

Evaluation of approximate methods to estimate maximum inelastic displacement demands

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SUMMARY

Six approximate methods to estimate the maximum inelastic displacement demand of single-degree-of-freedom systems are evaluated. In all methods, the maximum displacement demand of inelastic systems is estimated from the maximum displacement demand of linear elastic systems. Of the methods evaluated herein, four are based on equivalent linearization in which the maximum deformation is estimated as the maximum deformation of a linear elastic system with lower lateral stiffness and with higher damping coefficient than those of the inelastic system. In the other two methods the maximum inelastic displacement is estimated as a product of the maximum deformation of a linear elastic system with the same lateral stiffness and the same damping coefficient as those of the inelastic system for which the maximum displacement is being estimated, times a modifying factor. Elastoplastic and stiffness-degrading models with periods between 0.05 and 3.0 s are considered when subjected to 264 ground motions recorded on firm sites in California. Mean ratios of approximate to exact maximum displacements corresponding to each method are computed as a function of the period of vibration and as a function of the displacement ductility ratio. Finally, comments on the advantages and disadvantages of each method when applied to practical situations are given. Copyright © 2001 John Wiley & Sons, Ltd.

KEY WORDS: displacement-based design; performance-based design; approximate methods; inelastic displacement; elastic displacement; equivalent-linear systems; inelastic displacement ratios

1. INTRODUCTION

Practically all structural damage and a large portion of the non-structural damage sustained in buildings as a result of earthquake ground motions is produced by lateral displacements. Thus, the estimation of lateral displacement demands is of primary importance in performance-based earthquake resistant design and in general when damage control is of interest.

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Furthermore, most structures will experience inelastic deformations when subjected to severe earthquake ground motions. Thus, of special interest is an adequate estimation of lateral displacement demands in structures that exhibit non-linear behaviour. This is particularly true in performance-based design in which a better prediction of seismic performance is desired.

When the full characterization of the ground motion is available maximum displacement demands can be computed through time-history analysis. However, in most cases the design of new structures or the evaluation and upgrading of existing structures is not carried out with time history analyses, instead seismic demands are specified with the maximum response of linear elastic single-degree-of-freedom (SDOF) using design linear elastic response spectra or uniform-hazard linear elastic response spectra. Thus, approximate methods to estimate the maximum inelastic displacement demands from the maximum displacement demand of linear elastic SDOF systems are particularly useful in these situations. For example, several recently proposed displacement-based methods [1–8] use the response of linear elastic SDOF systems to estimate the maximum inelastic displacements in bridge and building structures. Similarly, recently published design recommendations [9–11] include analysis procedures where global lateral displacement demands on structures, usually referred to as target displacements, are computed from maximum deformations of linear elastic SDOF systems. In all of these approximate methods a key step is the estimation of maximum inelastic displacement demand of SDOF systems from the maximum displacement demand of linear elastic SDOF systems. Moreover, these approximate analysis methods provide useful insight to the response of inelastic systems to earthquake ground motions that is difficult to obtain from the response time history computed from an individual record. Hence, the evaluation of available approximate methods to estimate maximum inelastic displacement demands of SDOF systems from maximum displacement demands of elastic SDOF systems is specially valuable for users of these recently proposed displacement-based procedures.

The objective of this work is to evaluate six approximate methods to estimate maximum inelastic displacement demands of SDOF systems from maximum displacement demands of elastic SDOF systems which form the underlying principle of most of the approximate analysis procedures used in recently proposed displacement-based design methods. It is beyond the scope of this work to evaluate the use of these methods to estimate inelastic displacement demands of multi-degree-of-freedom (MDOF) systems and to evaluate the actual implementation of the approximate SDOF methods in displacement-based design procedures.

Although there have been several studies [12–15] that have evaluated approximate methods to estimate maximum inelastic displacement demands of SDOF systems, their scope has been very limited. For example Jennings [12] summarized and compared six early proposals of equivalent linearization methods but results were primarily based on harmonic loading and provided no conclusions regarding the accuracy of any of the methods. Hadjian [15] performed an evaluation of seven equivalent linear methods. Although his study provided interesting comparisons and comments regarding various methods when used under harmonic and earthquake loads, the study did not provide any quantitative results regarding the accuracy of the results that can be expected from the user of these methods. Perhaps the best evaluation conducted to date was done by Iwan [13, 14] who evaluated various approximate methods. However, the results only considered 12 earthquake ground motions, three levels of inelastic behaviour and were restricted to nine mid-range periods between 0.4 and 4.0 s, hence no evaluation for short period structures was provided. Error measures provided for

each method were averaged through various periods, thus, information regarding errors for specific periods of vibration was not provided. Furthermore, the error measure used, although provided quantitative results of the size of the averaged errors corresponding to each method, did not provide information whether the approximate method tended to overestimate or underestimate the maximum displacements. Moreover, some of the approximate methods that are being implemented in recent displacement-based design recommendations were not available when these previous evaluation studies were made. Of particular interest to practicing engineers is to know which approximate methods, that provide the basis in recently proposed analysis procedures, produce better results for specific periods of vibration or at least for specific spectral regions as well to know which methods provide better results for specific levels of inelastic behaviour expected to occur in the structure. For example, some methods may provide better results in the short period region than others. Similarly some methods may provide better results for higher levels of inelastic behaviour than others. Equally important to practicing engineers is to be aware of the limitations of these approximate methods to estimate maximum inelastic displacements, and in particular to be aware of the level of errors that can be produced while using these approximate methods.

2. APPROXIMATE METHODS TO ESTIMATE MAXIMUM INELASTIC DISPLACEMENT DEMANDS

Many approximate methods to estimate maximum inelastic displacement demands from maximum elastic displacement in SDOF systems have been proposed. In general, approximate methods commonly used can be classified into two main groups. The first group comprises methods based on equivalent linearization in which the maximum deformation is estimated as the maximum deformation of an equivalent linear elastic system with lower lateral stiffness (higher period of vibration) and with higher damping coefficient than those of the system for which the maximum inelastic displacement is being estimated. The second group includes methods in which the maximum inelastic displacement is estimated as a product of the maximum deformation of a linear elastic system with the same lateral stiffness and same damping coefficient than that of the inelastic system for which the maximum displacement is being estimated times a displacement modification factor.

2.1. *Methods based on equivalent linearization*

The concept of equivalent viscous damping was first proposed by Jacobsen [16] to obtain approximate solutions of the steady forced vibration of damped SDOF systems with linear force–displacement relationships but with damping forces proportional to the n th power of the velocity of motion when subjected to sinusoidal forces. In this pioneering study, the stiffness of the equivalent system was set equal to the stiffness of the real system and the equivalent viscous damping ratio was based on equating the dissipated energy per cycle of the real damping force to that of the equivalent damping force. Years later, the same author extended the concept of equivalent viscous damping to yielding SDOF systems [17] by considering simultaneously an equivalent viscous damping ratio and a period shift. When a period shift is used many different values of equivalent viscous damping can be obtained depending on the selection of the period shift. As noted by Jennings [12] and Hadjian [15] if the equal energy

dissipation principle is employed, different methods of treating the period shift are the reasons for the different equivalent viscous damping ratios given in the literature. Additionally, for a given period shift, variations in the hysteretic model considered will also yield variations in the equivalent damping ratio.

The equation of motion of a SDOF with inelastic hysteretic behaviour under earthquake excitation is given by

$$\ddot{x} + 2\xi_0\omega_0\dot{x} + \frac{F(x)}{m} = -\ddot{x}_g \quad (1)$$

where x is the lateral displacement of the mass relative to the ground; \ddot{x}_g is the ground acceleration; and m , ξ_0 , and $F(x)$ are the mass, damping ratio and the restoring force of the system. The circular frequency of vibration, ω_0 , is given by

$$\omega_0^2 = \sqrt{\frac{k_0}{m}} = \frac{2\pi}{T} \quad (2)$$

where k_0 and T are the initial stiffness and period of vibration of the system.

In equivalent linearization methods the maximum response of the system (whose exact solution is computed with Equation (1)) is approximated with the maximum response of an equivalent linear system whose response x_{eq} is computed with the following equation

$$\ddot{x}_{\text{eq}} + 2\xi_{\text{eq}}\omega_{\text{eq}}\dot{x}_{\text{eq}} + \omega_{\text{eq}}^2 x_{\text{eq}} = -\ddot{x}_g \quad (3)$$

where ξ_{eq} and ω_{eq} are the viscous damping ratio and circular frequency of vibration of the equivalent linear system, which are higher and lower than those of the original system, respectively.

Of the many methods based on harmonic loading available in the literature only the method proposed by Rosenblueth and Herrera [18] will be evaluated here. The method is considered here only for historic reasons because it was the first equivalent linear method to propose the secant stiffness at maximum deformation as the basis for selecting the period shift. The selection of the secant stiffness is considered on many of the approximate analysis methods based on equivalent linearization that are implemented in recently proposed displacement-based design procedures. In Rosenblueth and Herrera's equivalent linearization method, also referred to as geometric stiffness method, the circular frequency of vibration is given by

$$\omega_{\text{eq}} = \sqrt{\frac{k_s}{m}} = \frac{2\pi}{T_{\text{eq}}} \quad (4)$$

where k_s is the secant stiffness at maximum deformation, and T_{eq} is the period of vibration of the equivalent system. For a bilinear system with a post yield stiffness of α times the initial stiffness, the relationship between the period of vibration of the equivalent system to that of the original system is given by

$$\frac{T_{\text{eq}}}{T} = \sqrt{\frac{k_0}{k_s}} = \sqrt{\frac{\mu}{1 - \alpha + \alpha\mu}} \quad (5)$$

where μ is the displacement ductility ratio defined as the ratio of the maximum absolute value of the response to the yield displacement. Similarly, the viscous damping ratio in the

equivalent linear elastic system is given by

$$\zeta_{\text{eq}} = \zeta_0 + \frac{2}{\pi} \left[\frac{(1-\alpha)(\mu-1)}{\mu - \alpha\mu + \alpha\mu^2} \right] \quad (6)$$

For elastoplastic systems ($\alpha=0$) Equations (5) and (6) reduce to

$$\frac{T_{\text{eq}}}{T} = \sqrt{\frac{k_0}{k_s}} = \sqrt{\mu} \quad (7)$$

$$\zeta_{\text{eq}} = \zeta_0 + \frac{2}{\pi} \left(1 - \frac{1}{\mu} \right) \quad (8)$$

Gülkan and Sozen [19] noted that under earthquake loading most of the time the displacement would be significantly smaller than the maximum response, thus the equivalent damping ratio computed with Equations (6) or (8), which are based on harmonic loading, would result in an overestimation of the equivalent viscous damping and hence would lead to an underestimation of the response. Using the Takeda hysteretic model [20] and experimental shake table results of small-scale reinforced concrete frames Gülkan and Sozen developed the following empirical equation to compute the equivalent damping ratio:

$$\zeta_{\text{eq}} = \zeta_0 + 0.2 \left(1 - \frac{1}{\sqrt{\mu}} \right) \quad (9)$$

The empirical method proposed by Gülkan and Sozen [19] was later on extended to MDOF in the well-known substitute structure method [21]. Equations (5) or (7) and Equation (9) are also used in the substitute structure method except that in Equation (9) the displacement ductility ratio is replaced by a damage ratio. For elastoplastic systems these parameters are equal to each other. Here only the method of Gülkan and Sozen is evaluated.

Using a hysteretic model derived from a combination of elastic and Coulomb slip elements together with results from time history analyses using 12 recorded earthquake ground motions, Iwan [14] derived empirical equations to estimate the period shift and equivalent damping ratio as follows:

$$\frac{T_{\text{eq}}}{T} = 1 + 0.121(\mu - 1)^{0.939} \quad (10)$$

$$\zeta_{\text{eq}} = \zeta_0 + 0.0587(\mu - 1)^{0.371} \quad (11)$$

More recently, Kowalsky [22] used the secant stiffness at maximum deformation for defining the period shift together with the Takeda hysteretic model [20] to derive an equation for the equivalent viscous damping ratio. For an unloading stiffness factor of 0.5 and a post yield to initial stiffness ratio, α , the equivalent damping ratio is given by

$$\zeta_{\text{eq}} = \zeta_0 + \frac{1}{\pi} \left(1 - \frac{1-\alpha}{\sqrt{\mu}} - \alpha\sqrt{\mu} \right) \quad (12)$$

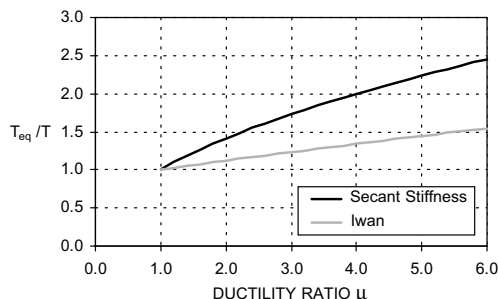


Figure 1. Comparison of periods shifts in equivalent linearization methods.

For systems with post yield stiffness equal to zero Equation (12) reduces to

$$\zeta_{eq} = \zeta_0 + \frac{1}{\pi} \left(1 - \frac{1}{\sqrt{\mu}} \right) \quad (13)$$

Figure 1 shows a comparison of the period shift based on the secant stiffness at maximum deformation as used in Rosenblueth and Herrera, Gülkan and Sozen and Kowalsky's methods compared to that used in Iwan's equivalent linearization method with $\alpha=0$. It can be seen that the period shift based on secant stiffness is larger than that used in Iwan's method and that the difference increases with increasing level of non-linearity. For a displacement ductility ratio equal to four the period shift based on secant stiffness is approximately 50 per cent larger than that used in Iwan's method. A comparison of equivalent damping ratio of the various methods, with ζ_0 and α set to zero, is shown in Figure 2. It can be seen that the equivalent damping ratio in the Rosenblueth and Herrera method, which is based on equating the energy dissipated per cycle of steady response to harmonic excitation in the non-linear and equivalent linear SDOF systems (Equation (8)), leads to the highest value of the equivalent damping ratio. Equivalent damping ratios in Gülkan and Sozen, Iwan's and Kowalsky's methods, Equations (9), (11) and (13) yield significantly lower values. Furthermore, from Equations (9) and (13) it can be seen that when ζ_0 is equal to zero the equivalent damping ratio used in the Kowalsky's method is approximately 1.6 times larger than that in the Gülkan and Sozen method which is very similar to that in the Iwan's method for all ductility levels.

The period shift in Rosenblueth and Herrera, Gülkan and Sozen and Kowalsky's methods is based on secant stiffness so Figure 2 provides enough information to conclude that for a given earthquake ground motion and given level of inelastic behaviour the highest response will be predicted by the Gülkan and Sozen method which uses the smallest equivalent damping ratio of the three methods, followed by Kowalsky's method and then the Rosenblueth and Herrera which uses much higher values of equivalent damping ratio. However, the ratio of energy dissipated per cycle in the non-linear system and energy dissipated per cycle in the equivalent linear system depends on the product of the equivalent damping ratio and the equivalent stiffness, thus for methods that use different period shifts, information provided by Figure 2 is insufficient to provide a comparison of the equivalent damping ratio. In order to provide a better comparison, Hadjian [15] proposed normalizing the equivalent damping ratio by the ratio of initial to equivalent stiffness. Normalized equivalent damping ratios for all four methods are shown in Figure 3. It can be seen that normalized equivalent damping

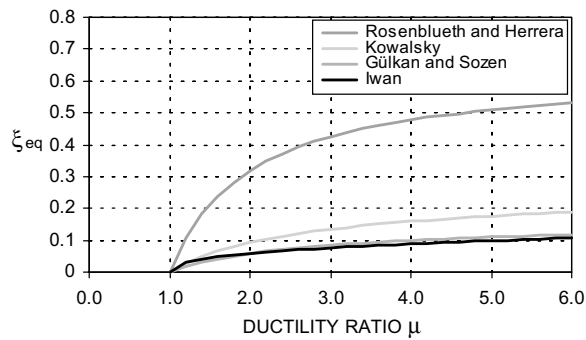


Figure 2. Comparison of equivalent viscous damping ratios in four approximate methods.

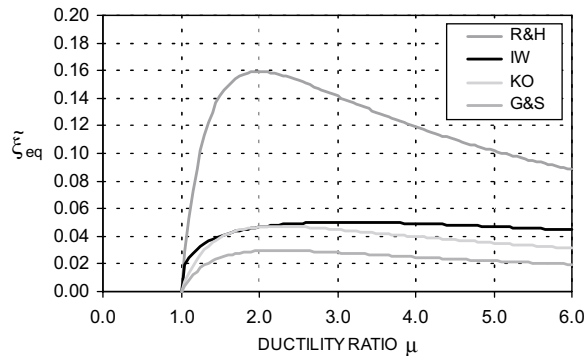


Figure 3. Comparison of normalized equivalent viscous damping ratios in four approximate methods.

ratios in Rosenblueth and Herrera method, which is based on steady-state harmonic response, are significantly higher than those of the other three methods which have been developed specifically for seismic loading. The smallest normalized equivalent damping ratios are those corresponding to the Gülkan and Sozen method while the normalized equivalent damping ratios in Kowalsky's and Iwan's methods are relatively close to each other for displacement ductility ratios smaller than 6. For this level of inelastic deformations, normalized equivalent damping ratios in the Rosenblueth and Herrera method are at least 4.5 times higher than those in the Gülkan and Sozen method.

2.2. Methods based on a displacement modification factor

In this second group of methods the maximum response of the inelastic SDOF system, Δ_i , is estimated as a product of the maximum deformation of a linear elastic system, Δ_e , with the same lateral stiffness and same damping coefficient as that of the inelastic system (i.e. k_0 and

ξ_0) times a displacement modification factor, C , as follows:

$$\Delta_i = C \Delta_e \quad (14)$$

This type of methods have their origin in the study by Veletsos and Newmark [23] who first studied the ratio of the maximum deformation of elastoplastic systems to the maximum deformation of elastic systems having the same initial stiffness and same damping ratio. This and other studies [24] provided the basis for the well-known Newmark and Hall [25] method for estimating inelastic response spectra from elastic response spectra. In this method the displacement modification factor varies depending on the spectral region in which the initial period of vibration of the SDOF system is located in the following manner:

$$C = \mu, \quad T < T_a = 1/33 \text{ s} \quad (15a)$$

$$C = \frac{\mu}{(2\mu - 1)^\beta}, \quad T_a \leq T < T_b = 0.125 \text{ s} \quad (15b)$$

$$C = \frac{\mu}{\sqrt{2\mu - 1}}, \quad T_b \leq T < T_c \quad (15c)$$

$$C = \frac{T_c}{T}, \quad T_c \leq T < T_c \quad (15d)$$

$$C = 1, \quad T \geq T_c \quad (15e)$$

where

$$\beta = \frac{\log(T/T_a)}{2 \log(T_b/T_a)} \quad (16)$$

$$T_c = \frac{\sqrt{2\mu - 1}}{\mu} T_c \quad (17)$$

For an elastic response spectrum based on peak ground acceleration, peak ground velocity and peak ground displacement of 1 g, 121.92 cm/s (48 in/s) and 91.44 cm (36 in), respectively, together with spectral amplification factors corresponding to a 50th percentile and $\xi_0=5$ per cent, the corner period T_c is equal to 0.57 s. The displacement modification factors computed with Equations (15a)–(15e) are shown in Figure 4 for six different levels of inelastic displacement demands. In the Newmark and Hall method Equation (15c) corresponds to the equal energy concept (the absorbed energy is the same in linear and elastoplastic systems at maximum deformation) while Equation (15e) corresponds to the equal displacement approximation, which sometimes is also referred to as the ‘equal displacement rule’.

More recently, Miranda [26] conducted a statistical study of ratios of maximum inelastic to maximum elastic displacements computed from ground motions recorded on firm soils. In that study Miranda concluded that, with the exception of ground motions influenced by forward directivity, the ratio of maximum inelastic to maximum elastic displacement demands was not significantly affected by earthquake magnitude nor by the distance to the source. Similarly, the study concluded that for sites with average shear wave velocities higher than

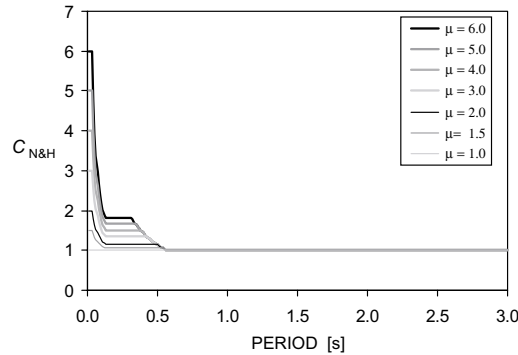


Figure 4. Displacement modification factors in the Newmark and Hall method.

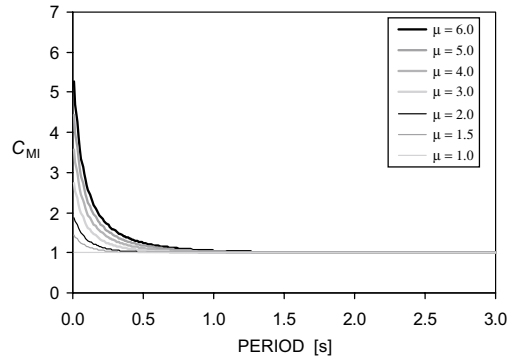


Figure 5. Displacement modification factors in the Miranda method.

180m/s in the upper 30m of the site profile inelastic displacement ratios were not significantly affected by local site conditions and proposed the following simplified expression to compute the displacement modification factor:

$$C = \left[1 + \left(\frac{1}{\mu} - 1 \right) \exp(-12 T \mu^{-0.8}) \right]^{-1} \quad (18)$$

Displacement modification factors computed with Equation (18) are shown in Figure 5. It can be seen that the general trend of the displacement modification factors in Miranda's equation is similar to that in the Newmark and Hall method. Both methods lead to inelastic displacements larger than elastic displacements for short periods, and inelastic displacements equal to elastic displacement in the intermediate and long period spectral regions. Furthermore, both Newmark and Hall and Miranda's methods have adequate limiting values of the displacement modification factors, namely $C = \mu$ as $T \rightarrow 0$ and $C = 1$ as $T \rightarrow \infty$. However, in contrast with the Newmark and Hall method in which the period limiting the equal displacement approximation T_c is constant regardless of the level of inelastic deformation, in the Miranda's

method this period increases with increasing displacement ductility ratio. Moreover, in the short period region the Newmark and Hall has a spectral region (for periods between T_b and T_c') where the displacement modification factor remains constant with changes in the period of vibration while in the Miranda's method in the short period spectral region the displacement amplification increases monotonically with decreasing periods of vibration.

It is important to notice that approximate methods included in the large majority of current building codes usually belong to this second group of methods based on displacement modification factors. However, the displacement modification factors currently specified in US codes [27, 28] are, unfortunately, period independent.

3. EVALUATION OF APPROXIMATE METHODS

3.1. *Systems and ground motions considered*

Evaluation of the approximate methods previously described requires the comparison between 'exact' results computed with non-linear time history analyses with those computed with the approximate methods. However, it is well-known that the maximum response of an inelastic SDOF system is influenced by the hysteretic load–deformation behaviour. Thus consideration on various hysteretic models similar to those used in the development of these approximate methods is necessary. Approximate methods by Gülkan and Sozen and Kowalsky's were developed based on results of inelastic SDOF models with the Takeda hysteretic model [20] while Newmark and Hall and Miranda's methods were developed based on results from the seismic response of elastoplastic SDOF systems. On the other hand, empirical equations in Iwan's method were developed by fitting the results of the seismic response of a model consisting of a combination of a linear and an elastoplastic elements which was used to produce six types of bilinear SDOF systems with various degrees of pinching. In this evaluation three types of hysteretic load–deformation behaviour are considered: the elastoplastic model (EP), the modified Clough stiffness-degrading model [29, 30] (MC) and the Takeda hysteretic model [20] (TA). In all cases the post elastic stiffness was set equal to zero ($\alpha=0$) and the damping ratio to 5 per cent. This damping ratio, although different to that considered in the original studies by Gülkan and Sozen and by Iwan's, is selected here because that is the damping ratio used in the other studies and is equal to the one assumed in most building codes for specifying seismic demands on structures. A set of 50 periods of vibration between 0.05 and 3.0 s were considered with period increments equal to 0.05 s for periods smaller than 2.0 s and equal to 0.1 s for periods between 2.0 and 3.0 s.

A total of 264 earthquake acceleration time histories recorded in the state of California in 12 different earthquakes with surface wave magnitudes ranging from 5.8 to 7.7 were used in this evaluation. All the ground motions selected were recorded on sites with average shear wave velocities higher than 180 m/s (600 ft/s) in the upper 30 m (100 ft) of the site profile which correspond to site classes A, B, C and D according to current U.S. codes [27, 28]. The ensemble of ground motions used in this study is the same previously used by Miranda [26]. For a complete list of all ground motions including peak ground accelerations, earthquake magnitude, site class at the recording station, epicentral distance and distance to the horizontal projection of the fault rupture, the reader is referred to that study.

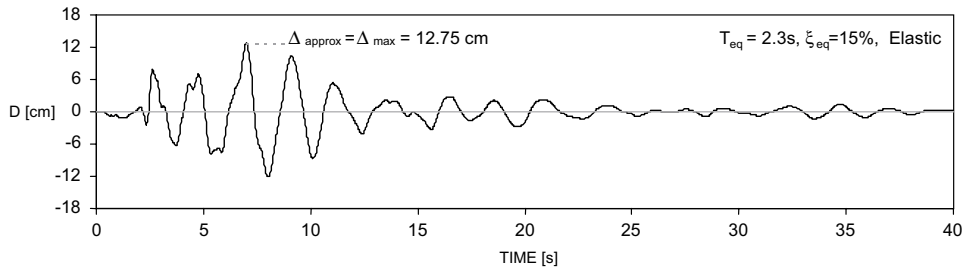


Figure 6. Approximate inelastic displacement computed with equivalent linearization.

3.2. Evaluation procedure and results

The approximate methods were evaluated using the following steps:

STEP 1: Estimation of the maximum inelastic displacement using approximate methods.
1A. For equivalent linearization methods the maximum inelastic displacement is estimated with the following steps:

- Step a. Using the displacement ductility ratio and the period of vibration of the system for which the inelastic displacement want to be estimated calculate the period of vibration of the equivalent system using Equation (7) for Rosenblueth and Herrera, Güllkan and Sozen or Kowalsky's methods or with Equation (10) for Iwan's method.
- Step b. Using the displacement ductility ratio compute the damping ratio of the equivalent linear system using the following equations: for Rosenblueth and Herrera method with Equation (8); for Güllkan and Sozen method with Equation (9); for Iwan's method with Equation (11); and for Kowalsky's method with Equation (13).
- Step c. Compute the displacement time history of the equivalent system with a linear time history analysis using Equation (3) using the equivalent period of vibration and equivalent damping ratios computed in steps a and b.
- Step d. Calculate the approximate maximum inelastic displacement as the maximum absolute value of the time history displacements computed in step c.

An example using the Güllkan and Sozen equivalent linear method for a system with period of vibration of 1.15 s, a damping ratio 5 per cent when undergoing a displacement ductility demand of 4 when subjected to the north–south component of the Corralitos ground motion recorded during the 1989 Loma Prieta, California earthquake is shown in Figure 6. In this case Equation (7) leads to an equivalent period of vibration of 2.3 s which is twice of the original system and Equation (9) leads to an equivalent damping ratio of 15 per cent. As shown in this figure, the Güllkan and Sozen equivalent linear method estimates the maximum inelastic displacement demand on the original system as 12.75 cm. The same example in spectral form is shown in Figure 7.

1B. For methods based on displacement modification factors the maximum inelastic displacement is estimated with the following steps:

- Step a. Compute the linear elastic displacement time history of the original system ($T = 1.15$ s and $\xi_0 = 5$ per cent) using Equation (1) by setting the lateral strength equal

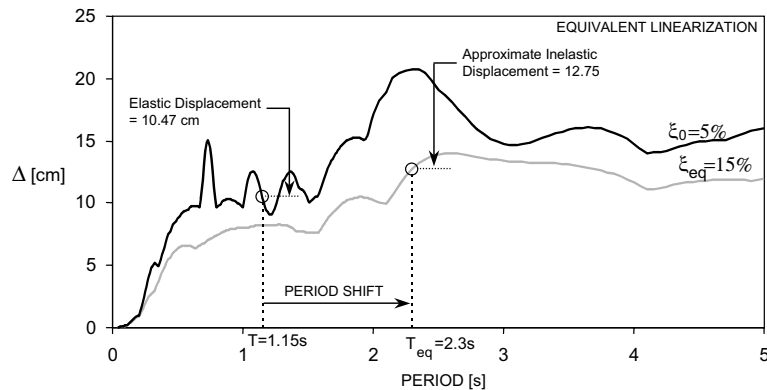


Figure 7. Approximate inelastic displacement based on equivalent linearization.

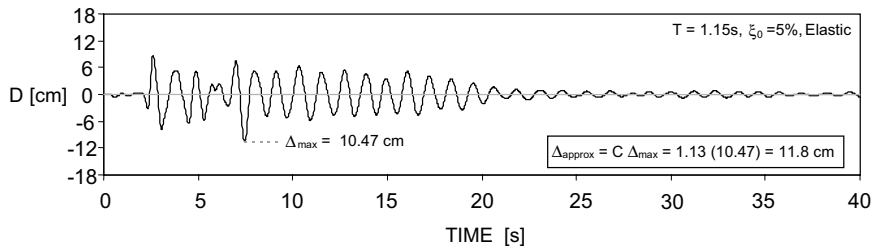


Figure 8. Approximate inelastic displacement computed with a displacement modification factor.

to a very large value to guarantee that the system remains elastic during the total duration of the ground motion.

- Step b. Calculate the maximum elastic displacement demand of the SDOF system, Δ_e , (i.e. the elastic spectral ordinate) as the maximum absolute value of the displacement time history.
- Step c. Compute the displacement modification factor, C , using Equations (15) for the Newmark and Hall method or using Equation (18) for the Miranda method.
- Step d. Calculate the maximum inelastic displacement using Equation (14).

An example using the Miranda method for the same system with period of vibration of 1.15 s, a damping ratio 5 per cent when undergoing a displacement ductility demand of 4 when subjected to the north–south component of the Corralitos ground motion recorded during the 1989 Loma Prieta, California earthquake is shown in Figure 8. It can be seen that maximum elastic displacement in this case is 10.47 cm. For $T = 1.15$ s and $\mu = 4$, the displacement modification factor computed with Equation (18) is 1.13. The approximate maximum inelastic displacement is computed as the maximum elastic displacement times the displacement modification factor, which in this case leads to an approximate maximum inelastic displacement of 11.8 cm. Figure 9 shows the same example in spectral form.

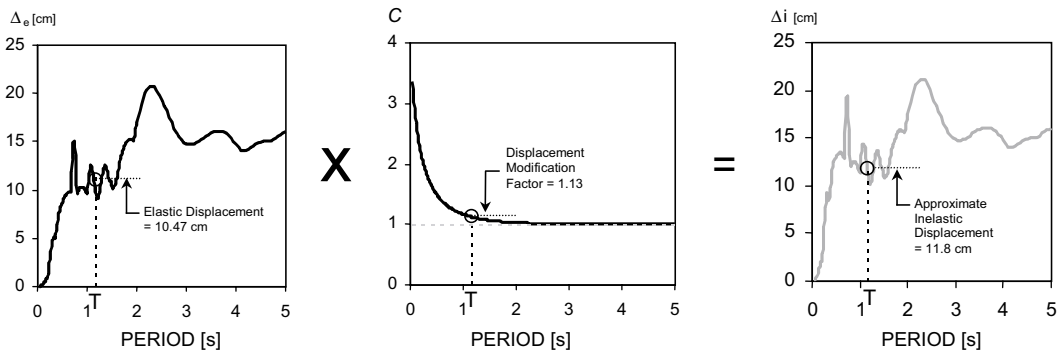


Figure 9. Approximate inelastic displacement computed using displacement modification factors.

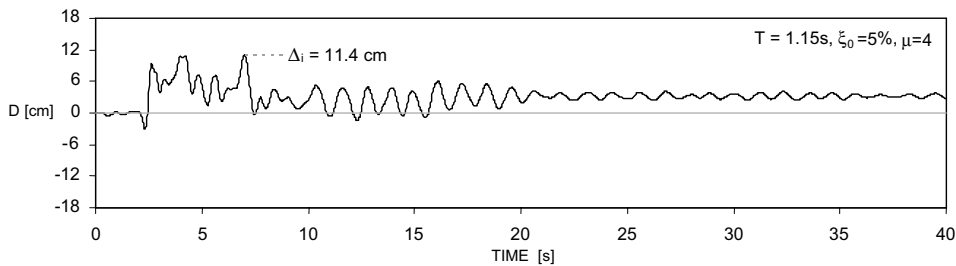


Figure 10. Exact maximum inelastic displacement computed with non-linear time history analysis.

STEP 2: Estimation of the exact maximum inelastic displacement. Exact maximum inelastic displacement demands Δ_i corresponding to specific values of μ were computed through nonlinear time history analyses by iteration on the lateral strength of the system using Equation (1) until the displacement ductility demand was, within a tolerance, equal to the desired displacement ductility ratio. The tolerance was chosen such that Δ_i was considered satisfactory if the computed ductility demand was within 1 per cent of the specified displacement ductility. For each earthquake record, each hysteretic models and each period of vibration, 'exact' maximum inelastic displacements were computed for six levels of inelastic deformation, corresponding to the following displacement ductility ratios: 1.5, 2, 3, 4, 5 and 6. A total of 237 600 'exact' inelastic displacement demands were computed by iteration as part of this investigation.

Figure 10 shows the displacement time history for the system with elastoplastic behaviour computed with nonlinear time history analysis. It can be seen that the maximum inelastic displacement demand is equal to 11.4 cm. Thus, in this example the Gülkan and Sozen equivalent linear method estimates a maximum displacement that is 12 per cent larger than the exact value and the Miranda method produces an estimate that is 3.5 per cent larger than the exact maximum inelastic displacement.

STEP 3: Compute ratio of approximate to exact maximum displacement. These ratios were computed for all combinations of ground motion, period of vibration, level of inelastic behaviour, hysteretic behaviour.

STEP 4: Compute mean and standard deviation of the ratios. For each hysteretic model, each period and each level of inelastic deformation, mean and standard deviation of approximate to exact displacement ratios corresponding to each method were averaged over the 264 ground motions.

Figure 11 shows mean approximate to exact displacement ratios corresponding to each approximate method with exact values computed with the elastoplastic hysteretic model. In these plots, values smaller than one indicate that the approximate method underestimates on average the 'exact' maximum displacement in the inelastic system and values larger than one mean that the approximate method overestimates on average the 'exact' maximum inelastic displacement. It can be seen that, as expected, the largest errors are produced by the Rosenblueth and Herrera method which on average produces important underestimations of the maximum inelastic displacement, particularly for periods of vibration larger than about 0.3 s where the Rosenblueth and Herrera method estimates maximum displacements that on average around half of 'exact' values.

The Gülkan and Sozen method tends to produce conservative estimates of the maximum inelastic displacement, that is, it produces estimates of the maximum inelastic displacement that on average are larger than the 'exact' values. In general, the level of overestimation increases as the level of inelastic deformation increases. This is particularly true for periods smaller than 0.6s where mean overestimations larger than 40 per cent are produced for ductility ratios larger than two. For periods larger than 1.5s this method produces overestimations that on average are between 10 and 20 per cent. The mean approximate to exact displacement ratios corresponding to the Kowalsky's method follow the same trend than the Gülkan and Sozen method, however the overestimations in the short period spectral region are smaller than those of the Gülkan and Sozen method. This is to be expected because in both methods the period shift is based on the secant stiffness and because of the similarities in Equations (9) and (13) for computing the equivalent damping ratio. Furthermore, since the equivalent damping ratio of the Gülkan and Sozen method is smaller than that of the Kowalsky's method (see Figures 2 and 3), then the displacement estimates of the Kowalsky's method are smaller those of the Gülkan and Sozen method. For periods longer than 1.0 s the estimations are in general very good for all levels of inelastic deformation, with average errors smaller than 10 per cent.

With the exception of periods smaller than 0.2 s, the Iwan's method produces very good estimations of the maximum displacement of elastoplastic systems. In general, this method tends to underestimate the maximum displacement. However, for periods longer than 0.2 s underestimations are on average smaller than 15 per cent. Underestimations increase with increasing displacement ductility ratios.

Mean approximate to exact displacement ratios for the Newmark and Hall and Miranda's methods have a similar trend. For periods longer than 1.0 s both methods produce very good results with estimations that on average are only slightly conservative (overestimations smaller than 10 per cent). In particular, for periods longer than 1.5 s both methods produce the same results since both procedures use the equal displacement approximation in this spectral region. For periods smaller than 1.0s the Miranda's methods produces better estimations than those of the Newmark and Hall method. The largest errors in the Miranda's method are produced for periods around 0.2s where the method produces overestimations that increase from 8 per cent for displacement ductility ratio of 1.5 to 26 per cent for a displacement ductility ratio of 6.

Figure 12 presents mean approximate to exact displacement ratios produced by the 6 methods while estimating the maximum displacement of SDOF systems with the modified Clough

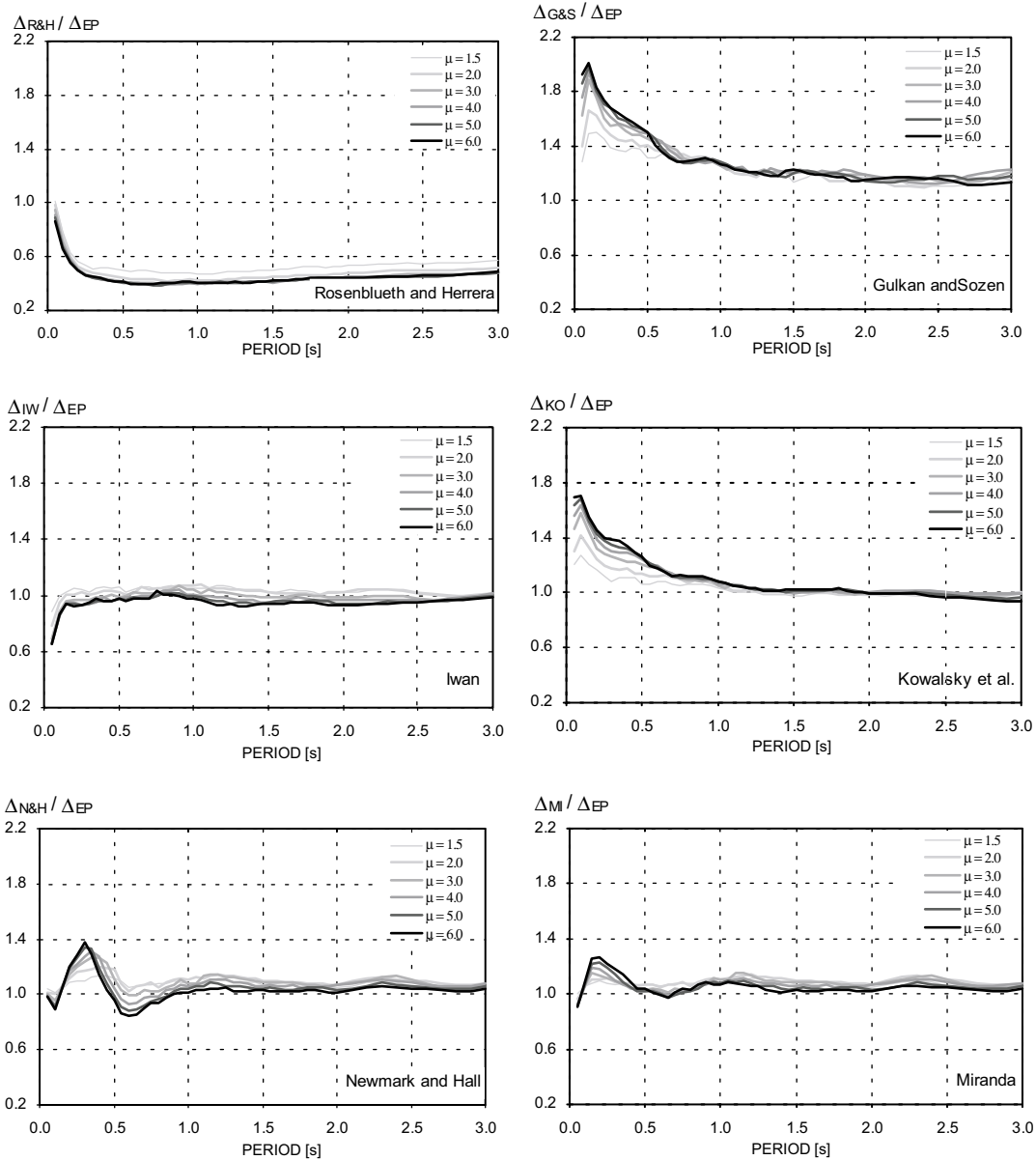


Figure 11. Mean approximate to exact displacement ratios for systems with elastoplastic hysteretic model.

hysteretic model. Again the largest errors are produced by the Rosenblueth and Herrera method whose equivalent damping ratio was derived based on equating the energy dissipated per cycle in the nonlinear system to that of the equivalent linear system using the steady state response to harmonic loads. For systems with periods larger than 0.5 s the best estimations

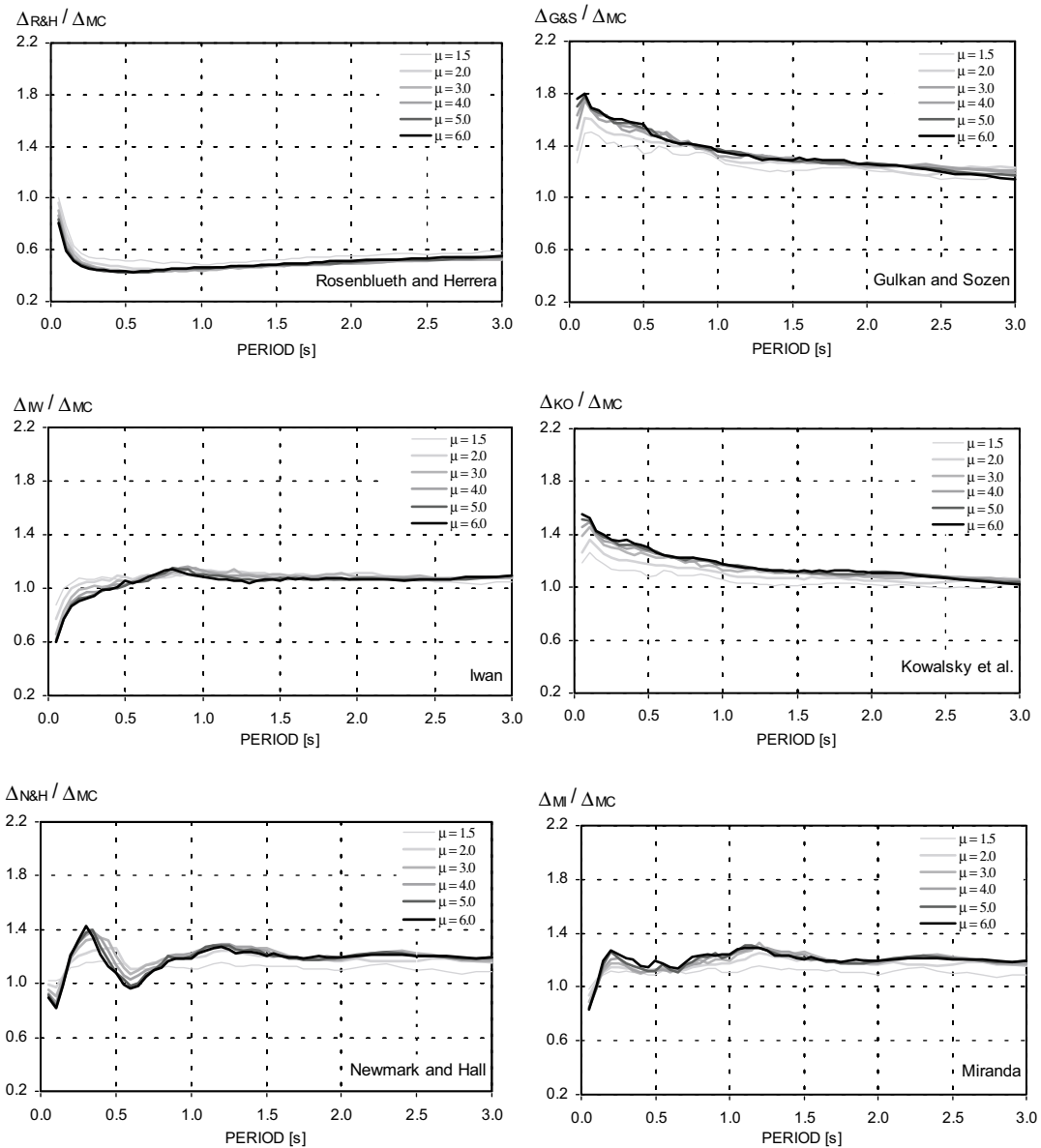


Figure 12. Mean approximate to exact displacement ratios for systems with modified Clough hysteretic model.

are produced by the Iwan's method, that in this spectral region on average tends to overestimate the maximum displacements by about 10 per cent. Errors for the Gülkan and Sozen and Kowalsky's methods decrease with increasing periods and with decreasing displacement ductility ratio. Again, on average Kowalsky's method produces better results than the Gülkan

and Sozen method. For periods larger than 1.0 s Kowalsky's method produces errors that on average are smaller than 15 per cent. In this spectral region both the Newmark and Hall and Miranda's method tend to overestimate the maximum inelastic displacement of systems with the modified Clough model from about 9 per cent for displacement ductility ratios of 1.5 to about 25 per cent for a displacement ductility ratio of 6. For periods between 0.1 and 0.5 s the Gülkan and Sozen, Kowalsky's, Newmark and Hall and Miranda's methods overestimate on average the maximum displacement while the Rosenblueth and Herrera and Iwan's methods underestimate the maximum displacement.

Mean values of approximate to exact displacement ratios produced by the six methods while estimating the maximum displacement in SDOF systems with the Takeda hysteretic model are presented in Figure 13. For periods smaller than 0.7 s the smallest errors are those of the Miranda's method that tends to overestimate the maximum displacement. For periods larger than 0.7 s the smallest errors are those of the Iwan's method, which on average overestimates the maximum displacements by less than 8 per cent. In the short period region the underestimations are more important and increase with decreasing periods and as the level of inelastic deformation increases. In the Gülkan and Sozen and Kowalsky's methods mean errors increase as the period of vibration decreases. Errors produced while estimating maximum displacements of systems with the Takeda model are in general very similar to errors produced while estimating the maximum response of systems with the modified Clough hysteretic model.

The standard deviation of the relative error for the different methods when estimating the maximum response of SDOF systems with elastoplastic behaviour are shown in Figure 14. It can be seen that with the exception of the Rosenblueth and Herrera method, the standard deviation of the error increases as the level of deformation increases. The smallest standard deviations of relative errors are those of the Rosenblueth and Herrera method, however, it is the method with the largest errors. For periods longer than 1.0 s the standard deviation of the relative error is approximately period independent. The Newmark and Hall and Miranda's methods have an increase in standard deviation of the relative error for periods smaller than 0.5 s.

In general all methods produce better results in the intermediate and long period regions than in the short period region. For periods longer than 1.0 s the Iwan's, Kowalsky's, Newmark and Hall, and Miranda's methods produce estimations of the maximum displacement of inelastic SDOF systems that on average are relatively good, however, the latter two methods are much easier to use in practical situations since they are based on the equal displacement approximation, thus elastic results can be directly used as a relatively good estimation of the maximum inelastic displacement. This should not be interpreted as the Newmark and Hall, and Miranda methods are preferred methods or that will produce better results, simply that in these spectral regions are easier to use. Furthermore, standard deviations of relative errors of these four methods are significant indicating that when applied to individual records any of these methods could produce significant errors, particularly for large levels of inelastic behaviour. For periods of vibration smaller than 1.0 s both the Miranda's and Iwan's methods produce relatively good results with errors that on average are smaller than 20 per cent.

Results shown in Figures 11–14 correspond to systems where the post yield to initial stiffness ratio, α , is equal to zero. The effect of α on the equivalent damping ratio and on the estimation of displacement demands using equivalent linearization methods was recently studied by Borzi *et al.* [31]. Similarly, a limited number of results of the influence of α on

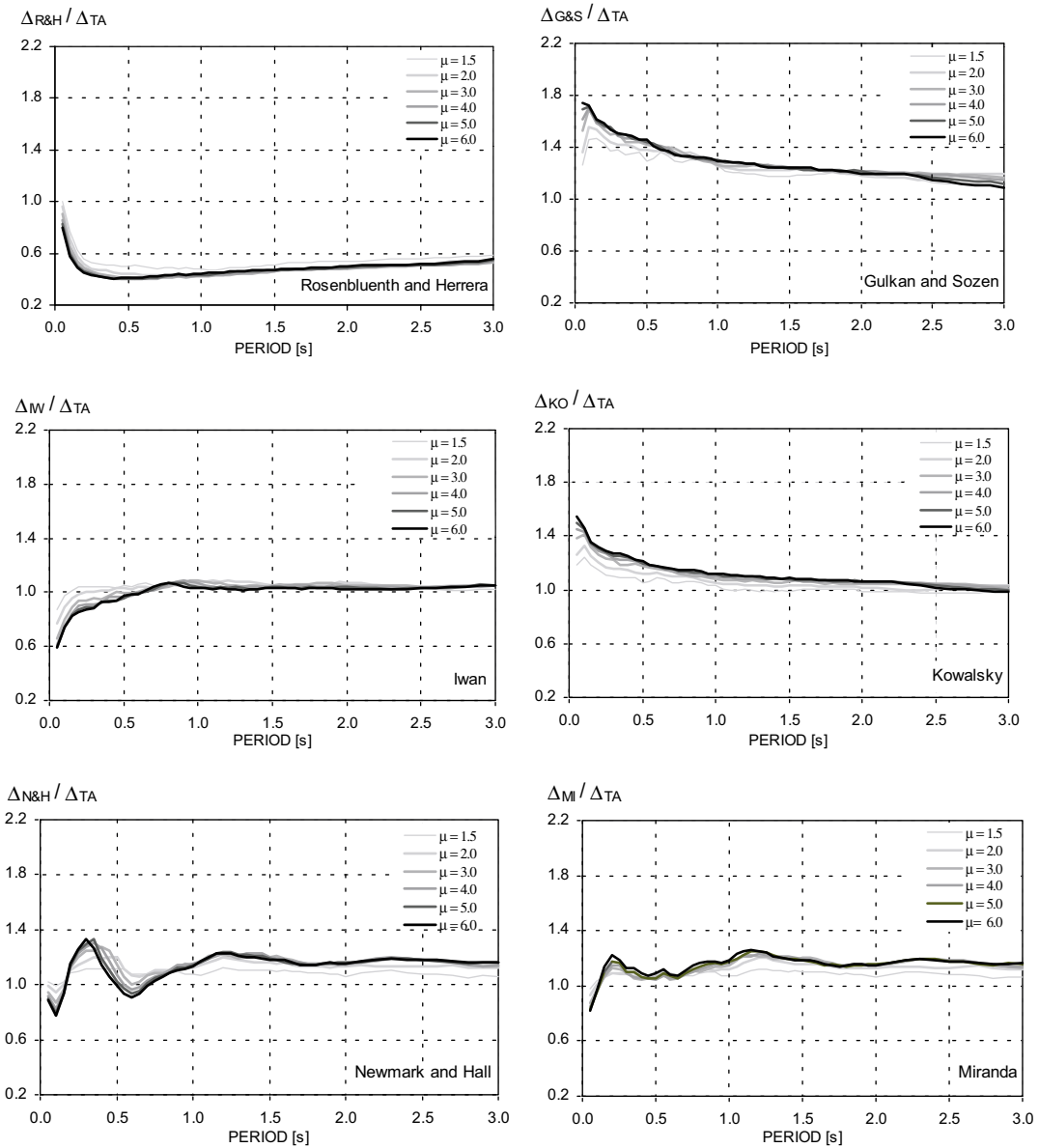


Figure 13. Mean approximate to exact displacement ratios for systems with Takeda hysteretic model.

the estimation of displacement demands using methods on displacement modification factors was studied by Rahnema *et al.* [32]. In general both studies conclude that hardening ($\alpha > 0$) tend to produce small decreases in displacement demands and that softening ($\alpha < 0$) produces increases in displacement demands.

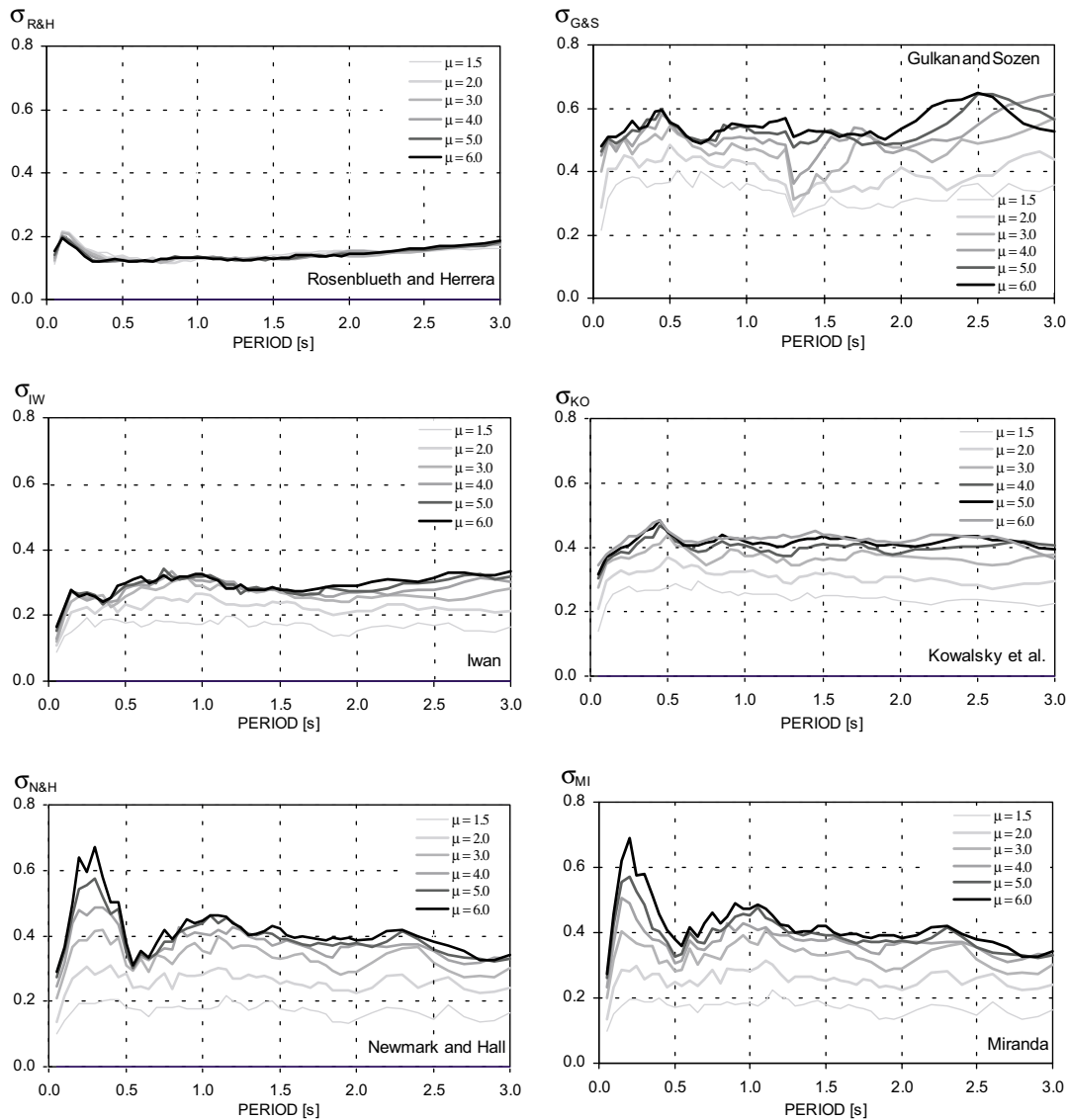


Figure 14. Standard deviation of relative errors for systems with elastoplastic hysteretic model.

Besides the various factors evaluated in this study there are other factors that may influence the estimation of inelastic displacements and that need to be considered in the implementation of any of the methods evaluated herein. Others factors to consider include, but are not limited to, the uncertainty on the estimation of the elastic displacement demand, the uncertainty on the estimation of the yield displacement and initial stiffness, the uncertainty on the estimation of the lateral strength and the displacement ductility ratio, the convergence of iterative methods, etc.

4. CONCLUSIONS

Evaluation of six approximate methods that are used to estimate the maximum inelastic displacement demand of single-degree-of-freedom systems when subjected to earthquake ground motions has been presented. In all six methods, the maximum displacement demand of inelastic systems is estimated from the maximum displacement demand of linear elastic systems. Four methods evaluated in this study, Rosenblueth and Herrera, Gülkan and Sozen, Iwan's and Kowalsky's are based on equivalent linearization in which the maximum deformation is estimated as the maximum deformation of a linear elastic system with lower lateral stiffness and with higher damping coefficient than those of the inelastic system. In the other two methods, Newmark and Hall and Miranda's, the maximum inelastic displacement is estimated as a product of the maximum deformation of a linear elastic system with the same lateral stiffness and same damping coefficient than those of the inelastic system for which the maximum displacement is being estimated times a displacement modification factor. The approximate methods were used to estimate the maximum response of SDOF systems with elastoplastic, modified Clough and Takeda and stiffness-degrading hysteretic load-deformation models with periods between 0.05 and 3.0 s undergoing six different levels of maximum displacement ductility demands when subjected to 264 ground motions recorded on firm sites in 12 California earthquakes. For each method mean ratios of approximate to exact maximum displacement and dispersion of relative errors were computed as a function of the period of vibration and as function of the displacement ductility ratio. The following conclusions can be drawn from the results of this study:

The use of period shifts based on the secant stiffness at maximum deformation together with equivalent ductility ratios based on equating the energy dissipated per cycle in non-linear and equivalent linear SDOF systems subjected to harmonic loads as done in the Rosenblueth and Herrera method produce significant underestimations of the maximum inelastic displacement for all three types of hysteretic models considered in this study. Maximum displacements predicted by this method are on average about half of the maximum displacements computed through time history analyses.

The Gülkan and Sozen, Iwan's and Kowalsky's methods are also based on equivalent linearization but consider equivalent damping ratios significantly smaller than those of the R&M method, thus produce much better results. Mean relative errors in these methods, in general, increase with increasing displacement ductility ratios and with decreasing periods of vibration. In general these methods produce more accurate in the intermediate and long period regions than in the short period region. In the short period spectral region the Gülkan and Sozen and Kowalsky's methods tend to significantly overestimate the maximum displacement while the Iwan's methods underestimates the maximum displacement, particularly for periods smaller than 0.4 s.

Both methods based on displacement modification factors, the Newmark and Hall and Miranda's methods, tend on average to produce small overestimations of the maximum displacements. For periods longer than 0.5 s overestimations are slightly larger when estimating the maximum response of stiffness degrading systems than when estimating the maximum response of elastoplastic systems.

In the intermediate and long period spectral regions (periods of vibrations longer than about 1.2 s) the Iwan's, Kowalsky's, Newmark and Hall and Miranda's methods produce estimations of the maximum displacement of inelastic SDOF systems that, on average, are

relatively good for sites on rock or firm soil deposits. However, in these spectral regions the Newmark and Hall and Miranda's methods are much easier to use in practical situations since they are based on the equal displacement approximation, thus elastic results can be directly used as a good average estimation of the maximum inelastic displacement. This should not be interpreted as the Newmark and Hall and Miranda methods are preferred methods or that will produce better results, simply that in these spectral regions are easier to use. In the short period region the Miranda's and Iwan's methods yield the best estimations of maximum displacements. Nevertheless, for users of any of these approximate methods, or for users of analyses procedures based on these methods, it is important to realize that despite having relatively small mean errors, dispersion of the results in some cases is substantial, particularly for large levels of inelastic behaviour. Hence, when applied to individual earthquake ground motion records, any of these deterministic methods could lead to significant errors in the estimation of the maximum displacement.

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