig. 4. The parameters with subscripts *iy* characterize the damage due to the moment m_{iy} and so on.

3. Damage model

3.1. Flexibility matrix and state law

The model proposed in this paper is described using the notation of continuum mechanics, i.e. in terms of state laws and evolution laws. A formal thermodynamic formulation of models of damage lumped at inelastic hinges can be found in Cipollina et al. [3] and will not be discussed in this paper.

The state law that relates generalized stresses and deformations can be expressed in the same way as in the case of planar frames ([2,4]):

$$\mathbf{\Phi} - \mathbf{\Phi}_p = \mathbf{F}(\mathbf{D}^+)\langle\mathbf{M}\rangle_+ + \mathbf{F}(\mathbf{D}^-)\langle\mathbf{M}\rangle_- \quad (1)$$

where $\mathbf{F}$ is the flexibility matrix of the member and the symbols $\langle m\rangle_+$ and $\langle m\rangle_-$ are the positive and negative parts of the variable m, i.e.:

$$\langle m\rangle_+ = \begin{cases} m & \text{if } m \geq 0 \\ 0 & \text{otherwise} \end{cases} \langle m\rangle_- = \begin{cases} m & \text{if } m \leq 0 \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

As shown in the case of planar frames [4], the flexibility matrix depends on the damage variables in the following way:

$$\mathbf{F}(\mathbf{D}) = \mathbf{F}^0 + \mathbf{C}(\mathbf{D}) \quad (3)$$

where $\mathbf{F}^0$ is the conventional elastic flexibility matrix, as it can be found in text books of structural analysis, and $\mathbf{C}(\mathbf{D})$ represents the additional flexibility that results from concrete cracking.

The hinge flexibility term $\mathbf{C}(\mathbf{D})$ is a diagonal matrix that is obtained by generalization of the case of planar frames [2,4]:

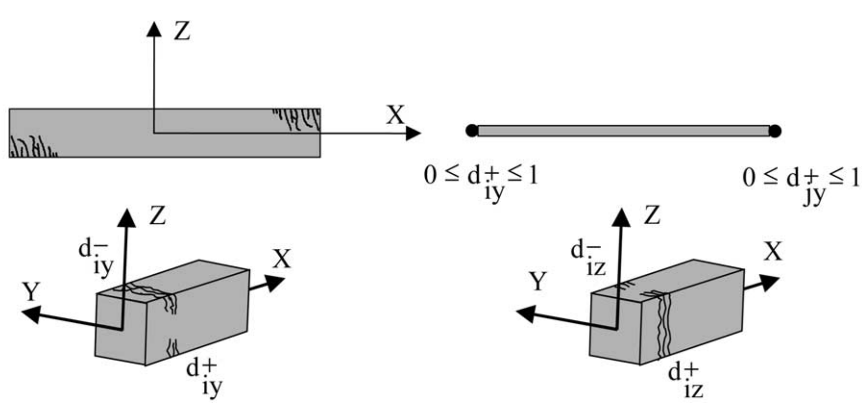

Fig. 4. Damage variables.

$$C_{11}^{+} = \frac{d_{iy}^{+}F_{11}^{0}}{1-d_{iy}^{+}};\ C_{11}^{-} = \frac{d_{iy}^{-}F_{11}^{0}}{1-d_{iy}^{-}};\ C_{22}^{+} = \frac{d_{jy}^{+}F_{22}^{0}}{1-d_{jy}^{+}};$$
$$C_{22}^{-} = \frac{d_{jy}^{-}F_{22}^{0}}{1-d_{jy}^{-}}C_{33}^{+} = C_{33}^{-} = C_{66}^{+} = C_{66}^{-} = 0 \tag{4}$$
$$C_{44}^{+} = \frac{d_{iz}^{+}F_{44}^{0}}{1-d_{iz}^{+}};\ C_{44}^{-} = \frac{d_{iz}^{-}F_{44}^{0}}{1-d_{iz}^{-}};\ C_{55}^{+} = \frac{d_{jz}^{+}F_{55}^{0}}{1-d_{jz}^{+}}$$
$$C_{55}^{-} = \frac{d_{jz}^{+}F_{55}^{0}}{1-d_{jz}^{+}}$$

It can be noticed that the flexibility of the hinge increases with damage evolution. In this way, stiffness degradation due to concrete cracking is represented.

The state law (1) and the flexibility matrices are not enough to analyze a frame or define a constitutive model since additional sets of variables (plastic deformations and damage) have been introduced. Equations that describe the evolution of these internal variables must be added and are introduced in the next sections.

3.2. The generalized Griffith criterion

In the case of planar frames [2], a generalized form of the Griffith criterion was used to describe damage evolution in a hinge. It is proposed in this paper to follow the same approach in the biaxial case.

It is perhaps useful to recall that the classic Griffith criterion, used in fracture mechanics, states that crack propagation can occur only if the energy released upon crack growth is high enough to provide the energy required for that propagation. The Griffith criterion is usually written in the following way:

$$G = R \tag{5}$$

Where G is the energy release rate that is computed in a continuum in the following way:

$$G = -\frac{\partial U^{*}}{\partial a} \tag{6}$$

The symbol U^{*} represents the complementary elastic strain energy and a is the crack length (Fig. 5). The term

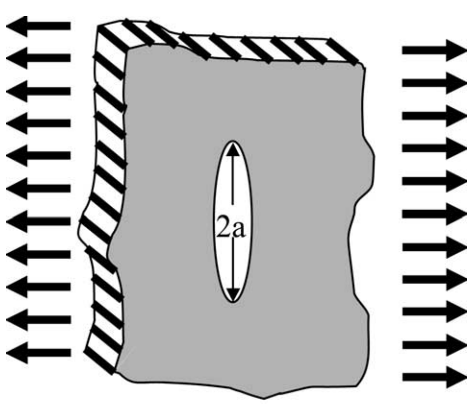

Fig. 5. Crack in a continuum.

R is called crack resistance and it is assumed to be a material property, although it can depend on the crack extension ([12]). If the energy release rate G is lower than the crack resistance R, there is no crack propagation.

In the case of a frame member, the complementary strain energy can be determined from the state law (1) giving:

$$U^{*} = \frac{1}{2}\langle \mathbf{M}\rangle_{+}^{\mathrm{T}}\mathbf{F}(\mathbf{D}^{+})\langle \mathbf{M}\rangle_{+} + \frac{1}{2}\langle \mathbf{M}\rangle_{-}^{\mathrm{T}}\mathbf{F}(\mathbf{D}^{-})\langle \mathbf{M}\rangle_{-} \tag{7}$$

In the case of frames with lumped inelasticity, the damage parameters represent, in a global way, the extension of the member cracks. Therefore, four different energy release rates can be obtained for each hinge of the frame member. For instance, for the hinge i, the expressions of the energy release rates are:

$$\begin{aligned} G_{iy}^{+} &= -\frac{\partial U^{*}}{\partial d_{iy}^{+}} = \frac{F_{11}^{0}}{2}\left(\frac{\langle m_{iy}\rangle_{+}}{1-d_{iy}^{+}}\right)^{2}; \\ G_{iy}^{-} &= -\frac{\partial U^{*}}{\partial d_{iy}^{-}} = \frac{F_{11}^{0}}{2}\left(\frac{\langle m_{iy}\rangle_{-}}{1-d_{iy}^{-}}\right)^{2} \\ G_{iz}^{+} &= -\frac{\partial U^{*}}{\partial d_{iz}^{+}} = \frac{F_{44}^{0}}{2}\left(\frac{\langle m_{iz}\rangle_{+}}{1-d_{iz}^{+}}\right)^{2} \\ G_{iz}^{-} &= -\frac{\partial U^{*}}{\partial d_{iz}^{-}} = \frac{F_{44}^{0}}{2}\left(\frac{\langle m_{iz}\rangle_{-}}{1-d_{iz}^{-}}\right)^{2} \end{aligned} \tag{8}$$

The explicit form of the crack resistance of an inelastic hinge has been obtained on the basis of experimental results [3]. It has been found [5], that the crack resistance depends on the cross-member properties of the frame member, the value of the damage variable and the axial force. The procedure for the computation of the crack resistance R is described in detail in Perdomo et al. [5] and will not be discussed in this paper. Then, the generalized Griffith criterion for hinge i can be written as:

$$\begin{aligned} &G_{iy}^{+} = R(d_{iy}^{+};n);\ G_{iy}^{-} = R(d_{iy}^{-};n); \\ &G_{iz}^{+} = R(d_{iz}^{+};n);\ G_{iz}^{-} = R(d_{iz}^{-};n) \end{aligned} \tag{9}$$

As in the continuum mechanics case, the generalized Griffith criterion states that there may be crack propagation in a plane only if the corresponding energy release rate keeps the value of the crack resistance term:

$$\begin{cases} \dot{d}_{iy}^{+} = 0 \text{ if } G_{iy}^{+} < R(d_{iy}^{+}) \text{ or } \dot{G}_{iy}^{+} < \dot{R}(d_{iy}^{+}) \\ \dot{d}_{iy}^{+} > 0 \text{ if } G_{iy}^{+} = R(d_{iy}^{+}) \text{ and } \dot{G}_{iy}^{+} = \dot{R}(d_{iy}^{+}) \end{cases} \tag{10}$$

where the dot over the damage, energy release rate and crack resistance terms represents time derivatives.

It can be noticed that the value of damage variables can only increase or be constant. The last condition in the first line of (10) characterizes an elastic unloading. It is therefore stated that there is no damage evolution during elastic unloading whatever the value of the energy release rate. The last condition in the second line of (10) asserts that the energy release rate can never be greater than the crack resistance.

It can be noticed by inspection of (9) that the uncoupling of the damage processes has been assumed, i.e. it is assumed that the extension of the cracks due to moments in the *XY* plane have no influence in the damage evolution due to moments in the *XZ* plane and vice versa. Probably, this assumption is only acceptable up to some limited degree of cracking. However, this is the simplest approach to biaxial damage and the numerical examples carried out so far show that the uncoupling hypothesis leads to reasonably good results, even in cases with relatively high levels of damage. So far, the necessity of a more sophisticated approach including damage coupling has not been found.

3.3. Yield function

In the case of planar frames ([2,5]), the yield function of a damaged hinge was obtained from the perfectly plastic one by the consideration of two concepts: the kinematic hardening idea and the deformation equivalence hypothesis. The same procedure can be followed in the biaxial case. The function proposed by Bresler [13] can be used as a starting point:

$$f_i(\mathbf{M}) = \left[\max\left(\frac{\langle m_{iy}\rangle_+}{M^+_{0y}(n)};\frac{\langle -m_{iy}\rangle_+}{M^-_{0y}(n)}\right)\right]^{\nu} + \left[\max\left(\frac{\langle m_{iz}\rangle_+}{M^+_{0z}(n)};\frac{\langle -m_{iz}\rangle_+}{M^-_{0z}(n)}\right)\right]^{\nu} - 1 \tag{11}$$

where $M^+_{0y}(n)$, $M^-_{0y}(n)$, $M^+_{0z}(n)$ and $M^-_{0z}(n)$ are the yielding moments under loadings in the *XZ* and *XY* planes, positive and negative. The parameter ν depends on the properties of the member cross-section.The surface defined as $f_i = 0$ represents, in the moments and axial force space, the conventional interaction surface of the classic RC theory [14] (Fig. 6).

The idea of kinematic hardening comes from the theory of plasticity in continuum media and is used to represent the Baushinger effect. This concept states that during plastic evolution, the yield function does not remain motionless in the stress space, as the perfectly plastic one, but experiences a movement. Specifically, it is assumed that the center of the interaction surface displaces following some evolution law. Therefore, the yield function of a hinge with kinematic hardening is obtained by substitution of the generalized stresses $\mathbf{M}$ with the term $\mathbf{M}-\mathbf{X}$, where the matrix $\mathbf{X}$ represents the position of the interaction surface center. The simplest option is to assume that the position of the center is proportional to the plastic rotations, then the matrix $\mathbf{X}$ is given by:

$$\mathbf{X} = (c_y\phi^p_{iy}, c_y\phi^p_{jy}, 0, c_z\phi^p_{iz}, c_z\phi^p_{jz},) \tag{12}$$

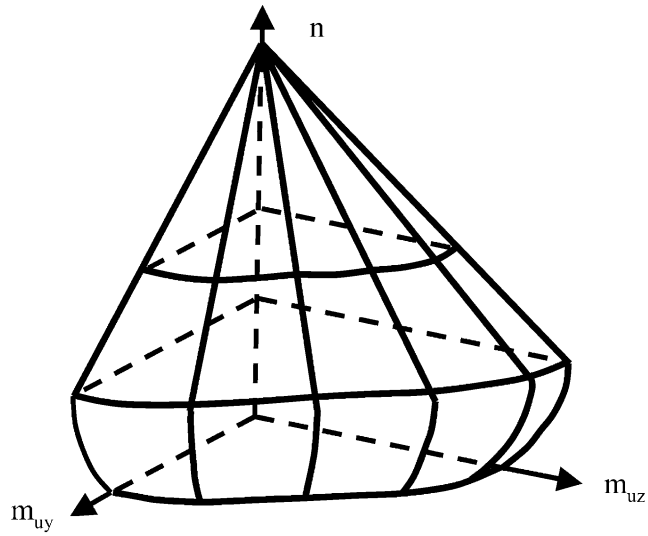

Fig. 6. Interaction surface.

Where the terms c_{iy}, c_{jy} and so on are the proportionality parameters that depend on the cross-member properties, the sign of the moment and the axial force. Therefore the yield function of a hinge with kinematic hardening, but still without damage, becomes:

$$\begin{gathered} f_i = [\max\left(\frac{1}{M^+_{0y}(n)}\langle m_{iy} - c^+_y(n)\phi^p_{iy}\rangle_+;\right. \\ \left.\frac{1}{M^-_{0y}(n)}\langle -m_{iy} + c^-_y(n)\phi^p_{iy}\rangle_+\right)]^{\nu} + \\ [\max\left(\frac{1}{M^+_{0z}}\langle m_{iz} - c^+_z(n)\phi^p_{iz}\rangle_+;\right. \\ \left.\frac{1}{M^-_{0z}}\langle -m_{iz} + c^-_z(n)\phi^p_{iz}\rangle_+\right)]^{\nu} - 1 \end{gathered} \tag{13}$$

The hypothesis of deformation equivalence states that the behavior of a damaged hinge can be described by the same yield function of the intact one, if the moment on the hinge is substituted by another variable called effective moment. The effective moments on a biaxial hinge can be obtained by the generalization of the planar frames case ([2,5]):

$$\begin{gathered} \bar{m}^+_{iy} = \frac{m_{iy}}{1-d^+_{iy}};\ \bar{m}^-_{iy} = \frac{m_{iy}}{1-d^-_{iy}}; \\ \bar{m}^+_{iz} = \frac{m_{iz}}{1-d^+_{iz}};\ \bar{m}^+_{iz} = \frac{m_{iz}}{1-d^-_{iz}} \end{gathered} \tag{14}$$

After substitution of (14) into (13), the yield function of a damaged hinge is finally obtained:

$$f_i = \left[\max\left(\frac{1}{M_{0y}^{+}(n)}\left\langle\frac{m_{iy}}{1-d_{iy}^{+}} - c_y^{+}(n)\phi_{iy}^{p}\right\rangle_{+};\right.\right.$$
$$\left.\left.\frac{1}{M_{0y}^{-}(n)}\left\langle\frac{-m_{iy}}{1-d_{iy}^{-}} + c_y^{-}(n)\phi_{iy}^{p}\right\rangle_{+}\right)\right]^{\nu} +$$
$$\left[\max\left(\frac{1}{M_{0z}^{+}}\left\langle\frac{m_{iz}}{1-d_{iz}^{+}} - c_z^{+}(n)\phi_{iz}^{p}\right\rangle_{+};\right.\right. \tag{15}$$
$$\left.\left.\frac{1}{M_{0z}^{-}}\left\langle\frac{-m_{iz}}{1-d_{iz}^{-}} + c_z^{-}(n)\phi_{iz}^{p}\right\rangle_{+}\right)\right]^{\nu} - 1$$

There is no plastic rotation evolution in the hinge, if the yield function f_i takes negative values. It can be noticed that the use of the deformation equivalence hypothesis allows for the consideration of the strength degradation effect, i.e. for higher values of damage, lower flexural moments are needed to start the yielding of the hinge.

In the particular case of a monotonic loading with moments in only one plane, for instance, positive moments in the *XZ* plane, the yield function (15) can be written as:

$$f_i = \left\langle\frac{m_{iy}}{1-d_{iy}^{+}} - c_y^{+}(n)\phi_{iy}^{p}\right\rangle_{+} - M_{0y}^{+}(n) \tag{16}$$

It can be noticed that in such a case, the yield function takes the form described in Perdomo et al. [5]. The interpretation of the terms $c_y^{+}(n)$ and $M_{0y}^{+}(n)$ is, therefore, the same as indicated in that reference and, more important, the procedure for the computation of those parameters is also similar. The only difference is that now, the procedure must be used four times for the determination of the constants $c_y^{+}(n)$, $M_{0y}^{+}(n)$, $c_y^{-}(n)$, $M_{0y}^{-}(n)$, $c_z^{+}(n)$, $M_{0z}^{+}(n)$, $c_z^{-}(n)$ and $M_{0z}^{-}(n)$.

3.4. Plastic rotations evolution law

The yield function (15) determines when the plasticity in a hinge becomes active. However, it does not indicate how this plasticity takes place, specifically it does not indicate the ratio between the plastic rotations ϕ_{iy}^{p} and ϕ_{iz}^{p}. The plastic rotation evolution laws can be obtained via the normality rule ([15]). This assumption comes also from the theory of plasticity and states that the rate of plastic deformations forms a vector normal to the interaction function in the moment space (Fig. 7).

Therefore, the plastic rotations evolution law for hinge *i* is:

$$\dot{\phi}_{iy}^{p} = \dot{\lambda}_i\frac{\partial f_i}{\partial m_{iy}};\ \dot{\phi}_{iz}^{p} = \dot{\lambda}_i\frac{\partial f_i}{\partial m_{iz}}; \tag{17}$$
$$\begin{cases}\dot{\lambda}_i = 0 \ \text{if}\ f_i < 0 \ \text{or}\ \dot{f}_i < 0\\ \dot{\lambda}_i > 0 \ \text{if}\ f_i = 0 \ \text{and}\ \dot{f}_i = 0\end{cases}$$

Where the term λ_i is called in the plasticity literature, the plastic multiplier.

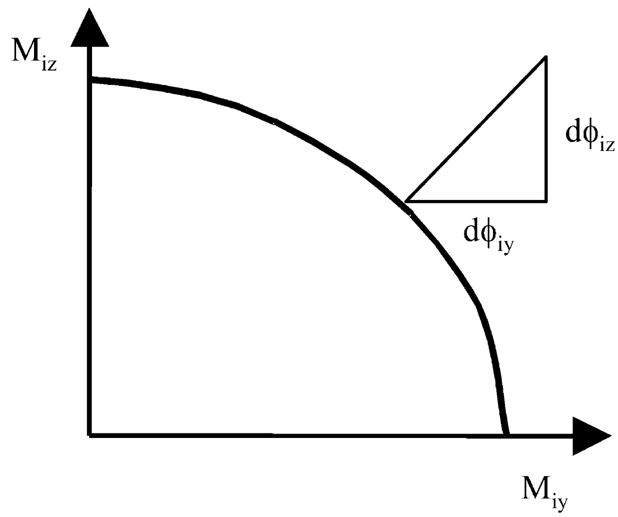

Fig. 7. Normality rule in plasticity.

The state law (1), the generalized Griffith criteria (10) and the plastic rotation evolution law (17) for hinges *i* and *j*, constitute a damage model for RC frame members under biaxial bending.

4. Numerical examples

4.1. Biaxial tests by Bousias et al. [16]

Bousias et al. [16] carried out a complex experimental program on the behavior of RC frame members under biaxial bending. The specimens consisted of reinforced concrete columns built as a cantilever into a heavily reinforced foundation. The columns were subjected to axial load and two lateral actions, in some cases in a force-controlled mode, in the other cases in a stroke-controlled way. The results of twelve of these tests are reported in [16]. The results of the numerical simulation of five of these tests are shown in the next sections. Other simulations can be found in Marante [17].

4.2. Simulation of test S0

This is a conventional cyclic uniaxial test under constant axial load. The loading path is shown in Fig. 8a. It can be noticed that the test consisted of three sequences of deflection cycles in only one lateral direction. Each sequence consists of 13 cycles, the first seven have increasing amplitude and the next ones are decaying cycles. The experimental results registered during each sequence can be seen in Fig. 8b, d and f. The corresponding numerical simulation is presented in Fig. 8c, e and g. The good agreement between model and test can be noticed. The simulation could have been improved by the consideration of the low cycle fatigue effects as indicated in Thomson et al. [18] and Picón [19].

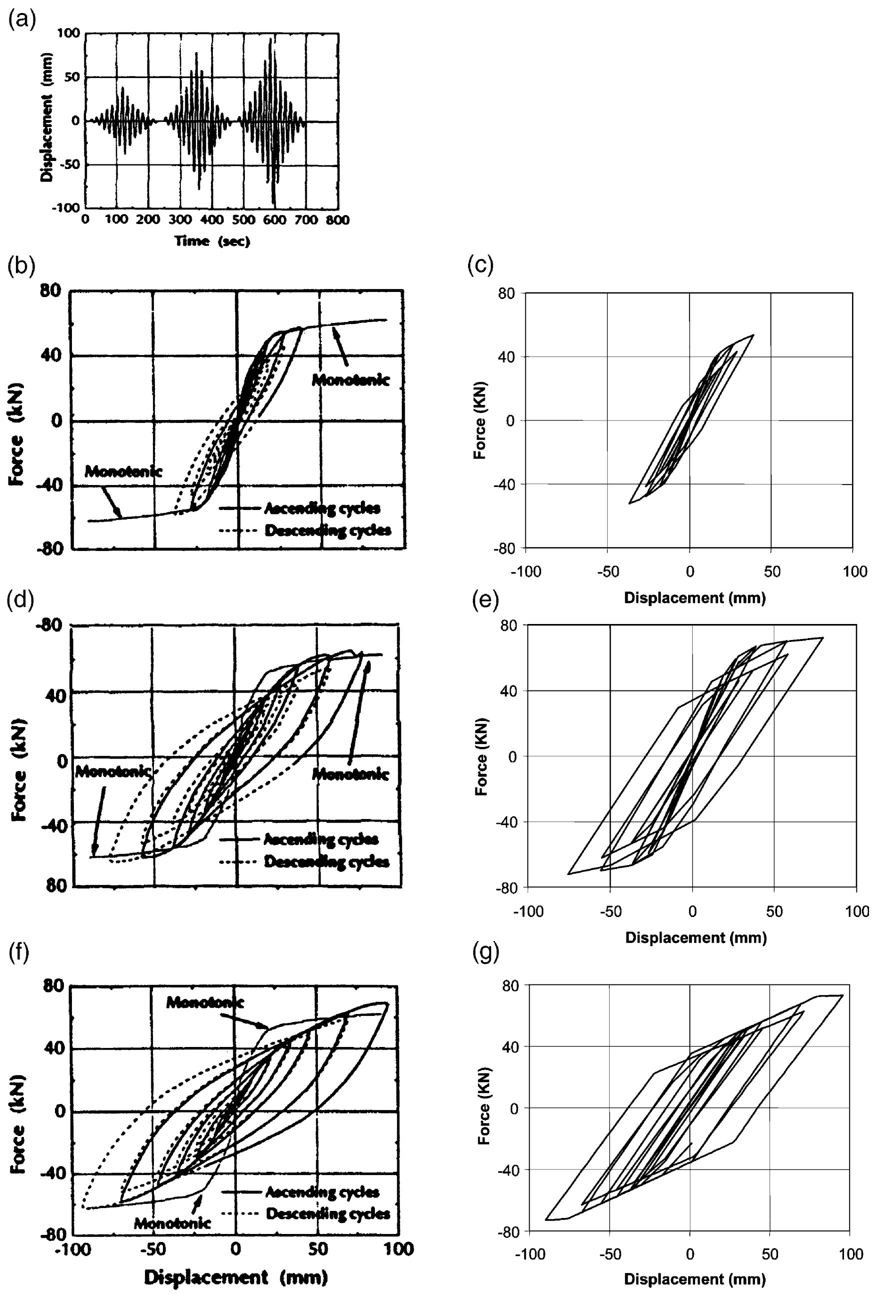

Fig. 8. Experimental results on a RC column after Bousias et al. [16] and numerical simulation: test S0.

The quality of the simulation always depends, as in any model, on the parameters chosen for the particular test under consideration. As indicated in the previous sections, in this model, the parameters needed to compute the crack resistance of the hinge are the cracking, plastic and ultimate moment. All these values can be computed using standard RC theory. However, errors of 10–15 % can be expected in such a case. In the simulations presented in Fig. 8, the computed values of these parameters were adjusted taking into account the experimental results. That is the reason why the simulation is so good. The results would not have been so good if the computed parameters had been used without correction.

4.3. Simulation of test S1

In this test, uniaxial displacement cycles in pairs of linearly increasing amplitude are alternately applied in the two transverse directions (Fig. 9a). Fig. 9b shows the hysteresis loops in both directions corresponding to the experimental test.

As it can be observed, the hysteresis curves obtained through experimental tests in both orthogonal directions are almost identical. In Boussias et al. [16], it is indicated that only a very slight stiffness degradation in the direction of subsequent loading due to the damage caused by preceding cycle in the orthogonal direction was

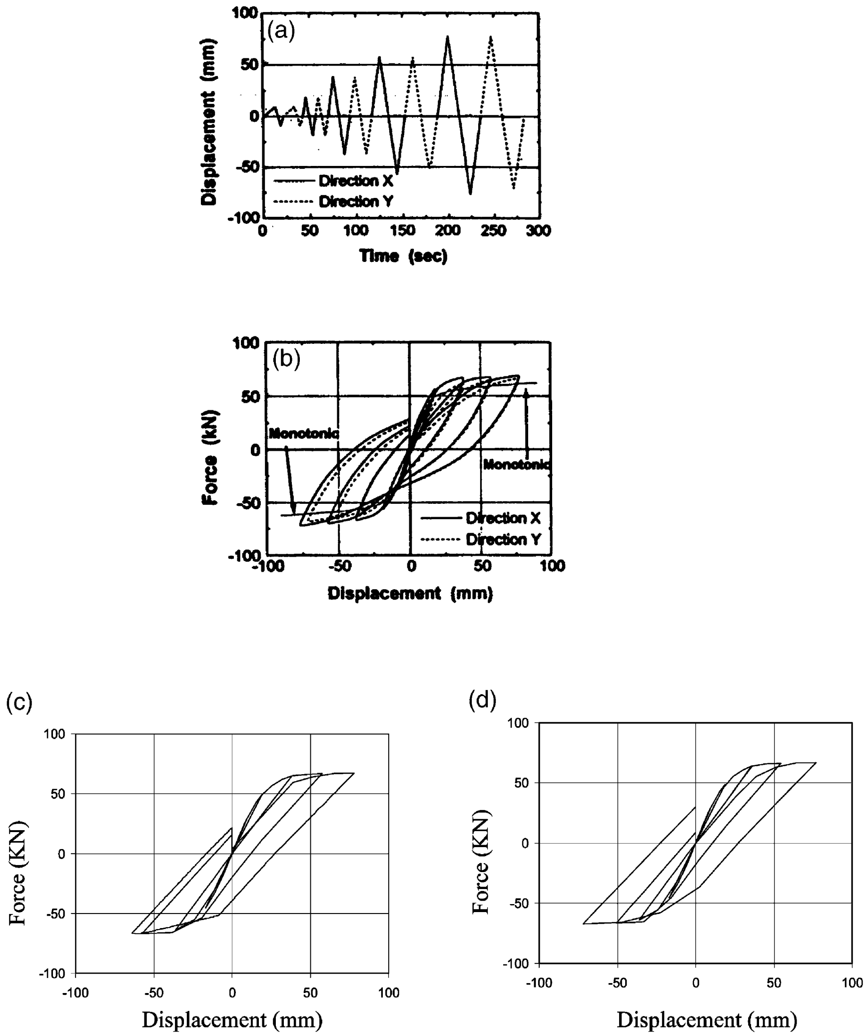

Fig. 9. Experimental results on a RC column after Bousias et al. [16] and numerical simulation: test S1.

observed. This observation appears to justify the hypothesis of the uncoupling of the cracking processes in the present model.

4.4. Simulation of test S3

This third simulation corresponds to a mixed controlled test: displacement controlled in the *Y*-direction and force controlled in the *Z*-direction. The constant level of *Z*-force is gradually increased, while in the orthogonal direction the same set of three deflection cycles with linearly increasing amplitude is applied for each level of the *Y*-force. The loading path is indicated in Fig. 10a, experimental results in Fig. 10b and d, and simulation in Fig. 10c and e.

4.5. Simulation of test S4

This test is similar to S3 except that the imposed force in *Z*-direction changes sign at each sequence with the repetition of three cycles of *Y*-displacement. The loading path is indicated in Fig. 11a, experimental results in Fig. 11b and d and simulation in Fig. 11c and e.

4.6. Simulation of test S6

In this last example, nested butterfly shaped deflection paths are imposed. In Bousias et al. [16], the loading of the test is described in the following terms: the right-hand half of the displacement path is a mirror image of the left-hand half with respect to the origin, but is traced in the opposite sense. More specifically, the *Y*-deflections increase uniaxially from zero up to point 1 (Fig. 12a), and are held constant while the *Z*-deflections increase from zero up to a maximum value, equal to the half of the currently applied *Y*-direction (point 2). Then, the *Y*- and *Z*-deflections are simultaneously returned to zero, and reversed up to the mirror image of point 2, keeping their ratio constant (2:1). The deflections go back to zero independently, first in the *Z*-direction (while holding the *Y*-deflection constant up to the point 4) and then uniaxially in the *Y*-direction to zero and beyond,

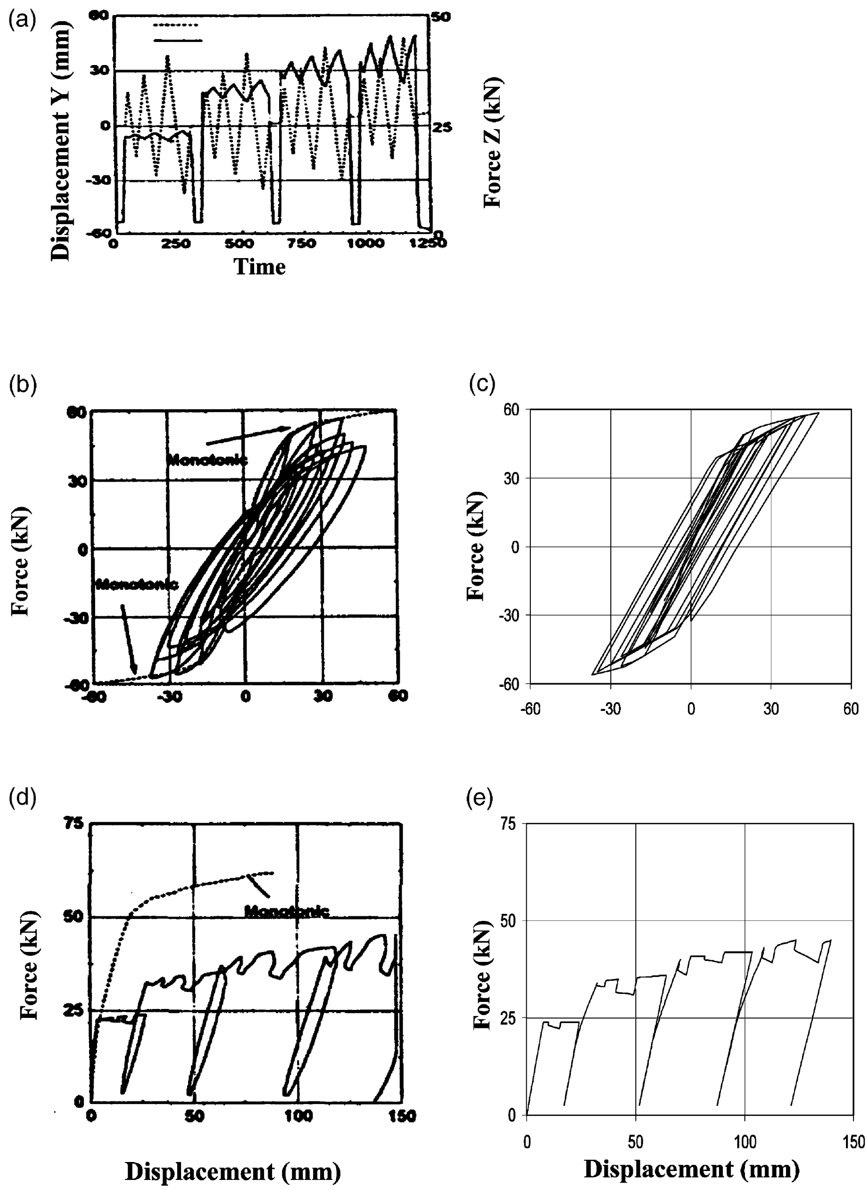

Fig. 10. Experimental results on a RC column after Bousias et al. [16] and numerical simulation: test S3.

repeating the path at a larger size 1′2′3′4′ and so on. As can be observed, during some stages of the loading, both displacements change simultaneously. Again, the loading is indicated in Fig. 12a, experimental results in Fig. 12b and d and simulation in Fig. 12c and e.

4.7. Influence of variable axial forces

As mentioned in previous sections, the crack resistance and the yield functions depend on the level of the axial force n. The influence of variations of the axial force on the flexural damage evolution and the general behavior of the frame member was discussed in Perdomo et al. [5]. Although this analysis was carried out for the planar frame case, the conclusions of that study are also applicable in the present context. Fig. 13 was taken from that paper and shows the numerical simulation results of a rectangular column subjected to cyclic lateral loading and constant axial force (Fig. 13b) and variable axial force (Fig. 13c). In the latter case, the axial force was increased for negative lateral displacements and decreased for positive ones, as in the case of real columns subjected to seismic loadings. It can be noticed that the maximum lateral force of both signs differs significantly. This is due exclusively to the variation of the axial force since the cross-section of the column has a symmetric reinforcement. This effect has been observed experimentally as reported in Abrams [20].

In all the numerical simulations of biaxial behavior, the axial load was assumed as constant since there is no specific information on this matter in Bousias et al [16]. However, most probably the axial force fluctuated during

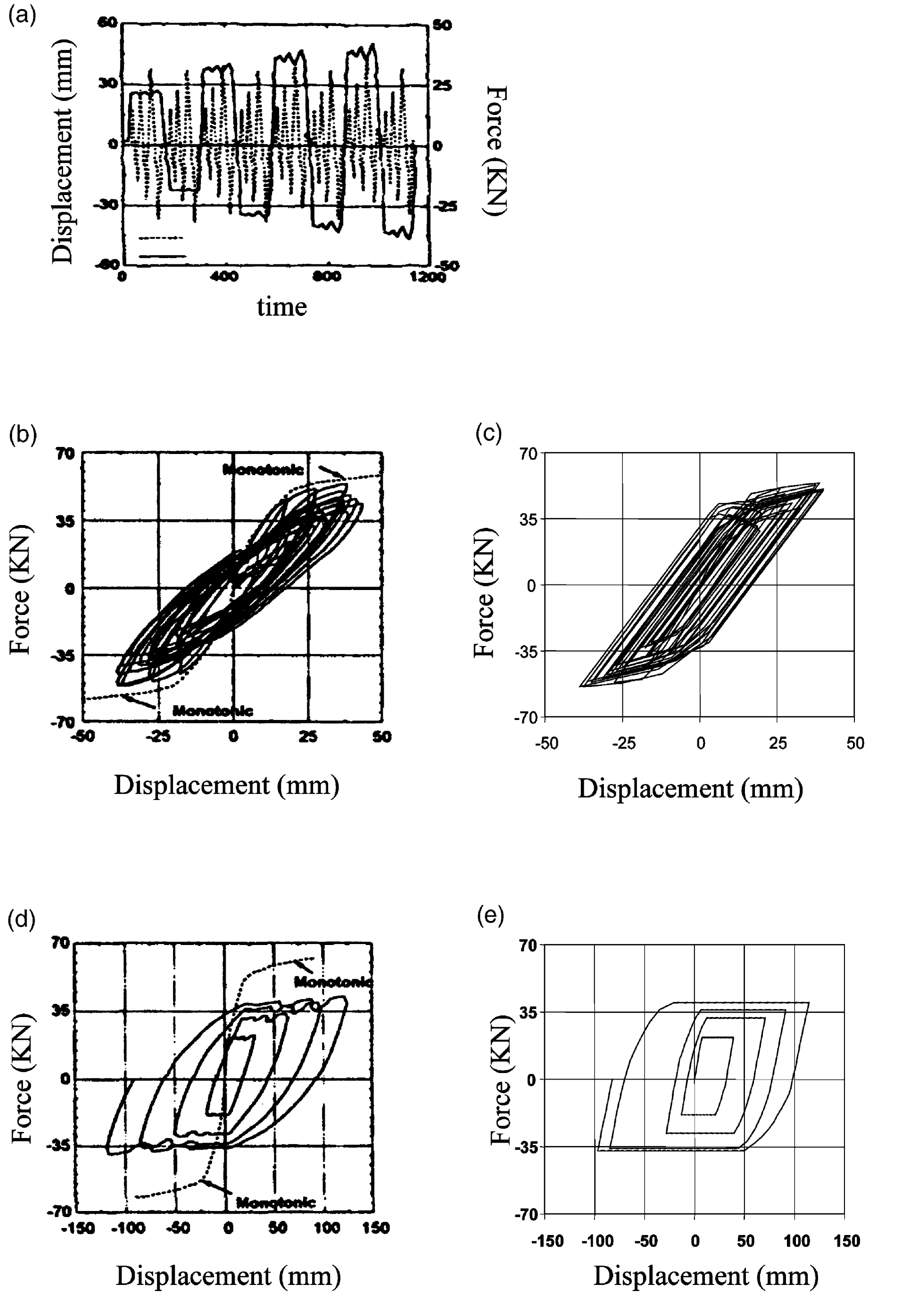

Fig. 11. Experimental results on a RC column after Bousias et al. [16] and numerical simulation: test S4.

the test. This can be observed in Fig. 12d where for positive lateral displacements higher forces are observed than for negative ones. In the simulation (Fig. 12e), this effect could not be reproduced due to the lack of information on the axial force fluctuation.

5. Final remarks and conclusions

It has been shown that LDM can be used to describe the behavior of frame members under biaxial flexure and axial load. LDM can be considered as a branch of fracture mechanics, and integrates concepts such as plastic hinge, damage variable (that is not the same thing as a damage index), energy release rate, deformation equivalence hypothesis and RC standard theory.

In the proposed model, it is assumed that the damage processes are uncoupled in both orthogonal directions. This hypothesis seems to be justified though experimental results. The damage-uncoupling hypothesis allows the computation of the model parameters and the crack resistance terms in a very simple way by considering four monotonic loadings. This is very important in the case of real engineering applications.

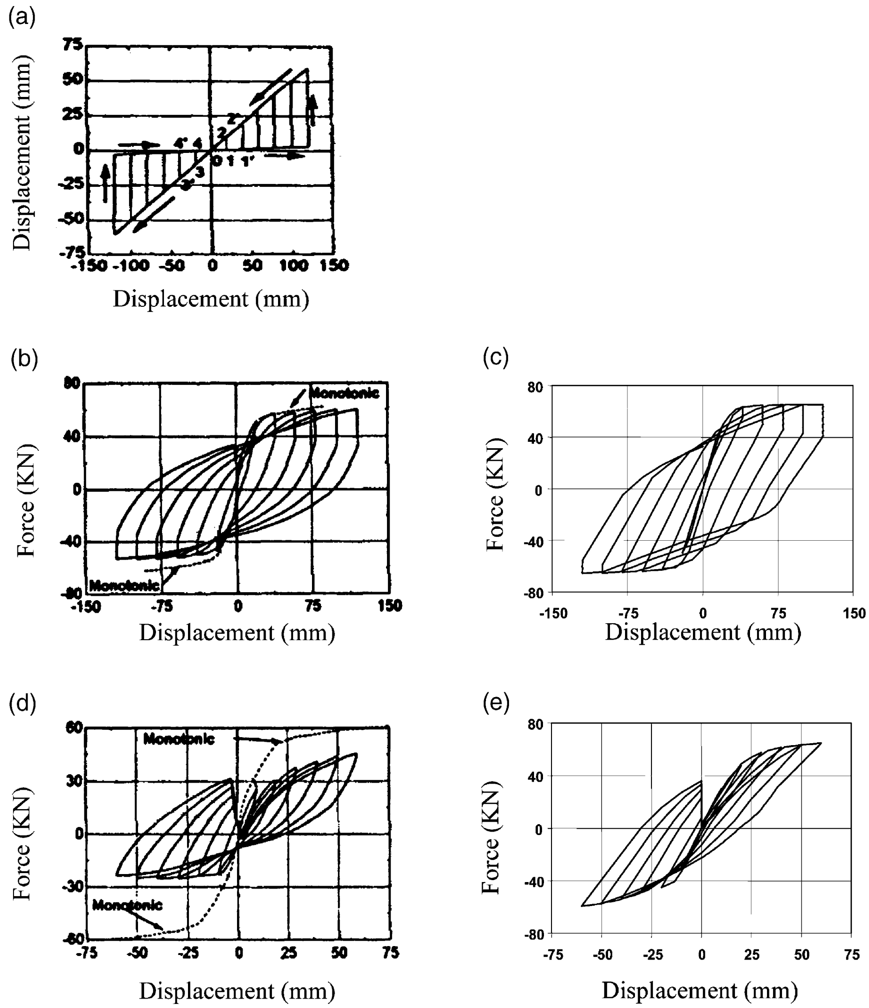

Fig. 12. Experimental results on a RC column after Bousias et al. [16] and numerical simulation: test S6.

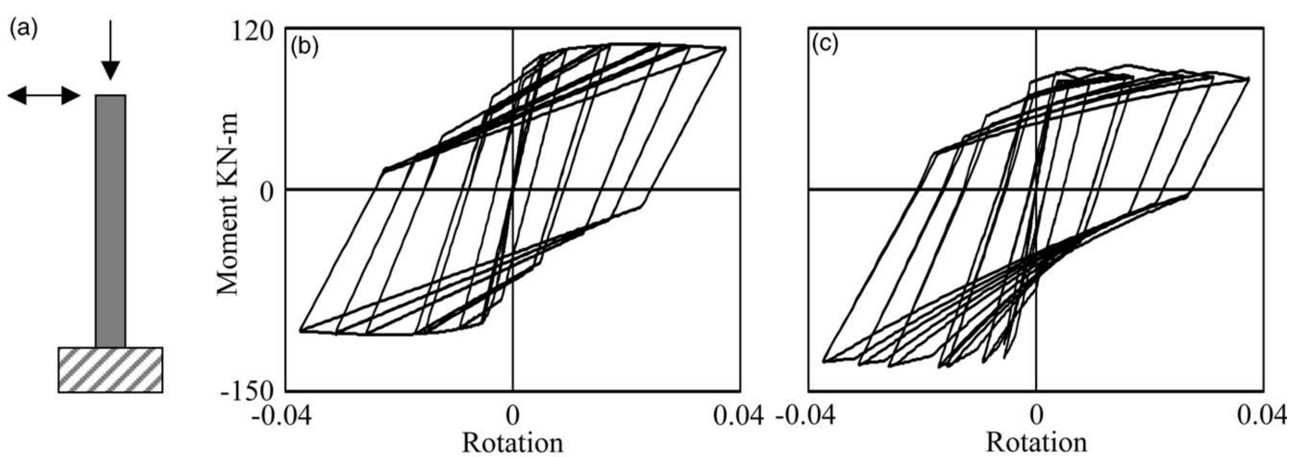

Fig. 13. (a) Column under axial force and lateral displacements. (b) Response under constant axial force. (c) Response under variable axial force.

The resulting model is conceptually as simple as the two-dimensional one and nevertheless gives good enough results for engineering purposes.

The main purpose of the model is to describe in a simple and effective way the behavior of RC frame members. As concrete cracking is one of the main phenomena that take place in the frame member during flexure, the use of fracture mechanics concepts as well as those of standard RC theory has been adopted. As a result a damage variable is needed and has been introduced. However, the damage variable can also been used as a damage index and a relationship with other currently used indices can be established. This aspect of the damage variable has not been discussed in the present paper but some ideas can be found in Alarcón et al. [7].

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