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COMPUTER PROGRAMS FOR CONCRETE COLUMN DESIGN

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ABSTRACT

A suite of computer programs has been written in BASIC for IBM personal computers for the design and analysis of rectangular and circular concrete columns and analysis of arbitrary concrete columns under uniaxial or biaxial bending, with or without an applied axial load.

Three programs have been written using the ACI equivalent rectangular stress block for the design and analysis of rectangular sections under uniaxial and biaxial bending and circular sections. These comply with the conditions set out in NZS 3101: 1982.

Three programs have been written using the Mander model for concrete stresses. Two are for the design and analysis of rectangular sections under uniaxial bending and circular sections. The other is for the analysis of symmetrical and general arbitrary sections under uniaxial or biaxial bending. All three are able to do a moment-curvature analysis on the section.

Some comparisons have been made between the design charts presented in the New Zealand Reinforced Concrete Design Handbook and the programs. Generally the ACI programs agree well with the charts and the Mander model programs give a more economical design because of the more realistic concrete stresses.

Full instructions for the use of the programs have also been presented.

TABLE OF CONTENTS

NOTATION

a	depth of equivalent rectangular stress block
a	distance along the x axis to the neutral axis
A_c	area of concrete core
A_g	gross area of the section
A_s	area of longitudinal steel
A_{sh}	area of hoops
b	width of section
b	distance along the y axis to the neutral axis
b_c	width of concrete core
C_c	concrete compressive force
C_s	steel compression force
c	neutral axis depth from extreme compression fibre
d_b	diameter of the main reinforcing bars
d_c	depth of concrete core
d_s	diameter of spiral or hoop stirrups
eps	iteration accuracy
E_c	initial Young's modulus of elasticity for concrete
E_s	Young's modulus of elasticity for steel
E_{sec}	$f'_{cc}/\varepsilon'_{cc}$
f_c	concrete stress
f'_c	concrete cylinder strength
f'_{cc}	maximum strength of confined concrete
f'_{co}	insitu unconfined concrete strength

f'_ℓ lateral confining stress

f_s steel stress

f_{su} maximum steel stress after strain hardening

f_y yield stress of steel

f_{yh} yield stress of confining steel hoops

k_e confinement effectiveness coefficient

m coefficient for defining steel stress-strain curve

M_i ideal moment capacity of the section

M'_y moment when the steel first yields or concrete reaches a strain of 0.002

N number of segments

P applied axial load

R constant for the Mander model (set to 5)

r $E_c/(E_c - E_{sec})$

r $\varepsilon_{su} - \varepsilon_{sh}$

s centre line spacing between confining hoops

s' clear distance between confining hoops

T steel tension force

w average centre line spacing between longitudinal bars

w' average clear distance between longitudinal bars

x,y reinforcing steel coordinates

y_{gi}	distance to the centre line of segment i from the extreme compression fibre
y_i	distance to the end of segment i from the extreme compression fibre
β_1	depth of equivalent rectangular stress block as a proportion of the neutral axis depth
ε_c	concrete strain
ε'_{cc}	concrete strain at f'_{cc}
ε'_{co}	concrete strain at f'_{co} - usually 0.002
ε_{cu}	ultimate concrete compression strain
ε_{max}	maximum concrete strain
ε_{min}	minimum practical concrete strain
ε_s	steel strain
ε_{sh}	steel strain at the onset of strain hardening
ε_{spall}	spalling strain of cover concrete
ε_{su}	steel strain at maximum stress
ε_y	steel yield strain
ρ_{cc}	volumetric ratio of longitudinal steel to concrete core
ρ_{max}	maximum allowable percentage of steel according to NZS 3101
ρ_{min}	minimum allowable percentage of steel according to NZS 3101
ρ_s	volumetric ratio of confining steel to concrete core
ϕ_i	curvature at M_i
ϕ_y	curvature at "first yield"

ϕ'_y curvature when the extreme steel strain reached yield or when the concrete strain reaches 0.002

Δ increment for the finite difference method of solving partial differential equations.

CHAPTER I

INTRODUCTION

1.1 AIM

The aim of this report is to produce a suite of computer programs for concrete column analysis and design, for use on a microcomputer in a design office situation. The programs are to cover various section shapes including rectangular, circular, symmetrical arbitrary sections and general arbitrary sections subject to both uniaxial and biaxial bending. Different models for calculating concrete stresses, including the Mander model and ACI equivalent rectangular stress block, are to be used.

1.2 BACKGROUND

The usual method for designing columns is by using design charts or a hand analysis. Design charts are available from the American Concrete Institute (ACI) and Concrete Publications Limited, New Zealand. The New Zealand Reinforced Concrete Design Handbook has used the equivalent rectangular stress block to produce a series of charts for designing rectangular and circular sections with uniform reinforcing steel.

The New Zealand Concrete Research Association has produced a suite of reinforced concrete design programs for circular sections with uniform steel and for rectangular sections with symmetrical steel. These programs will design the area of reinforcing steel required for a given axial load and moment using the equivalent rectangular stress block to NZS 3101.

Recent research by Brøndum-Nielsen and Kawakami et al has produced papers outlining programs for biaxial bending of arbitrary section shapes. The programs suggested by Brøndum-Nielsen are suitable for hand-held calculators and are limited to an equivalent rectangular stress block for the concrete stresses. Kawakami et al use a more realistic stress-strain curve for concrete stresses. These programs are not readily available in New Zealand.

Most columns designed for New Zealand conditions require some level of confinement to be able to withstand plastic hinging under earthquake loading. Confining the concrete core in columns can enhance the concrete strength considerably. Several models for confined concrete have been developed over the years but recently Mander et al have produced a model which is suitable for all section shapes. It is reasonably simple to use and produces realistic stresses for both confined and unconfined concrete.

The scope of existing charts and programs covers only regular section shapes under uniaxial bending with symmetrical reinforcement using a simple model for concrete stresses. Computer programs are required which can cope with non-uniform reinforcing arrangements, irregular section shapes under both uniaxial and biaxial bending. The use of a more realistic model for concrete stresses such as the Mander model, which takes confinement into account, may also result in a more economical design.

1.3 SCOPE

The writing of seven programs was undertaken to cover the different conditions required. These are illustrated in Fig. 1.1 and listed as follows:

(1) Uniaxial bending of rectangular sections using the ACI stress block.

(2) Biaxial bending of rectangular sections using the ACI stress block.

(3) Circular sections using the ACI stress block.

(4) Uniaxial bending of rectangular sections using the Mander model.

(5) Circular sections using the Mander model.

(6) Symmetrical arbitrary sections with bending in a direction parallel to the axis of symmetry using the Mander model.

(7) General arbitrary sections with uniaxial or biaxial bending using the Mander model.

The first five programs listed are written in a similar style and are almost identical to run. Each has the facility to design the required area of steel for a given axial load and moment and to analyse the section for a given axial load and area of steel. The programs using the Mander model also have a moment-curvature analysis routine.

The arbitrary section programs only have the facility for a section analysis and a moment-curvature analysis.

1.4 SIGN CONVENTION

Compression forces are positive.

Tension forces are negative.

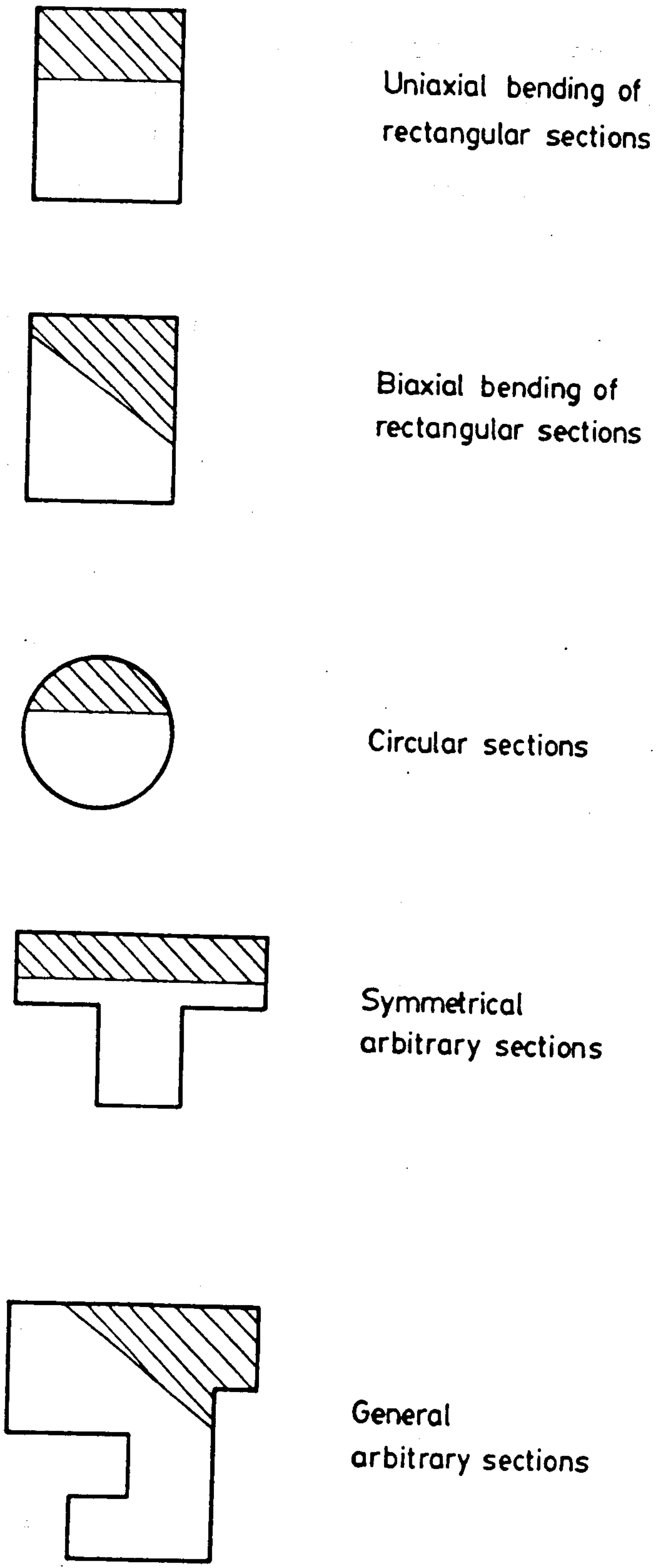


Fig. 1.1 The Different Conditions Considered.

CHAPTER II

MATERIAL PROPERTIES

2.1 THE ACI EQUIVALENT RECTANGULAR STRESS BLOCK

The New Zealand Standard NZS3101 (2) suggests using the rectangular stress block to calculate the concrete compression force C_c at an ultimate strain of 0.003. This method would be the most commonly used in design practice.

The parameters for rectangular compressed areas are shown in Fig. 2.1a and the equations are as follows:

$$C_c = 0.85f'_c ab \qquad (2.1)$$

where $a = \beta_1 c$

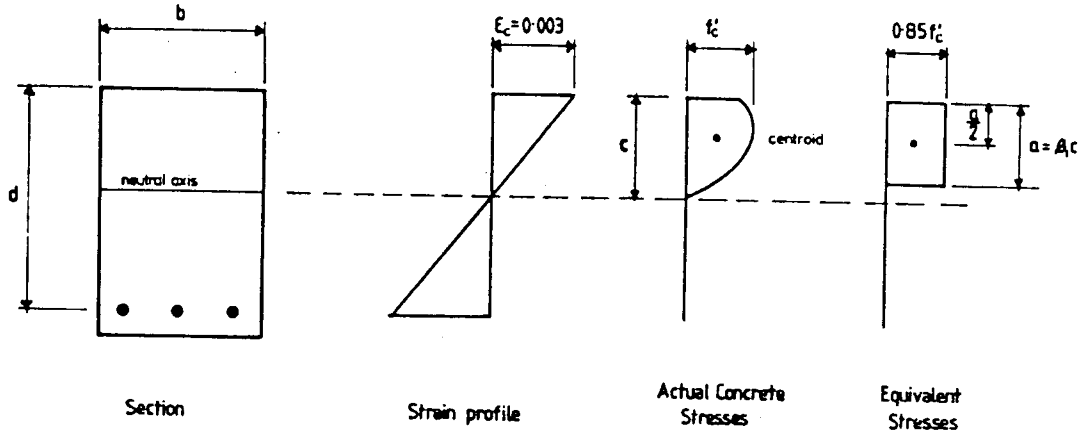
$$\beta_1 = 0.85 \text{ for } f'_c \leq 30 \text{ MPa}$$

$$\beta_1 = 0.85 - 0.008(f'_c - 30) \text{ for } f'_c > 30 \text{ MPa}$$

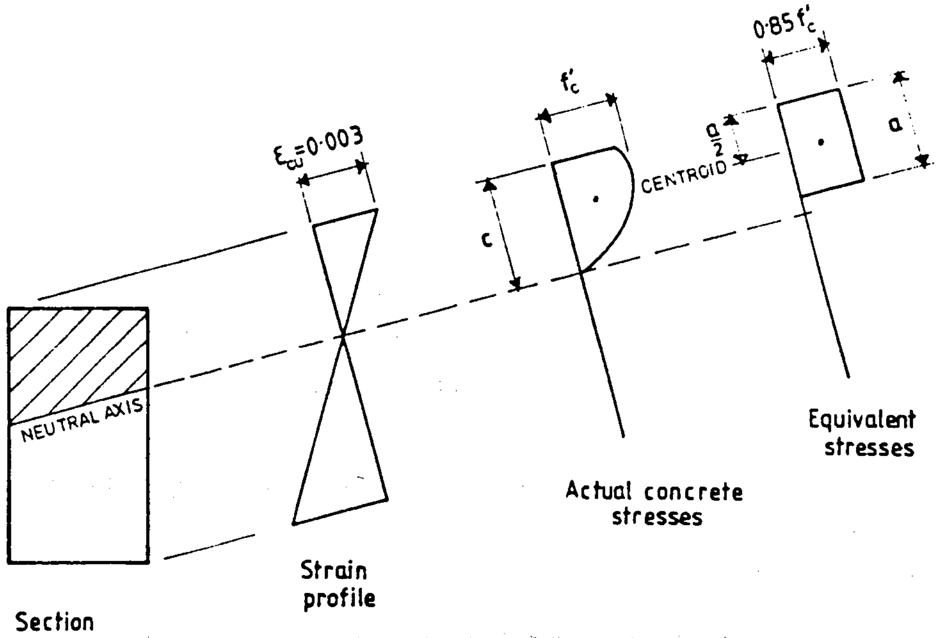
but not less than 0.65

The depth to the line of force from the extreme compression fibre is $\frac{a}{2}$.

For non-rectangular compressed areas (Fig. 2.1b) the same parameters are used. Although they are not strictly applicable, the method is probably sufficiently accurate for design purposes.



(a) Rectangular compression area.



(b) Non-rectangular compression area.

<u>Fig. 2.1</u> The ACI Rectangular Stress Block.

2.2 THE MANDER MODEL

2.2.1 Confined concrete

When a column contains stirrups or ties which are closely spaced, the strength of the concrete core is enhanced by the confining pressure provided by the ties. Mander et al (4) presented an analytical model to calculate the longitudinal stress in the concrete, f_c (see Fig. 2.2). An advantage of this model is that one equation defines the total curve.

$$f_c = \frac{f'_{cc}.x.r}{r-1+x^r} \tag{2.2}$$

where
$$f'_{cc} = f'_c\left(2.254\sqrt{1 + \frac{7.94f'_\ell}{f'_c}} - \frac{2f'_\ell}{f'_c} - 1.254\right) \tag{2.3}$$

and
$$f'_\ell = \tfrac{1}{2}k_e\rho_s f_{yh} \tag{2.4}$$

$$x = \frac{\varepsilon_c}{\varepsilon'_{cc}} \tag{2.5}$$

$$\varepsilon'_{cc} = \left\{R\left(\frac{f'_{cc}}{f'_c} - 1\right) + 1\right\}\varepsilon'_{co} \tag{2.6}$$

$$r = \frac{E_c}{E_c - E_{sec}} \tag{2.7}$$

$$E_c = 5000\sqrt{f'_c} \tag{2.8}$$

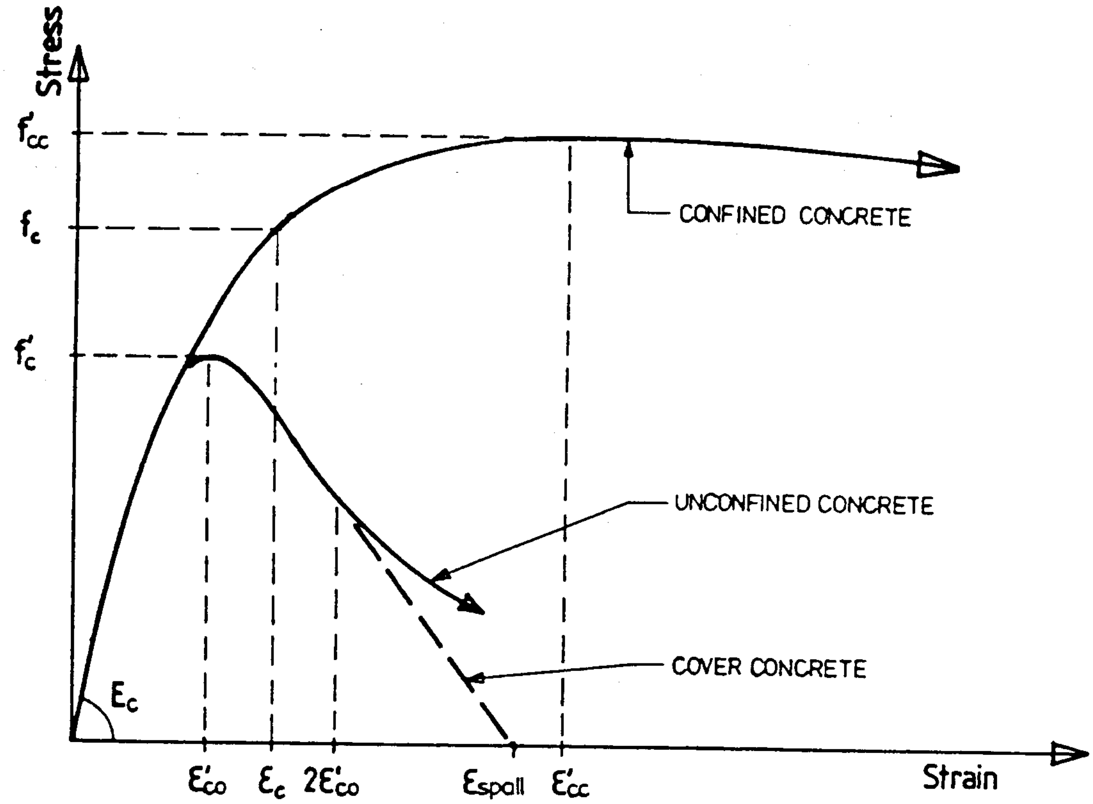


Fig. 2.2 Concrete Compressive Stress Strain Model as Proposed by Mander.

$$E_{sec} = \frac{f'_{cc}}{E'_{cc}} \qquad (2.9)$$

$$\varepsilon'_{co} = 0.002 \qquad (2.10)$$

$R = 5$ as a recommended average.

Here the cylinder strength of concrete, f'_c, has been used instead of the insitu unconfined concrete strength, f'_{co}. The f'_c is the value used in a design office and there appears to be only small differences in the values of each.

For circular sections: (see Fig. 2.3)

$$k_e = \frac{(1 - 0.5s'/d_s)^2}{(1 - \rho_{cc})} \quad \text{for circular hoops} \qquad (2.11)$$

$$k_e = \frac{(1 - 0.5s'/d_s)}{(1 - \rho_{cc})} \quad \text{for spirals} \qquad (2.12)$$

$$\rho_s = \frac{A_{sh}.\pi}{s.d_s} \qquad (2.13)$$

$$\rho_{cc} = \frac{4\ A_s}{\pi d_s^2} \qquad (2.14)$$

For rectangular sections: (see Fig. 2.4)

$$k_e = 1 - \frac{\text{Nbars}(W')^2}{6b_c d_c} \quad \frac{(1 - 0.5s'b_c)(1 - 0.5s'/d_c)}{(1 - \rho_{cc})} \qquad (2.15)$$

where Nbars = number of longitudinal bars

W' = average clear distance between bars.

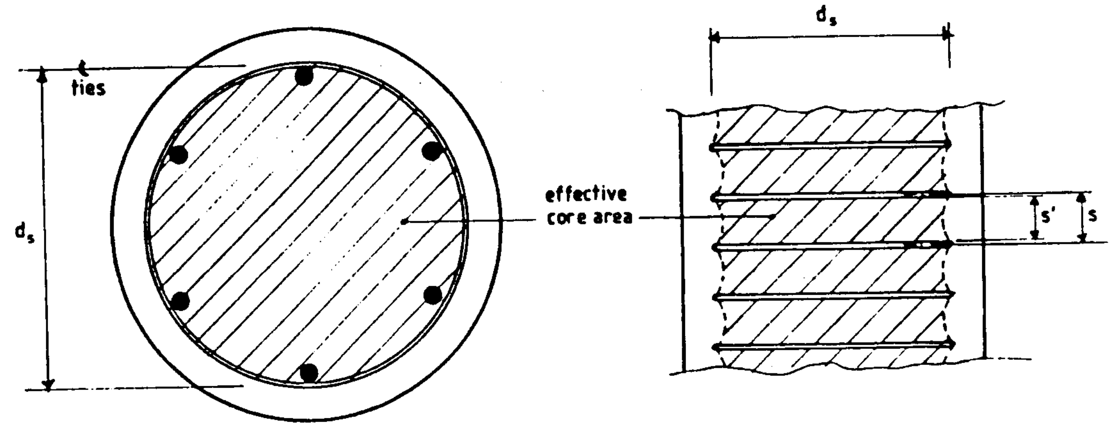


Fig. 2.3 Confinement of Concrete for Circular Sections.

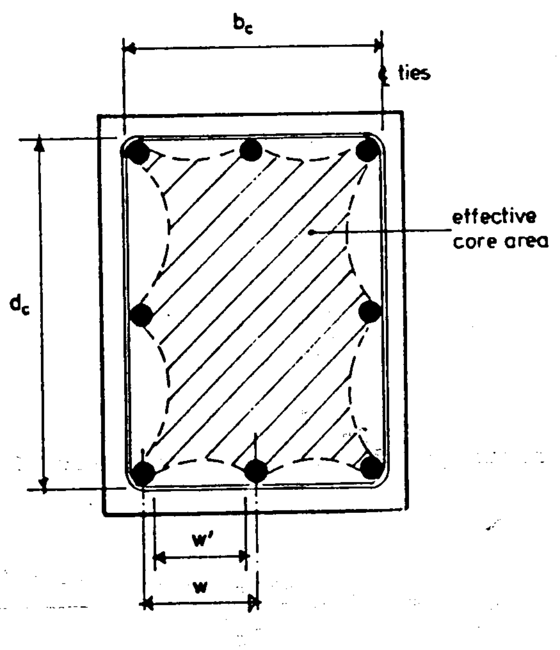


Fig. 2.4 Confinement of Concrete for Rectangular Sections.

$$\rho_s = \frac{A_{sh}(b_c + d_c)}{b_c d_c S} \tag{2.16}$$

$$\rho_{cc} = \frac{A_s}{b_c d_c} \tag{2.17}$$

For the program for rectangular sections, an average value for ρ_s has been used rather than the method proposed by Mander where the octahedral shear stresses are used to calculate f'_{cc}. Also, average values for the bar spacing have been used instead of the actual. This requires slightly less input and was considered sufficiently accurate for the purpose.

2.2.2 Cover concrete

The cover concrete stresses follow the curve for unconfined concrete where the confining stress $f'_\ell = 0$.

Hence
$$f'_{cc} = f'_c \tag{2.18}$$

$$\varepsilon'_{cc} = \varepsilon'_{co} \tag{2.19}$$

$$E_{sec} = \frac{f'_c}{\varepsilon'_{co}} \tag{2.20}$$

and
$$x = \frac{\varepsilon_c}{\varepsilon'_{co}} \tag{2.21}$$

The curve is linear between $2\varepsilon'_{co}$ and ε_{spall} where ε_{spall} = 0.0064. This value was taken as an average value from reference 4.

for $\varepsilon_c \leq 2\varepsilon'_{co}$
$$f_c = \frac{f'_c . r . x}{r-1+x^r} \tag{2.22}$$

for $2\varepsilon'_{co} < \varepsilon_c \leq \varepsilon_{spall}$
$$f_c = f'_c \left[\frac{2r}{r-1+2^r}\right] \left[1 - \frac{(\varepsilon_c - 2\varepsilon'_{co})}{\varepsilon_{spall} - 2\varepsilon'_{co}}\right] \tag{2.23}$$

for $\varepsilon_{spall} < \varepsilon_c$
$$f_c = 0 \tag{2.24}$$

2.2.3 Maximum concrete strain

The maximum concrete strain is calculated from the simplified equation rather than using the energy balance method.

$$\varepsilon_{max} = \varepsilon_{spall} + 0.04(K-1) \tag{2.25}$$

where $K = \dfrac{f'_{cc}}{f'_c}$

$\varepsilon_{spall} = 0.0064$

2.3 STEEL REINFORCEMENT STRESS STRAIN CURVES

2.3.1 For ACI method

A simple bi-linear relationship is assumed for the programs using the ACI rectangular stress block as shown in Fig. 2.5a.

where $f_s = E_s\varepsilon_s$ for $\varepsilon_s \le \varepsilon_y$ (2.26)

and $f_s = f_y$ for $\varepsilon_s > \varepsilon_y$. (2.27)

The yield strain $\varepsilon_y = \dfrac{f_y}{E_s}$

where f_y = yield stress.

2.3.2 <u>For the Mander Model</u>

In the programs using the Mander model, strain hardening of the steel can be taken into account as a user defined option. The shape of the curve is shown in Fig. 2.5b and has been taken from reference 1.

$$f_s = E_s \varepsilon_s \quad \text{for} \quad \varepsilon_s \leq \varepsilon_y \tag{2.28}$$

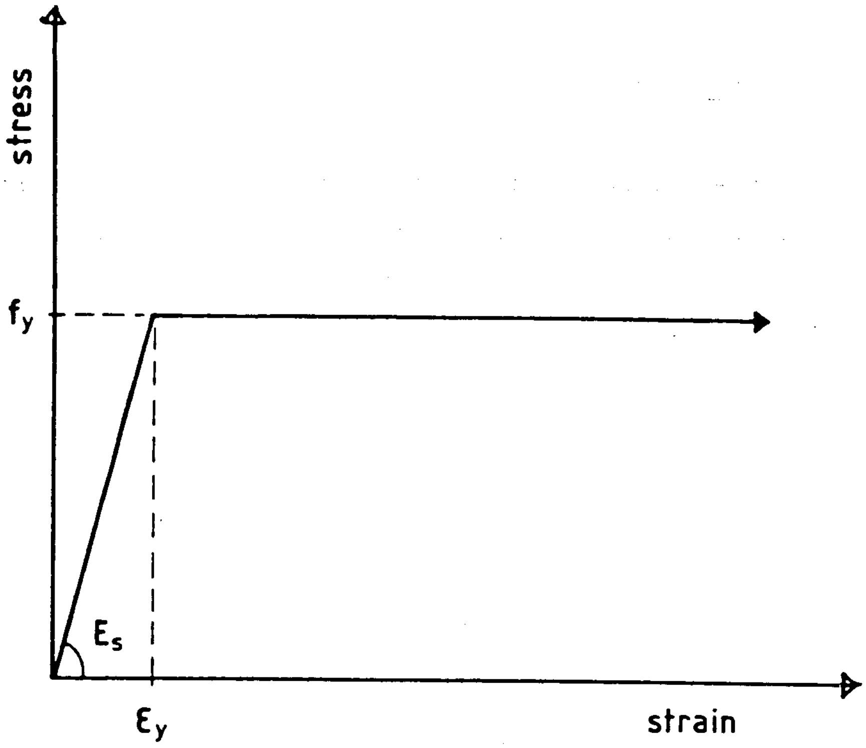
$$f_s = f_y \quad \text{for} \quad \varepsilon_y < \varepsilon_s \leq \varepsilon_{sh} \tag{2.29}$$

$$f_s = f_y \left[\frac{m(\varepsilon_s - \varepsilon_{sh}) + 2}{60(\varepsilon_s - \varepsilon_{sh}) + 2} + \frac{(\varepsilon_s - \varepsilon_{sh}((60 - m)}{2(30r + 1)^2} \right] \tag{2.30}$$

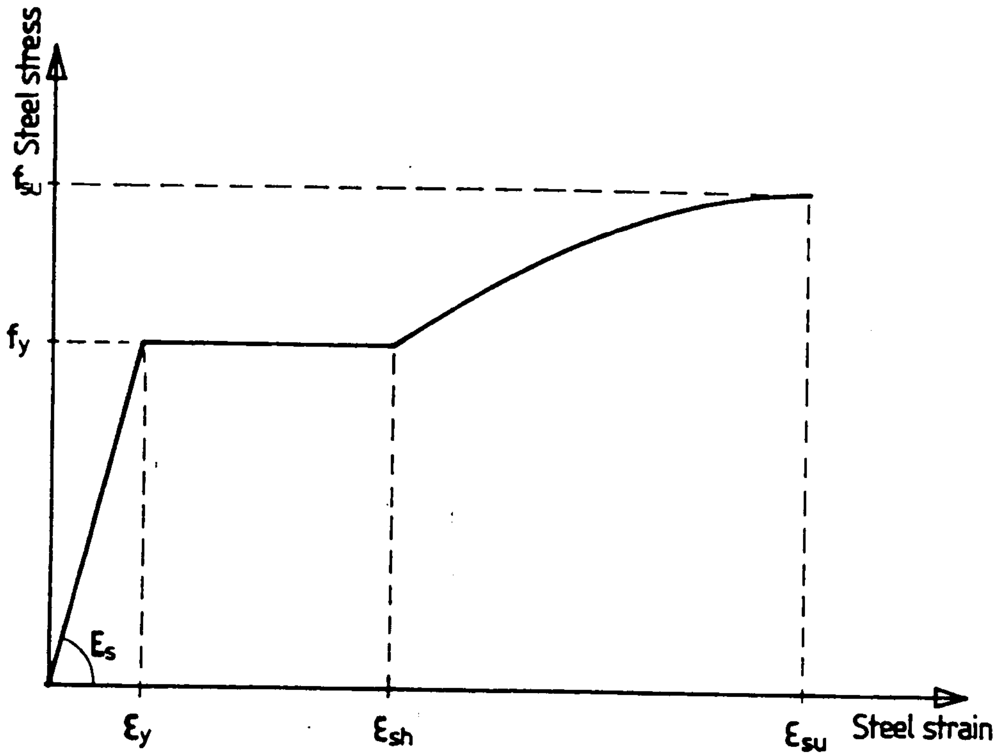
$$\text{for} \qquad \varepsilon_{sh} < \varepsilon_s \leq \varepsilon_{sh}$$

$$\text{where} \quad m = \frac{(f_{su}/f_y)(30r + 1)^2 - 60r - 1}{15r^2}$$

$$\text{and} \quad r = \varepsilon_{su} - \varepsilon_{sh} \quad .$$



(a) As used in the ACI programs.



(b) As used in the Mander model programs.

Fig. 2.5 Reinforcing Steel Stress Strain Curve.

2.3.3 Steel properties

Table 2.1 Steel Properties

		MILD STEEL	HIGH STRENGTH
YIELD STRESS	f_y (MPa)	as input	as input
MAXIMUM STRESS	f_{su} (MPa)	1.5 f_y	1.5 f_y
YOUNG'S MODULUS	E_s (MPa)	as input	as input
YIELD STRAIN	ε_y	f_y/E_s	f_y/E_s
STRAIN HARDENING	ε_{sh}	14 ε_y	3.24 ε_y
MAXIMUM STRAIN	ε_{su}	0.14 + ε_{sh}	0.12

The normal design values for f_y are 275 MPa for mild steel and 380 MPa for high strength steel. Young's Modulus is usually taken at 200,000 MPa.

CHAPTER III

THE DISCRETE ELEMENT APPROACH

3.1 GENERAL

Because the equation for concrete stress cannot be readily integrated, the column sections have to be divided into a number of elements. The concrete stress is then calculated using the strain at the centroid of each element and the stress is assumed to be constant over each element.

Since the stress-strain curves are different for confined and unconfined concrete, the areas of core and cover concrete have to be defined. The confined core of the section is taken to the centre-line of the confining ties or stirrups.

3.2 RECTANGULAR SECTIONS

For uniaxial bending, the neutral axis is always perpendicular to the direction of bending, so the elements or segments become thin strips. The cover concrete at the ends of the section is divided into three segments. The fourth segment has its centre-line coinciding with that of the main reinforcing steel and a length equal to the main reinforcing bar diameter d_b plus the tie diameter. The boundary between the 3rd and 4th segments is on the centre-line of the ties. This is illustrated in Fig. 3.1. Segments 4 to N-3 contain both core and cover concrete. Note that the area of reinforcing steel within a segment is subtracted from the core area of that segment.

3.2.1 Concrete force

To calculate the total concrete force, the strain at the centreline of each segment is determined using equation 3.1.

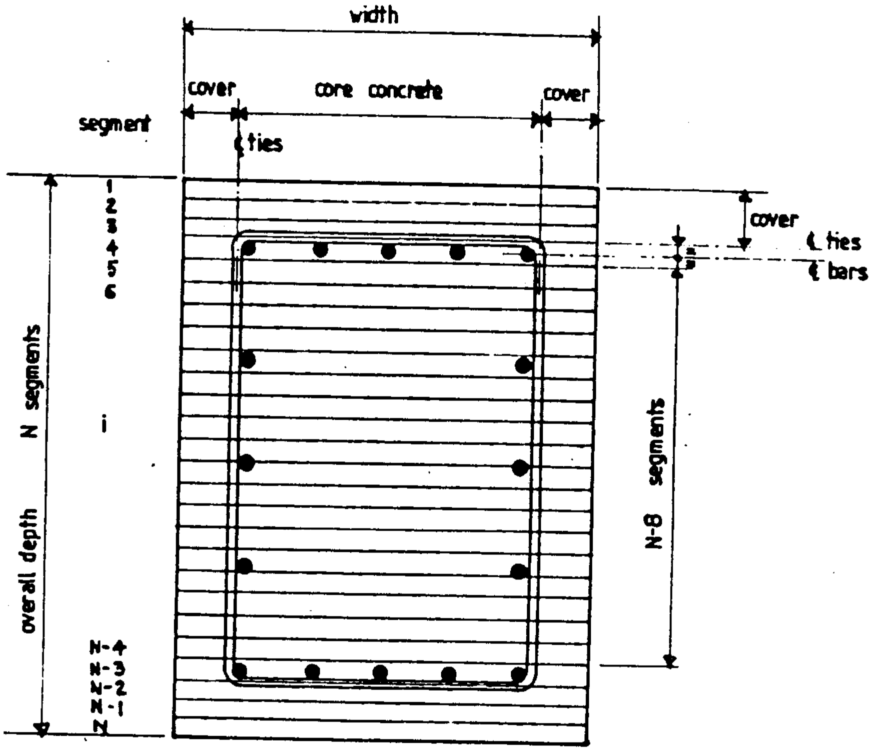


Fig. 3.1 Division of Rectangular Sections into Segments.

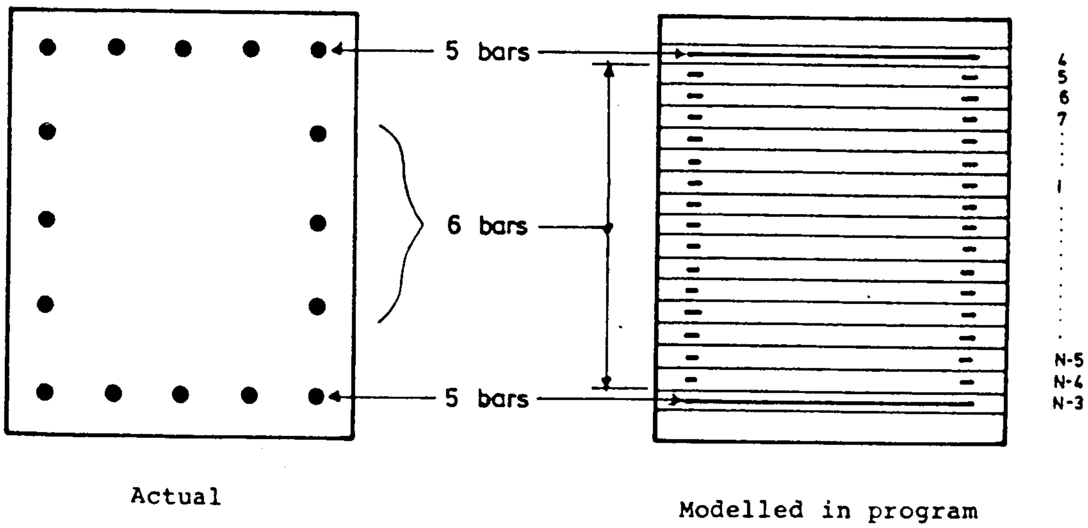


Fig. 3.2 Location of Reinforcing Steel in the Segments.

Concrete strain

$$\varepsilon_{ci} = \frac{\varepsilon_{cu}}{e}(c - y_{ci}) \; . \qquad (3.1)$$

The stresses for both confined and unconfined concrete are calculated and the total concrete force C_c for segment i equals

$$C_{ci} = A_{(core)i} \times f_{c(core)i} + A_{(cover)i} \times f_{c(cover)i} \qquad (3.2)$$

If the neutral axis lies within a segment then the forces are calculated using the part of the segment between the neutral axis and the previous segment.

3.2.2 Reinforcing steel

Using the segments set up, the reinforcing steel is, in effect, smeared around the perimeter of the section on the centre-line of the main reinforcing bars. The total steel at each end is lumped into segments 4 and N-3. The side steel remaining is distributed evenly amongst segments 5 to N-4. The area of steel in each segment acts at its centre-line. Fig. 3.2 illustrates this.

3.3 CIRCULAR SECTIONS

For circular columns the section is divided into segments in a similar manner to rectangular sections (see Fig. 3.3). The cover is divided into three elements and the other elements contain areas of both cover and core concrete.

3.3.1 Calculating the segment area

To calculate the area A of the shaded portion of the circle shown in Fig. 3.4 the following formula is used:

$$A = \left(\frac{dia}{2}\right)^2 \cos^{-1}\left(1 - \frac{2y}{dia}\right) - \left(\frac{dia}{2} - y\right)\sqrt{dia.y - y^2} \qquad (3.3)$$

If y is the distance to the end of a segment then the total area of segment i is:

$$A_{(segment)i} = A_i - A_{i-1} \quad (3.4)$$

$$\text{for dia} = \text{section diameter}$$

To calculate the core area of segment i:

$$dia = \text{diameter to centre-line of ties}$$

$$A_{(core)i} = A_i - A_{i-1} \text{ for segments 4 to N-3}$$

The cover area of segment i is then simply the total segment area minus the core area.

3.3.2 Concrete forces

The centroid of each segment is approximated to the centre-line and the strains and concrete forces are calculated using equations 3.1 and 3.2 as described for rectangular sections.

For the program using the ACI stress block the distance to the centre of compression from the extreme compression fibre is:

$$r - \frac{2}{3}\left\{r^3.\sin(\frac{\theta}{2}) - (r - y)^2 . \sqrt{2ry - y^2}\right\} /A \quad (3.5)$$

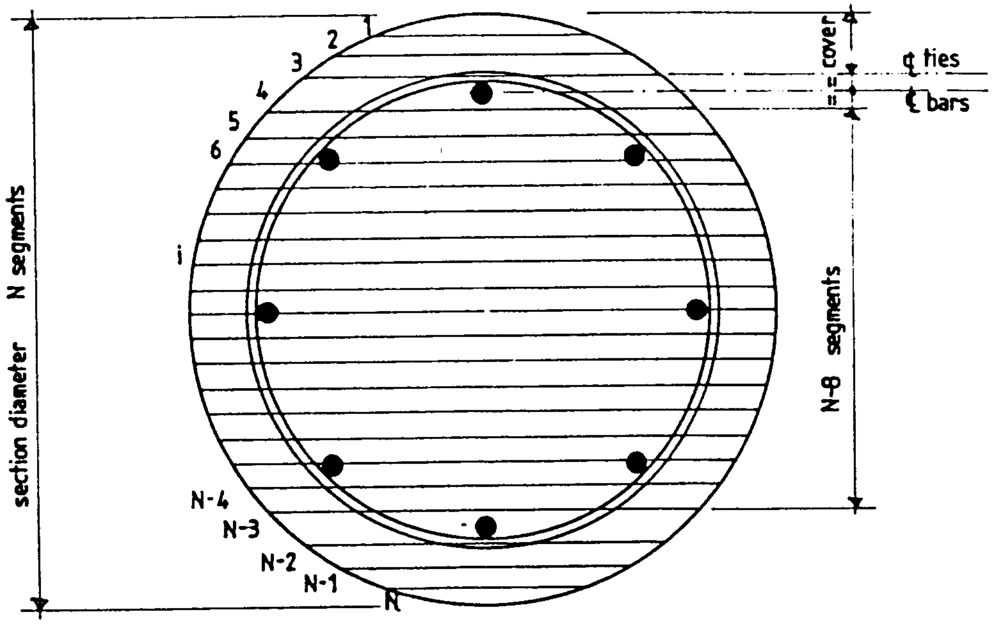
where

r = radius of section

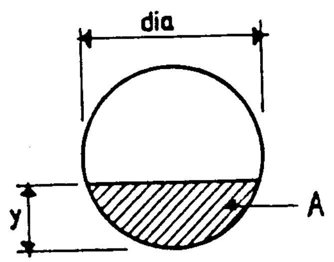
y = $\beta_1.c$

A = area calculated by Eqn. 3.3

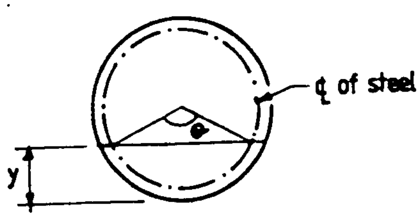
θ = $2\cos^{-1}(1 - \frac{y}{r})$



<u>Fig. 3.3</u> Dividing Circular Sections into Segments.



<u>Fig. 3.4</u> Area of a Sector of a Circle.



<u>Fig. 3.5</u> Definition of θ.

3.3.3 Reinforcing steel

The total area of reinforcing steel is "smeared" around the section at its centre-line. A proportion of the total steel area is put into each segment depending on the arc length, at the steel centre-line, within the segment. (See Fig. 3.5.)

$$\theta = 2 \cos^{-1}\left(1 - \frac{y - \text{cover} - d_b/2}{r}\right) \qquad (3.6)$$

r = radius to the steel centre-line.

The area of steel in segment i is therefore:

$$A_{s(\text{segment})i} = A_{s(\text{total})} \frac{(\theta_i - \theta_{i-1})}{2\pi} \qquad (3.7)$$

Although not strictly correct, the steel area in each segment is assumed to act at the centre-line of the segment.

3.4 ARBITRARY SECTIONS

For both the symmetrical and general arbitrary columns, the section is divided into a number of rectangular elements which form a grid between the extreme boundaries of the section (see Fig. 3.6).

The section will have a region of core concrete and may contain voids with cover concrete around the void itself. All these are defined by co-ordinates.

Once the grid has been set up, each element has to be checked to see where it lies in relation to the section. For example an element may lie outside the section, within a region of cover concrete, within the core or within a void. If the centroid of the element lies with the region of cover concrete, for example, then the whole element is said to lie within that region.

3.4.1 Method for determining position of elements

Fig. 3.7 shows the triangle ijk. The line ij represents two consective points, 1 and 2, on the boundary of the section shown in Fig. 3.7b and k represents the element A which lies outside the section.

Using the notation shown, the area of the triangle ijk is:

$$\text{Area} = \tfrac{1}{2}(a_k b_j - a_j b_k) \quad . \qquad (3.7)$$

If this area is positive then the point k is said to be on the left hand side of the line ij and if the area is negative then the point k is on the right hand side of the line ij.

Point k must be on the right hand side of <u>every</u> consective point on the section boundary to lie within the section. This is the case for element B.

Note that element A is on the right hand side of the boundary lines 2-3, 3-4 and 4-1 but is on the left hand side of line 1-2, so it must lie outside the section.

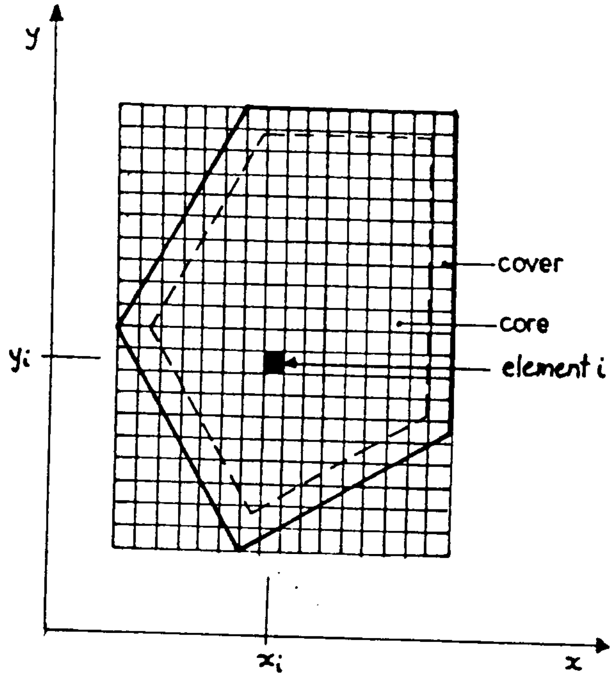
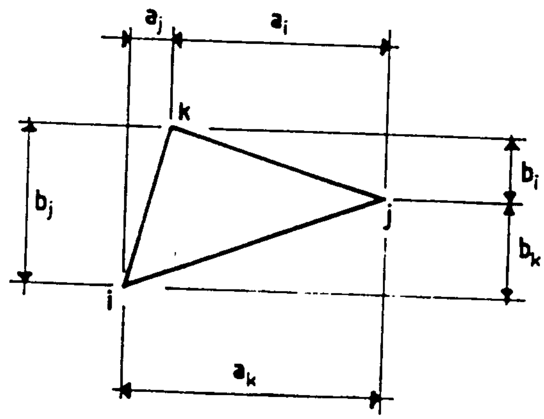
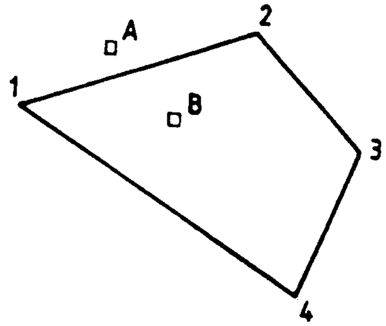


Fig. 3.6   Forming an Arbitrary Section into a Grid.



(a) Triangle ijk.



(b) Section 1-2-3-4.

Fig. 3.7   Determining the [illegible]

This method is used to determine the position of each element centroid in relation to the section.

If the section shape has a re-entrant such as a T or L section then the re-entrant must be input as a void. (See Fig. 3.8) If this is not done and the section is defined by the actual section co-ordinates, then only those elements within the shaded area will be recognized as being within the section. The void may have cover concrete around it.

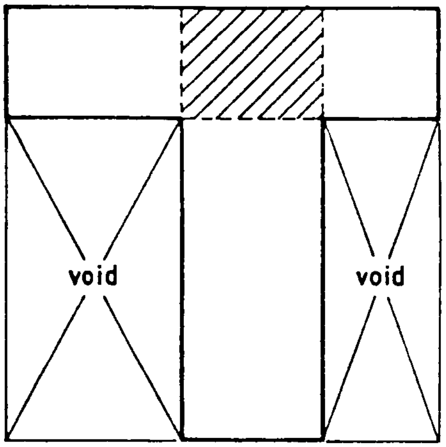


Fig. 3.8 A Re-entrant section

3.4.2 Concrete Forces

Concrete stresses are taken to be uniform over each element for the strain at the centroid of the element. Each element i has a corresponding area of cover concrete and core concrete. For elements within a region of cover concrete:

$$A_{(cover)i} = \text{Area of element}$$

$$A_{(core)i} = 0.$$

For elements within the core:

$$A_{(cover)i} = 0$$

$$A_{(core)i} = \text{Area of element.}$$

For elements outside the section or within a void:

$$A_{(cover)i} = 0$$

$$A_{(core)i} = 0.$$

The total concrete force:

$$C_c = \Sigma(A_{(core)i}.f_{c(core)i} + A_{(cover)i}.f_{c(cover)i}).$$

3.4.3 Reinforcing Steel

The reinforcing steel is defined by x and y coordinates with an area of steel at each co-ordinate. Either individual bars or groups of bars may be modelled.

3.5 CALCULATING THE NEUTRAL AXIS DEPTH

The main computational problem associated with the design and analysis of reinforced concrete columns is the calculation of the neutral axis depth. In general the neutral axis has to be in such a position that (i) internal equilibrium of the section is satisfied, and (ii) the moment capacity of the section in a direction perpendicular to the applied moment is zero.

3.5.1 Uniaxial Bending

For uniaxial bending of rectangular sections, all circular sections and the symmetrical arbitrary sections, the second criteria above can be ignored. The neutral axis is assumed to be perpendicular to the direction of applied moment and the symmetrical nature of the section ensures that this criteria is satisfied. When equilibrium has been reached then the neutral axis depth is said to be correct. Equilibrium is satisfied by the following equation:

$$C_c + C_s - T = P$$

where

C_c = concrete compressive forces

C_s = steel compressive forces

T = steel tensile forces

P = applied axial load

For these programs, the neutral axis is calculated by an iterative method using linear interpolation. The latest value for c is based on the previous two values and the axial load calculated from these.

$$c = c_2 + \frac{(P - P_2)(c_2 - c_1)}{(P_2 - P_1)} \qquad (3.9)$$

where c_1, c_2, P_1 and P_2 are the previous two values for the neutral axis depth and the axial loads calculated from them. For the rectangular and circular section programs the initial values for c_1 and c_2 are:

$$c_1 = \frac{\text{section depth}}{4}$$

$$c_2 = \frac{(P + A_s f_y/2)}{0.85^2 f_y.\text{section width}}$$

The resulting axial loads P_1 and P_2 are established and a new estimate c_3 is calculated using Equation 3.9. The axial load P_3 is then calculated. If c_3 is not the true value then $c_1 = c_2$, $P_1 = P_2$, $c_2 = c_3$ and $P_2 = P_3$ and c_3 calculated again.

The iterations stop when:

$$|\Delta P| < \text{eps } |P| \qquad (3.10a)$$

$$|\Delta c| < \text{eps} \qquad (3.10b)$$

where eps is the iteration accuracy.

When the applied load P is negative, i.e. tensile, the iterations may not converge. To avoid this problem the value of c_2 is halved or doubled accordingly until P lies between P_1 and P_2. For further calculations of c_3, c_1 and c_2 are selected so that P always lies between P_1 and P_2.

For symmetrical arbitrary sections, the initial value for c_1 is:

$$c_1 = \frac{(P + A_s f_y/2)}{0.85^2 f_y.\text{section width}}$$

and $\quad c_2 = 1.5c_1$

or $\quad = \frac{3}{4}c_1$

so that P lies between P_1 and P_2. Again, for further calculations of c_3, c_1 and c_2 are selected so that P lies between P_1 and P_2 so that convergence is assured.

3.5.2 Biaxial Bending of Rectangular Sections

This procedure for the calculation of the neutral axis position and the moment capacity has been taken directly out of reference 5. The notation used here is slightly different.

A two dimensional Newton-Raphson iteration is used to calculate a and b (see Fig. 3.9)

$$a_{i+1} = a_i - \frac{1}{D_i}\left(f_1 \frac{\partial f_2}{\partial b} - f_2 \frac{\partial f_1}{\partial b}\right)_i \qquad (3.11)$$

$$b_{i+1} = b_i - \frac{1}{D_i}\left(f_2 \frac{\partial f_1}{\partial a} - f_1 \frac{\partial f_2}{\partial a}\right)_i \qquad (3.12)$$

where $f_1 = (\Sigma A_{si} f_{si} y_i + f_c \int y_c dA - P\bar{y})\sin\theta$

$$- (\Sigma A_{si} f_{si} x_i + f_c \int x_c dA - P\bar{x})\cos\theta = 0 \qquad (3.13)$$

$$f_2 = \Sigma A_{si} f_{si} + f_c A - P = 0 \qquad (3.14)$$

$$D = \left\{\frac{\partial f_1}{\partial a}\frac{\partial f_2}{\partial b} - \frac{\partial f_1}{\partial b}\frac{\partial f_2}{\partial a}\right\} \qquad (3.15)$$

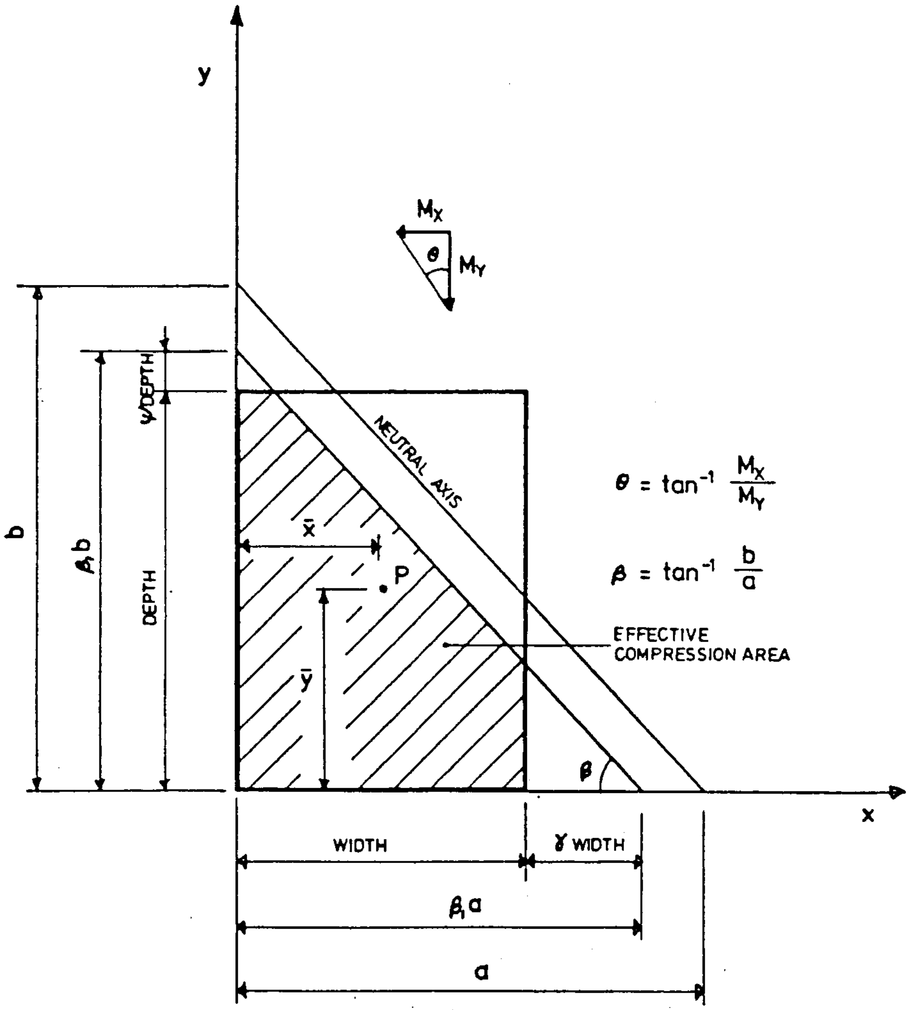


Fig. 3.9 Biaxial Bending of Rectangular Sections Using the ACI Stress Block.

The forward difference method is used to calculate the partial differential equations, for example:

$$\frac{\partial f_1}{\partial a} = \frac{1}{\Delta}(f_1(a+\Delta) - f_1(a)) \tag{3.16}$$

the area of compressed concrete

$$A = \frac{1}{2}\beta_1^2\ ab(1 - \gamma^2 - \psi^2) \tag{3.17}$$

and the integrals

$$\int y_c dA = \frac{1}{6}\beta_1^3\ ab^2[1 - \gamma^3 - \psi^2(3 - 2\psi)] \tag{3.18}$$

$$\int x_c dA = \frac{1}{6}\beta_1^3\ a^2b[1 - \psi^3 - \gamma^2(3 - 2\gamma)] \tag{3.19}$$

where

$$\gamma = 1 - \frac{b}{\beta_1 a} \nless 0 \tag{3.20a}$$

$$\psi = 1 - \frac{a}{\beta_1 b} \nless 0 \tag{3.20b}$$

The symbol $\nless$ means that if the expression is negative then it is set equal to zero.

This procedure applies directly to the ACI equivalent rectangular stress block method for calculating the concrete stresses.

The function f_2 is the equilibrium equation which satisfies the first criteria and the function f_1 satisfies the second criteria for the position of the neutral axis depth. Initial values for a and b are chosen so that the compressed area of concrete equals the applied axial load plus half the total steel force and in proportion to the applied moments.

$$b = \sqrt{\frac{2(P + A_s f_y/2)}{0.85^2 f'_c} \left[\frac{M_x}{M_y}\right]}$$

$$a = b\,\frac{M_y}{M}$$

Comments

Initially small values of Δ, in Equation 2.8 were used to solve the partial differential equations. The errors involved with this method are in the order of Δ^2, so it was felt that a value of 0.01 mm would be suitable for Δ. Problems with convergence were encountered though and with some experimenting, it was found that the most appropriate values for Δ are between 10 and 20 mm for typical sections sizes. It appears that the local slope of the curve is not a good estimate for the general slope.

Problems were also encountered as a or b approached infinity, i.e. uniaxial bending. To avoid division by zero errors and for a reasonably quick convergence, the ratio of the two moments has been selected to be not less than 1 to eps. This will give a finite value to both a and b and hence a moment in both directions but the moment in the direction perpendicular to the principal direction of bending is usually small and the results are accurate enough for the purpose.

Generally, convergence on the neutral axis position is quickest as the ratio of the moments M_x and M_y approach 1 to 1 and for average axial loads. The time for convergence is increased as uniaxial bending is approached, for very small or very large axial loads and with a greater number of reinforcing bars.

3.5.3 Biaxial Bending of General Arbitrary Sections

For general arbitrary sections a similar procedure outlined in section 3.5.2 can be applied using Equations 3.11 through to 3.15. The forward difference method is used to solve the partial differential equations. For example:

$$\frac{\partial f_1}{\partial a} = \frac{1}{\Delta}\left(f_1(a+\Delta) - f_1(a)\right) \qquad (3.21)$$

For each estimate of the neutral axis depth, the position of the most extreme compression fibre has to be established.

The distance z (shown in Fig. 3.10) is the minimum distance from the origin, perpendicular to the neutral axis, to any point x_i, y_i within the section boundary.

$$z = \min (y_i \cos\beta + x_i \sin\beta) . \qquad (3.22)$$

Once this has been calculated the strain at the origin $\varepsilon_{(0,0)}$ is:

$$\varepsilon_{(0,0)} = \varepsilon_{cu} \frac{a.\sin\beta}{(a.\sin\beta - z)} \qquad (3.23)$$

and the strain at point x,y is:

$$\varepsilon_{(x,y)} = \varepsilon_{(0,0)}(1 - (\frac{x}{a} + \frac{y}{b}) \qquad (3.24)$$

Using this strain profile the concrete and steel stresses can be calculated for each element and functions f_1 and f_2 calculated using equations 3.13 and 3.14.

For general sections the initial estimate for the neutral axis position has to be reasonably close. The program attempts to provide a good estimate using the following:

$$b = \sqrt{\frac{2(P + A_s f_y/2)}{0.85^2 f'_c} \quad \frac{M_y}{M_x}}$$

and

$$a = b \frac{M_x}{M_y}$$

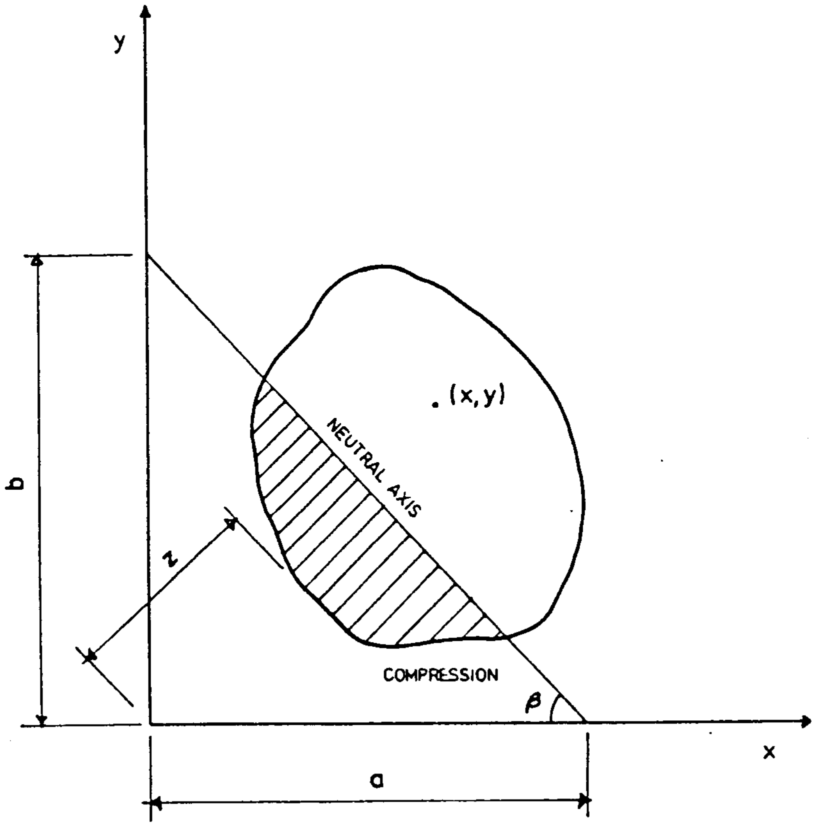


Fig. 3.10 Biaxial Bending of Arbitrary Sections.

Comments

If there are problems with the neutral axis iterations then it may be necessary to provide an option to input initial estimates to ensure convergence. There also may be problems if either a or b are negative. To avoid these the area of compression should be nearest the origin.

3.6 MOMENT CALCULATION

(a) Uniaxial bending

The moment capacity of the section is calculated about the extreme compression fibre.

For rectangular sections using the ACI stress block (see Fig. 3.11a):

$$M = -1(0.85f'_c a^2 b/2 + \Sigma f_{si} A_{si} y_{\varepsilon i} - P.d/2) \quad (3.25a)$$

and for rectangular sections using the Mander model (see Fig. 3.11b):

$$M = -1(\Sigma f_{ci} A_{ci} y_{\varepsilon i} + \overset{0}{\Sigma} f_{si} A_{si} y_{\varepsilon i} - P.d/2) \quad (3.25b)$$

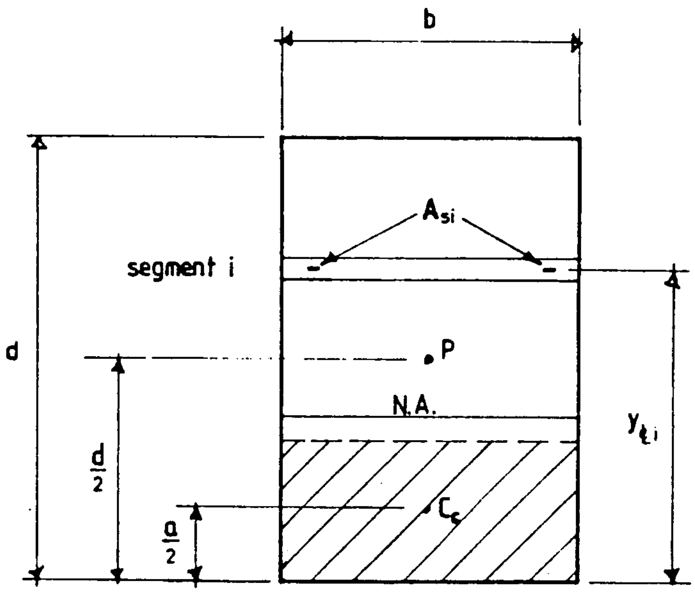
where d = overall depth of rectangular sections or overall diameter of circular sections.

For circular sections the same equations are used except that for the ACI program the distance to the line of concrete force is not $\frac{a}{2}$ but is as defined in section 3.3.2.

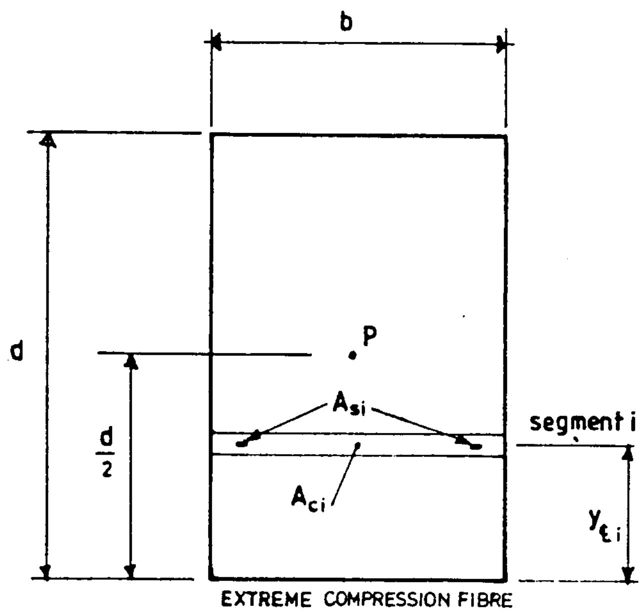
(b) Biaxial bending

For biaxial bending of rectangular and arbitrary sections the moment capacity of the section is calculated separately for the two directions.

For rectangular sections using ACI stress block:



(a) ACI Stress Block.



(b) Mander Model.

Fig. 3.11 Notation for Moment Calculation.

$$M_x = -1(0.85f'_c \int x_c dA + \Sigma f_{si} A_{si} x_i - P\bar{x}) \qquad (3.26a)$$

$$M_y = -1(0.85f'_c \int y_c dA + \Sigma f_{si} A_{si} y_i - P\bar{y} \qquad (3.26b)$$

where $\int y_c dA$ and $\int x_c dA$ are calculated using Equations 3.18 and 3.19.

For arbitrary sections the two moments are:

$$M_x = -1(\Sigma f_{ci} A_i x_i + \Sigma f_{si} A_{si} x_i - P.x_{plast}) \qquad (3.27a)$$

$$M_y = -1(\Sigma f_{ci} A_i y_i + \Sigma f_{si} A_{si} y_i - P.y_{plast}) \qquad (3.27b)$$

where x_{plast} and y_{plast} are the co-ordinates of the plastic centroid.

3.7 PLASTIC CENTROID

For arbitrary sections, the axial load P is taken to act at the plastic centroid. The plastic centroid is defined as "the centroid of resistance of the section if all the concrete is compressed to a maximum stress and the steel is compressed to the yield stress with uniform strain over the section"(1).

The confined concrete within the core is assumed to have a compressive stress of f'_{cc} and the stress of the cover concrete is calculated at a strain of ε'_{cc} using the stress-strain curve for cover concrete. The steel stress is taken equal to the yield stress.

$$x_{plast} = \frac{(\Sigma f_{si} A_{si} x_i + \Sigma f'_{cc} A_{core} x_i + \Sigma f_c A_{cover} x_i)}{(\Sigma f_{si} x_i + \Sigma f'_{cc} A_{core} + \Sigma f_c A_{cover})} \qquad (3.28)$$

and similarly for y_{plast}.

3.8 MOMENT-CURVATURE ANALYSIS

For the programs using the Mander model for calculating concrete stresses, a moment-curvature analysis can be done. This involves incrementing the concrete strain from approx. ε_{min} and calculating the moment and curvature for each strain increment. The analysis stops when one of the stopping conditions has been reached. At the present, the analysis is available for cracked sections only, i.e. the concrete has zero tensile strength, but tensile capacity could be added at a later date.

The minimum practical concrete strain is calculated by:

$$\varepsilon_{min} = \frac{P}{A_g E_c} \tag{3.29}$$

The programs start the analysis from a concrete strain of 0.0001 . If ε_{min} is greater than this then 0.0001 is added to the starting strain until it is greater than ε_{min}. This shortens the calculations slightly for the neutral axis iteration as the neutral axis depth is generally quite large for low strains and also keeps the strains in round figures.

3.8.1 Stopping conditions

There are five conditions which could stop the analysis. These are as follows:

(1) if the moment capacity falls below 80% of the maximum moment capacity of the section;

(2) if the concrete strain exceeds ε_{max} (Equation 2.25);

(3) if the moment capacity is negative;

(4) if the steel strain exceeds the upper limit of ε_{su};

(5) if the number of cycles exceeds that specified.

3.8.2 The Ideal Moment Capacity

At the end of the analysis the ideal moment, M_i, is calculated. Zahn (7) defined M_i "to be the maximum moment reached before the curvature exceeds five times ϕ_y". This definition is adopted for these programs.

Fig. 3.12 illustrates the two cases which could occur.

ϕ'_y and M'_y are the curvature and moment calculated when the extreme tension steel reaches the yield strain or when the extreme concrete compression strain reaches 0.002. This second condition applies for columns with high axial loads where the steel has not reached yield before the concrete has reached a strain of 0.002.

For case two, M_i is established by an iterative method. Once M'_y and ϕ'_y have been calculated, the first estimate:

$$\phi_{i1} = 1.25 \times 5\phi'_y \tag{3.30}$$

Linear interpolation between calculated points is used to determine M_{i1} for ϕ_{i1}. Then yield curvature ϕ_y is calculated by

$$\phi_{y1} = \frac{M_{i1}}{M'_y}\,\phi'_y \tag{3.31}$$

If ϕ_i is not equal to $5\phi_y$ then $\phi_{i2} = 5\phi_{y1}$, M_{i2} interpolated and the cycle continues until $\phi_i = 5\phi_y$ (± a tolerance).

For case one, M_i is equal to the maximum moment as it is greater than the moment at a curvature of $5\phi_y$.

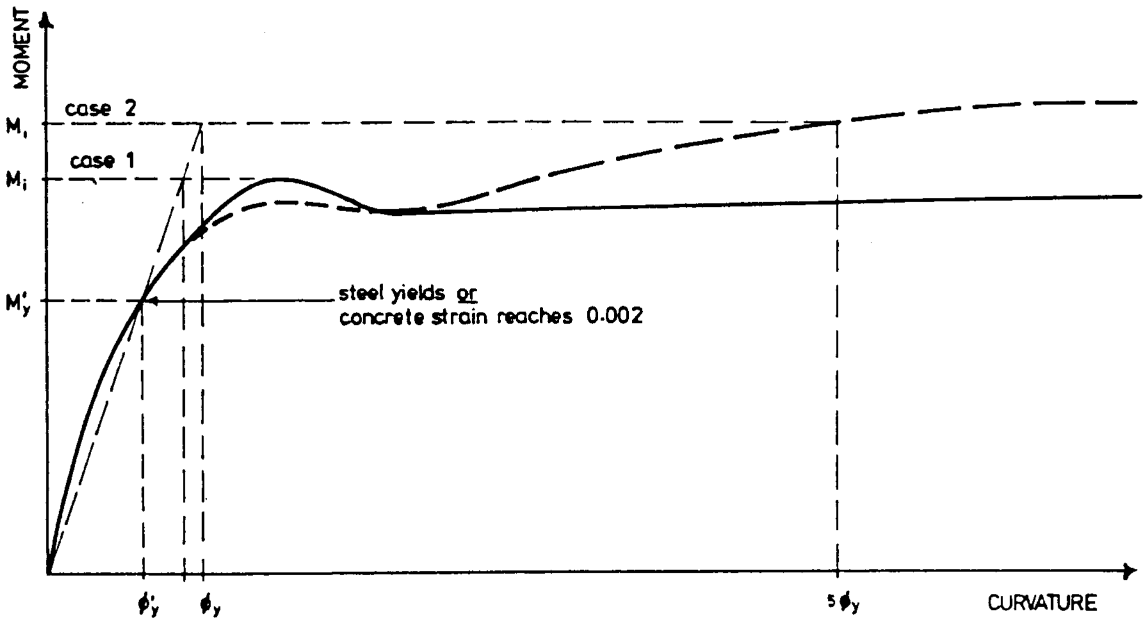


Fig. 3.12 The Ideal Moment Capacity.

CHAPTER IV

INSTRUCTIONS FOR USING ALL RECTANGULAR AND CIRCULAR PROGRAMS

4.1 GENERAL

The five programs covered under this chapter are:

(1) Uniaxial bending of rectangular sections - ACI stress block.

(2) Biaxial bending of rectangular sections - ACI stress block.

(3) Circular sections - ACI stress block.

(4) Uniaxial bending of rectangular sections - Mander model.

(5) Circular sections - Mander model.

These programs will calculate the required area of reinforcing steel for a given axial load and moment or calculate the moment capacity of the section for a given area of steel. The two programs using the Mander model also do a moment curvature analysis. There are also facilities to store and retrieve the input data on disk. Some code checks are available in the programs using the ACI stress block.

The units used are as follows:

Section Dimensions	-	mm
Material Properties	-	MPa
Loads	-	kN and kNm
Steel Areas	-	mm^2

The function key f10 is set up as an escape key. If the wrong menu option is selected, or to stop calculations, then press the f10 key and RETURN at a prompt or just f10 is the program is running to go back to the Main Menu.

4.2 MAIN MENU OPTIONS

The options available on the Main Menu are:

(1) Input Editor
(2) Print Input Data on Screen
(3) Print Input Data on Printer
(4) Section Design
(5) Section Analysis
(6) Moment Curvature Analysis (Mander model only)
(7) Exit

4.2.1 Input Editor

The following options are available on the Input Editor:

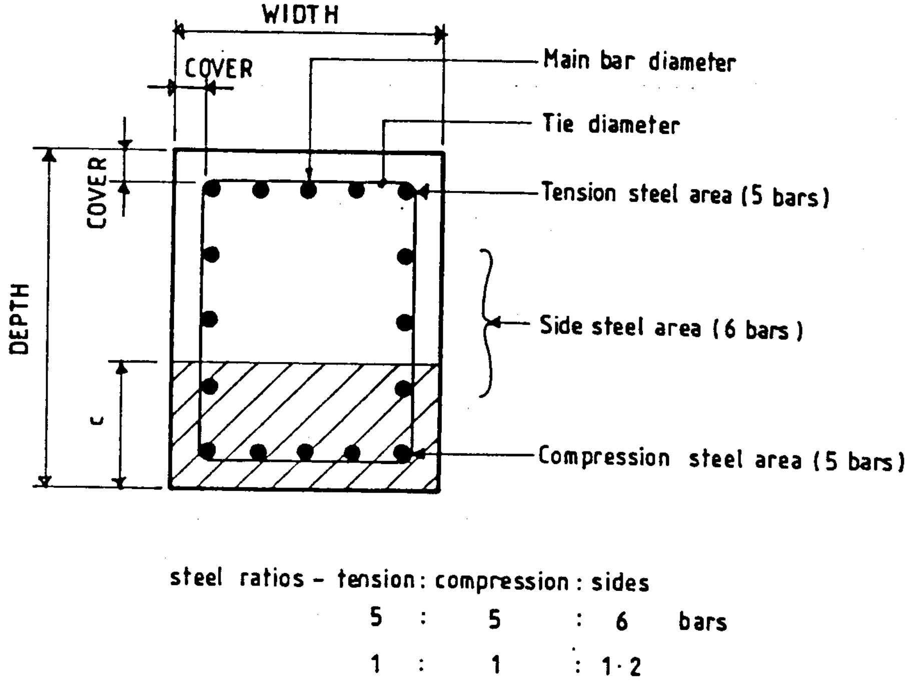
(1) Input section dimensions,
(2) Input material properties,
(3) Input loads,
(4) Input reinforcement details,
(5) Set control parameters,
(6) Store data on disk,
(7) Read data from disk,
(8) List and delete data files on disk,
(9) Exit to the Main Menu.

For entering data, the program will prompt the user for the required input. Within each prompt, the current value for the item is shown in square brackets. This is the default value and can be entered by pressing RETURN only, without typing in data. If the value is to be changed then type in the new data and press RETURN.

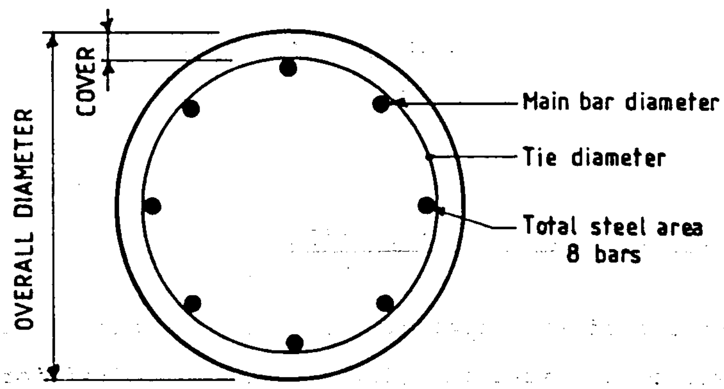
If a mistake is made then it can be corrected by deleting the required characters before RETURN or by going back to that optio from the INPUT EDITOR.

4.2.1.1 Input section dimensions

Input the section dimensions and cover to the main reinforcing steel as shown in Fig. 4.1.



(a) Rectangular sections.



(b) Circular sections.

Fig. 4.1 Input Parameters for the Programs.

4.2.1.2 Input material properties

The following values are initialized when the programs are run.

f_y = 275 MPa

E_s = 200,000 MPa

f'_c = 30 MPa

Identifying mild steel or high strength steel in the Mander model programs enables the relevant stress-strain curve to be set up to include strain hardening of the steel. The default option is mild stee

4.2.1.3 Input loads

Input the ideal axial load and moment applied to the section, i.e. the axial load and moment have been divided by the relevant strength reduction factor(s).

The applied moment is required for section design only.

For the biaxial bending program, the design moments in both the x and y directions are required.

4.2.1.4 Input reinforcement details

Details of the reinforcement must be entered before running an analysis on the section. They are not required for the design of the steel.

(a) Uniaxial bending of rectangular sections - ACI.

Enter the main bar diameter, tie diameter and the steel areas as illustrated in Fig. 4.1a.

(b) Biaxial bending of rectangular sections - ACI.

The total steel area is calculated using the number and diameter of the bars.

Note that the number of bars must be divisible by 4 as the program assumes an equal number of bars in each face.

(c) Circular sections - ACI.

Enter the main bar diameter, tie diameter and total longitudinal steel area as in Fig. 4.1b.

(d) Uniaxial bending of rectangular sections - Mander.

Enter values for the following:

(i) Number of bars,

(ii) Average bar diameter, (mm)

(iii) Average centreline distance between bars, (mm)

(iv) Steel areas, as per Fig. 4.2, (mm^2)

(v) Average number of tie legs per side,

(vi) Tie diameter, (mm)

(vii) Tie spacing, (mm)

(viii) Tie strength, (MPa).

This data is required to calculate the confining pressure f'_ℓ and k_e for the Mander model. Average values, rather than actual are adequate for the purpose.

(e) Circular sections - Mander.

Enter values for the bar diameter, total steel area, tie diameter, tie spacing and strength. Hoop and spiral ties also need to be distinguished between for the calculation of k_e.

4.2.1.5 <u>Set control parameters</u>

The program initializes the following values:

number of segments	-	30
iteration accuracy	-	0.001 (uniaxial bending)
(eps)		0.01 (biaxial bending)

These values can be changed if necessary.

4.2.1.6 Store data on disk

To store the input data, type in the file name (maximum of eight characters) and the disk drive required at the relevant prompts. If no drive is specified then the computer will default to the current drive. The program also provides a .DAT extension to the file name to identify it as a data file.

The data files are stored in a sub-directory under each program. This is to avoid trying to load the wrong data file into a program. If the sub-directory does not exist then the program will create one and then store the data.

If no characters are entered for the file name then the program will go back to the INPUT EDITOR.

4.2.1.7 Read data from disk

To load the input data, type in the file name and disk drive as in the procedure for storing data.

4.2.1.8 List and delete data files on disk

This will list all the .DAT files within the sub-directory of the current program. A prompt will then appear for the name of the file to be deleted. If no files are to be deleted just press RETURN to go back to the INPUT EDITOR.

4.2.2 Section Design

This option will calculate the required area of reinforcing steel for a given axial load and moment. Before the reinforcement can be designed, the section dimensions, material properties and loads must be entered.

Initially the minimum steel content of the section is 0.8% of the gross area and the moment capacity calculated. If this moment is greater than the input value then the results are printed on the screen and printer. Note that this coresponds to the minimum column reinforcement allowed by NZS 3101.

If minimum steel is not sufficient the steel area is incremented using linear interpolation until the moment capacity equals the input moment (plus or minus eps times the input moment).

The printed output includes information about the moment capacity and neutral axis depth at minimum steel, the required steel area, moment capacity, neutral axis depth and percentage of steel. Also the number of bars for a range of sizes is printed. For the ACI programs, the volumetric ratio for confining steel and information on the code checks is printed (see section 4.3).

(a) Uniaxial bending of rectangular sections - ACI

If the section is to be designed for seismic loads then type "Y" at the prompt. This is to determine whether to use section 6.4 or 6.5 in NZS 3101 for the code checks.

Estimates for the main bar diameter and tie diameter are required so the section can be divided into segments. These values are not too critical and could be several sizes out without making much difference to the final answer.

The ratios of the tension steel area to compression steel area to side steel area have to be entered. Press RETURN after entering each number. For uniform steel around the section ratios of between 1:1:1 and 1:1:2 will be suitable but depend on the total number of bars. A ratio of 1:1:1.2 will generally cover most situations, (see Fig. 4.1a). The design is not particularly sensitive to the side steel ratio.

The steel areas can be increased in one or more of the specified regions:

1. Tension Steel
2. Compression Steel
3. Side Steel

To increase the steel in the required region, type in the number . For example, the combination 123 will increase the steel uniformly over the whole section but 12 will increase the steel in the tension and compression regions only. The side steel will be left as it was for minimum steel. The numbers can be typed in any order. The program will not allow steel to be increased in the compression region only.

(b) Biaxial bending of rectangular sections - ACI

Type "Y" if the section is to be designed for seismic loading. Enter estimates for the number of bars (must be divisible by 4), bar diameter and tie diameter.

As the steel is assumed to be distributed evenly around the section, the total steel area is increased if the minimum moment is not sufficient until the moment capacity in both the x and y directions equals the applied moments (± eps x moment).

(c) Circular sections - ACI

Type "Y" if the section is to be designed for seismic loading. Enter estimates for the bar diameter and tie diameter.

The total steel area is increased if the minimum moment is not sufficient.

(d) Uniaxial bending of rectangular sections and circular sections - Mander

Enter estimates for the bar diameter, tie diameter, tie strength, f_{yh}, ρ_s (RHOs) and k_e.

The default value for f_{yh} is the same as the input value for f_y.

The default value for ρ_s is calculated using Equations 6.22 and 6.23 of NZS 3101 for circular sections, or 6.24 and 6.25 of NZS 3101 for rectangular sections, with f_{yh} equal to the input value.

The default value for k_e of 0.7 for rectangular sections is fairly typical but values for this could range from 0.5 to 0.8 depending on the shape of the section, the number of reinforcing bars and the tie spacing. As the section shape becomes long and thin k_e will decrease. Also the fewer bars there are, the lower the value for k_e.

For circular sections, the default for k_e is 0.9. Typically, values for k_e could range between 0.85 to 0.95, mainly depending on the spacing of the stirrups. The results are not particularly sensitive to k_e.

Enter the steel ratios for rectangular sections as outlined in section 4.2.2(a).

Options are available to include the full strain-hardened stress-strain curve for steel and to choose the ultimate concrete compressive strain.

4.2.3 Section Analysis

Before running a section analysis, the reinforcement details should be entered using the INPUT EDITOR.

The moment capacity of the section is calculated for the input reinforcement. An ultimate concrete strain of 0.003 is used for the ACI programs and the options of steel strain hardening and choosing the ultimate concrete strain are available on the two programs using the Mander model.

The actual values for ρ_s and k_e are calculated for the input reinforcement and these are printed along with the moment capacity, neutral axis depth, percentage of steel, extreme steel strain and section curvature.

4.3 CODE REQUIREMENTS

Chapter Six of NZS 3101: 1982 governs the design of members for flexure, with or without axial loads.

4.3.1 General

The following clauses of NZS 3101 regarding material strengths have been complied with:

6.3.1.3 Maximum usable strain at extreme concrete compression fibre shall be assumed equal to 0.003.

6.3.1.4 Stress in reinforcement below specified yield strength f_y for grade of reinforcement used shall be taken as E_s times steel strain. For strains greater than that corresponding to f_y, stress in reinforcement shall be considered independent of strain and equal to f_y.

6.3.1.6 Relationship between concrete compressive stress distribution and concrete strain may be assumed to be rectangular, trapezoidal, parabolic, or any other shape that results in prediction of strength in substantial agreement with results of comprehensive tests.

6.3.1.7 Requirements of 6.3.1.6 may be considered satisfied by an equivalent rectangular concrete stress distribution defined by the following:

(a) Concrete stress of $0.85 f'_c$ shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross-section and a straight line located parallel to the neutral axis at a distance $a = \beta_1 c$ from the fibre of maximum compressive strain.

(b) Distance c from fibre of maximum strain to the neutral axis shall be measured in a direction perpendicular to that axis.

(c) Factor β_1 shall be taken as 0.85 for concrete strengths f'_c up to and including 30 MPa. For strengths above 30 MPa, β_1 shall be reduced continuously at the rate of 0.04 for each 5 MPa of strength in excess of 30 MPa, but β_1 shall be not taken as less than 0.65.

4.3.2 For Members Not Designed for Seismic Loading

Checks are made against the following clauses of NZS 3101:

6.4.1.5 For columns and piers the maximum design axial load in compression at a given eccentricity shall not exceed 0.85 ϕP_o for members with transverse reinforcement conforming to either 6.4.7.1 (a) or 6.4.7.2 (a) and composite members conforming to 6.4.12.7, nor 0.80 ϕP_o for members with transverse reinforcement conforming to either 6.4.7.1 (b) or 6.4.7.2 (b) and composite members conforming to 6.4.12.8, where

$$P_o = 0.85 f_c' (A_g - A_{st}) + f_y A_{st} \quad \text{. . . . . . (Eq.6-1)}$$

6.4.6 *Limits for longitudinal reinforcement in columns and piers*

6.4.6.1 Area of longitudinal reinforcement for non-composite columns or piers shall not be less than 0.008 nor more than 0.08 times gross area A_g of section.

6.4.6.2 Minimum number of longitudinal reinforcing bars in columns and piers shall be six for bars in a circular arrangement and four for bars in a rectangular arrangement.

6.4.7 *Limits for transverse reinforcement in columns and piers*

6.4.7.1 Spiral or circular hoop reinforcement for columns and piers shall conform to 5.3.29 and 5.4.1 and shall be placed as follows:

(a) When a strength reduction factor ϕ of 0.9 is used:

(1) Volumetric ratio ρ_s shall be not less than the value given by

$$\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{yh}} \quad \text{. . . . . . (Eq. 6-3)}$$

where f_{yh} shall not exceed 400 MPa.

6.4.7.2 Hoop or tie reinforcement not of circular shape for columns and piers shall conform to 5.3.30 and 5.4.2 and shall be placed as follows:

(a) When a strength reduction factor ϕ of 0.9 is used:

(1) The total area in each of the principal directions of the cross-section within spacing s_h shall not be less than

$$A_{sh} = 0.3\, s_h h''\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{yh}} \quad \text{. . . . . (Eq. 6-4)}$$

where f_{yh} shall not exceed 400 MPa

For clause 6.4.1.5 the maximum allowable axial load is taken as 0.8 P_o for a strength reduction factor of 0.9. The maximum axial load of 0.85 P_o for a strength reduction factor of between 0.7 and 0.9 has not been considered as the applied axial load must be less than 0.1 $f'_c A_g$.

Note that the volumetric ratios calculated by Equations 6.3 and 6.4 of NZS 3101 are a maximum and may be reduced according to clauses 6.4.7.1(b) and 6.4.7.2(b).

4.3.3 Members Designed for Seismic Loading

Checks are made against the following clauses of NZS 3101:

6.5.1.5 For columns and piers the maximum design axial load in compression at a given eccentricity P_e shall not exceed 0.7 $\phi f'_c A_g$, unless it can be shown that P_e is less than 0.7 ϕP_o

where

$$P_o = 0.85 f'_c (A_g - A_{st}) + f_y A_{st} \quad \text{. . . . . . (Eq. 6-13)}$$

6.5.4.2 Longitudinal reinforcement in columns and piers shall be as follows:

(a) Area of longitudinal reinforcement shall be not less than 0.008 A_g

(b) Area of longitudinal reinforcement shall be not greater than 0.06 A_g for Grade 275 steel nor greater than 0.045 for Grade 380 steel, except that in the region of lap splices the total area shall not exceed 0.08 A_g for Grade 275 steel nor 0.06 A_g for Grade 380 steel

6.5.4.3 Transverse reinforcement in columns and piers shall conform to 5.5.4 and 5.5.5 and shall be placed as follows:

(a) In potential plastic hinge regions, as defined in 6.5.4.1, when spirals or circular hoops are used:

(1) Volumetric ratio ρ_s shall not be less than the greater of:

$$\rho_s = 0.45\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{yh}}\left(0.5 + 1.25\frac{P_e}{\phi f'_c A_g}\right) \quad \text{(Eq. 6-22)}$$

or

$$\rho_s = 0.12\frac{f'_c}{f_{yh}}\left(0.5 + 1.25\frac{P_e}{\phi f'_c A_g}\right) \quad \text{(Eq. 6-23)}$$

except that, where permitted by 6.5.4.3 (c), ρ_s may be reduced by one-half

(b) In potential plastic hinge regions, as defined in 6.5.4.1, when rectangular hoops with or without supplementary cross-ties are used:

(1) Total area of hoop bars and supplementary cross-ties in each of the principal directions of the cross-section within spacing s_h shall not be less than the greater of

$$A_{sh} = 0.3\, s_h h''\left(\frac{A_g}{A_c} - 1\right)\frac{f'_c}{f_{yh}}\left(0.5 + 1.25\frac{P_e}{\phi f'_c A_g}\right) \quad \text{(Eq. 6-24)}$$

or

$$A_{sh} = 0.12\, s_h h''\frac{f'_c}{f_{yh}}\left(0.5 + 1.25\frac{P_e}{\phi f'_c A_g}\right) \quad \text{(Eq. 6-25)}$$

except that where permitted by 6.5.4.3 (c) A_{sh} may be reduced by one-half

The confinement calculated by the programs is the <u>full</u> amount given by the appropriate equations in NZS 3101. Clause 6.5.4.3(c) allows a 50% reduction for columns of frames protected against plastic hinging by capacity design procedures.