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Dynamic response of pile groups under lateral loading

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Dynamic experiments on full-scale pile groups subjected to lateral loading were conducted in the field, first during winter when the surface was covered by a frozen soil layer and then in the summer under a completely unfrozen state. The results of the experiments are compared with theoretical predictions based on the approach of the dynamic interaction factors. Using frequency response curves of stiffness, damping and displacement amplitude of the pile group, the influence of pile-soil-pile interaction and the frozen soil layer on the dynamic behaviour of the pile group were studied. It is shown that by allowing for the boundary zone and free-pile length effects, close agreement can be achieved between the theoretical predictions and the measured response curves.

Key words: dynamics, vibration, pile group, full-scale testing, frozen soil, modelling.

INTRODUCTION

With distances of less than ten diameters between piles, the group effects, generally referred to as pile-soil-pile interaction, become pronounced. At present, the analysis of pile-soil-pile can be conducted in two ways: (1) direct analysis using techniques such as finite element where all piles of the group are considered to interact at the same time, and (2) approximate analysis based on the superposition of interaction factors where the interaction between each two piles is considered separately in forming the flexibility matrix of the entire group. The superposition method provides an approximate approach as its accuracy depends on how realistically the stiffness and damping of the single piles and the interaction factors can be determined. The primary advantage of this approach lies in its simplicity and ease of applicability; analysis of pile groups can be performed using theories and results developed for single piles. For static loads, interaction factors were introduced by Poulos and Davis [18] and by El-Sharnouby and Novak.[4] Dynamic interaction factors were presented in chart forms by Kaynia and Kausel[8] and Novak and Mitwally,[14] using dynamic interaction factors, developed closed-form solutions for the complex stiffness matrices. The previous studies indicate that the superposition method is in good agreement with the direct analysis for the case of floating piles in a homogeneous soil.

In recent years, with the developments in the offshore structures technology and the nuclear power plant industry and other applications, the dynamic behaviour of pile groups has received a renewed attention. Although there are a number of methods available for analyzing the dynamic response of pile groups, very little information is available on the field validation of these techniques. The lack of calibrated parameters makes it rather difficult to perform reliable analysis of practical projects using the relevant models.

Experimental studies on small-scale pile groups include the test of 102 small piles by El-Sharnouby and Novak[5] and Hassini and Woods[7] experiments on groups of two and four small-scale piles. Test of small groups of full-scale piles were reported by Blaney *et al.*,[1] and Crouse and Cheange.[2] A large group of 56 full-scale piles was tested by Masuda *et al.*[11] Since the nature of pile-soil-pile interaction depends on the actual confining pressure and the contact situation between the piles and soil, full-scale tests on piles are considered to provide valuable data for practical applications. However, since the circumstances and conditions for the dynamic experiment of pile group are complex, it is more difficult to explain the measured results using pile dynamics theory alone.

A comprehensive study involving both theoretical analysis and full-scale testing of a pile group is described in this paper. The measured dynamic response of the pile group under lateral loading is compared with the theor-

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etical predictions using the approach of dynamic interaction factors. The relevent soil properties and test data used in ths study were obtained using advance measurement techniques. The full-scale pile group was comprised of six cast-in-place reinforced concrete piles, 7·5 m long and 0·32 m in diameter. It is shown that by incorporating boundary zone effects and making provisions for the free pile length, good agreement can be obtained between the theoretical response curves and the measured results. The influence of pile-soil-pile interaction on the dynamic behaviour of pile group is expressed in terms of frequency response curves of stiffness, damping and displacement amplitude of the pile group.

Another aspect studied in the research performed involves the influence of a frozen soil layer on the pile group when subjected to dynamic loading. This is an important consideration in the design of many structures. Although a considerable body of analysis and field observations is now available in studying the response of piles in permafrost areas to static loads (e.g., Nixon[12] Nixon and McRoberts,[13] Morgenstern *et al.*[10] Crowther[3]) very little has been published on the dynamic response of pile groups in frozen soils. In order to investigate the influence of frozen soil layer on dynamic behaviour of the pile group, the dynamic experiments were conducted first during the winter time when the surface was covered by a frozen soil layer and then during the summer time when the frozen soil had completely thawed out.

DYNAMIC INTERACTION FACTORS APPROACH

For two identical and equally loaded piles i and j, an interaction α_{ij} is defined as

$$\alpha_{ij} = \frac{\delta_{dij}}{\delta_{sjj}} \tag{1}$$

where δ_{dij} is dynamic displacement of pile i due to load on pile j and δ_{sjj} is static displacement of pile j due to its own load. Both dynamic and static displacements are referred to the pile head.

The dynamic interaction factors to be used in the analysis are complex, i.e.

$$\alpha = \alpha_1 + i\,\alpha_2 \tag{2}$$

in which α_1 and α_2 are real and imaginary parts, respectively; $i = \sqrt{-1}$.

The value of dynamic interaction factors depends on dimensionless frequency, a_o, pile spacing ratio, s/d, pile slender ratio, L/d, and the modulus ratio, E_p/E_s. Here, d is diameter of pile, L is pile length, s is pile spacing, E_p is modulus of pile, E_s is modulus of soil and $a_o = \omega r/V_s$, where ω is circular frequency, r is radius of pile and V_s is shear wave velocity of soil. If the pile slender ratio, L/d, and the modulus ratio, E_p/E_s, are assumed to have relatively little effect on the frequency variation, at least in the low frequency range, the dynamic interaction factors may be expressed as

$$\alpha(a_o, s/d, L/d, E_p/E_s) = \alpha_{st}(a_o = 0, s/d, L/d, E_p/E_s)\,F(a_o, s/d) \tag{3}$$

in which α_{st} is static interaction factor and $F(a_o, s/d)$ represents the frequency variation; it is derived using the interpolation scheme from the dynamic interaction factors developed by Kaynia and Kausel.[8]

From eqn (1), the compatibility equations can be expressed in terms of flexibility as

$$\{\delta\} = f_{st}\,[\alpha]\,\{P\} = [f]\{P\} \tag{4}$$

in which f_{st} is the static flexibility of a single pile, $\{P\}$ and $\{\delta\}$ are the vectors of forces and displacements at the pile head, respectively; $[f]$ is the group flexibility matrix; $[\alpha]$ is the $n \times n$ matrix of interaction factors, where n is the number of piles. Equation (4) is directly applicable to vertical translation or pure horizontal translation of pile cap. For coupled lateral translation and rocking, the moments and rotations are included in $\{P\}$ and $\{\delta\}$ respectively, with the appropriate interaction factors included in resulting in a $2n \times 2n$ system of equations. Inversion of the flexibility matrix results in the group stiffness matrix.

For rigid foundations, the complex foundation stiffness in different modes of vibration is calculated by applying the pertinent boundary conditions. The boundary condition for vertical stiffness calculation is: vertical displacement, $V_i = 1$, for $i = 1, 2, \ldots, n$. The vertical stiffness of pile group is then given by

$$K_v = \bar{K}_v \sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_{ij}^{v} \tag{5}$$

where $\bar{K}_v$ is the vertical static stiffness of single pile, ε_{ij}^{v} are the elements of $[\boldsymbol{\alpha}]_v^{-1}$; $[\boldsymbol{\alpha}]_v$ is the interaction matrix of vertical displacements.

For horizontal stiffness, the boundary conditions are horizontal displacement $U_i = 1$ and rotation $\Psi_i = 0$, for $i = 1, 2, \ldots, n$. Then, the horizontal stiffness of pile group is

$$K_u = \bar{K}_u \sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_{2i-1,2j-1}^{H} \tag{6}$$

in which $\bar{K}_u$ is the horizontal static stiffness of single pile, $\varepsilon_{2i-1,2j-1}^{H}$ are the elements of $[\boldsymbol{\alpha}]_H^{-1}$ that correspond to the horizontal forces associated with horizontal displacements, and $[\boldsymbol{\alpha}]_H$ is the matrix of interaction coefficients for the horizontal translation and rotations. For rocking stiffness, the boundary conditions are: $\Psi_i = 1$ and $U_i = 0$, for $i = 1, 2, \ldots, n$.

The rocking stiffness of pile groups can, therefore, be expressed as

$$K_\varphi = \bar{K}_\varphi \sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_{2i,2j}^{H} + \bar{K}_v \sum_{i=1}^{n} \sum_{j=1}^{n} \varepsilon_{ij}^{v} X_i X_j \tag{7}$$

where, $\bar{K}_\varphi$ if the rocking static stiffness of single pile, $\varepsilon_{2i,2j}^{H}$

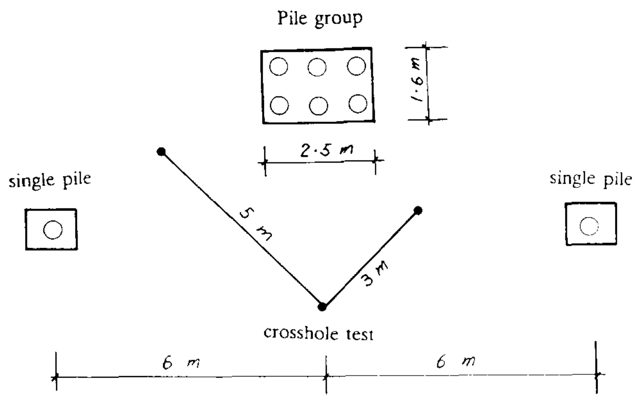

Fig. 1. Plan view of piles and crosshole test.

are the elements of $[\alpha]_H^{-1}$ that correspond to the moments associated with rotations; X_i and X_j are the distances of each pile from a reference point (usually the centre of gravity of the foundation).

The effects of pile-soil-pile interaction of stiffness and damping of the pile group are best illustrated by the group efficiency ratio, *GER*, defined for stiffness (and damping) as

$$GER = \frac{\textit{stiffness (damping) of pile group}}{n \times \textit{stiffness (damping) of a single pile}}$$

Table 1. Soil properties

Depth (m)	Bulk unit weight, γ(kN/m^3)	Void ratio e	Moisture content w (%)
2	20·2	0·58	21·8
4	18·5	0·75	23·7
6	19·7	0·60	20·5
8	20·0	0·59	20·2

where n is the number of piles in the group. With the stiffness and damping of pile group described as above, the coupled horizontal and rocking vibration of the pile group under lateral loading can be calculated using the computer program DYNA that was developed by Novak and his associates.

The stiffness of a single pile is embodied in the expressions for the pile group stiffness as shown in eqns (5), (6) and (7). These indicate that realistic dynamic analysis of pile groups depend largely on how accurately the impedances of single piles can be determined. To increase the accuracy in determining the impendence of single piles, the plane-strain model was extended to include a cylindrical annulus of softened soil (boundary zone) that may develop around the pile. The properties of the boundary zone are different from the homogeneous

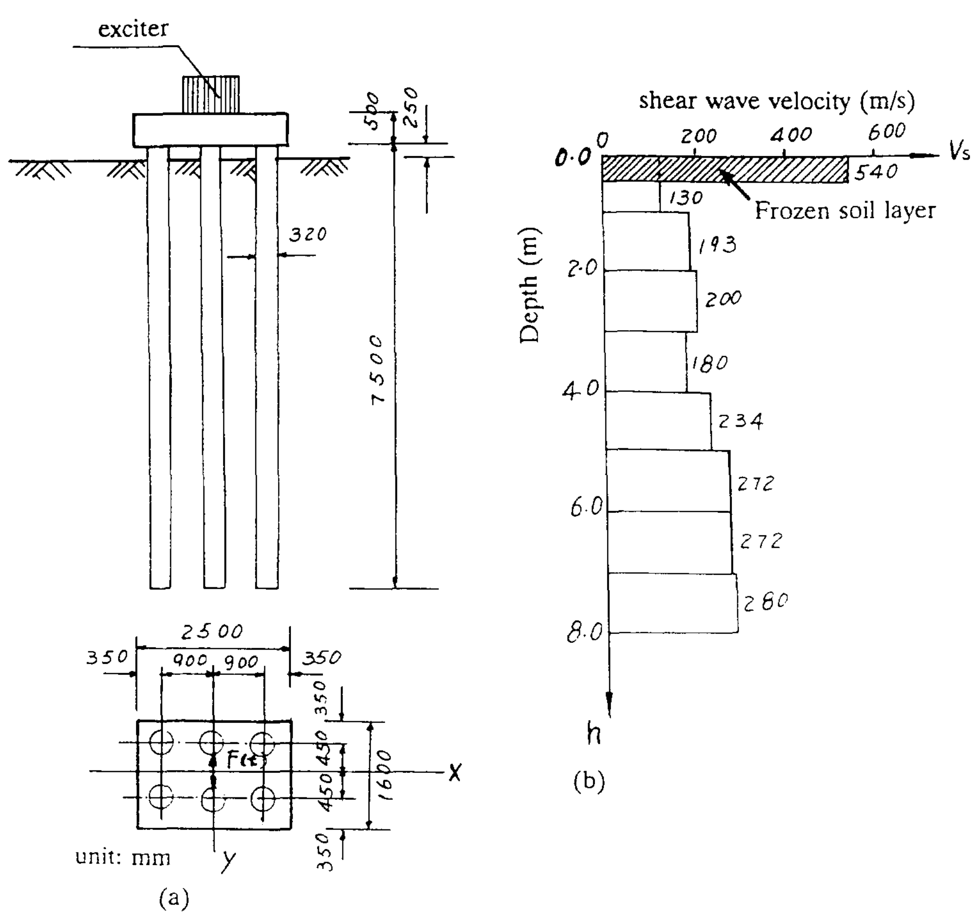

Fig. 2. Layout of pile group and soil conditions: (a) pile group; (b) shear wave velocity of soil.

Table 2. Pile properties

Unit weight, γ (kN/m^3)	Moment of inertia, I (m^4)	Young's modulus, E_p (MPa)	Poisson's ratio, v_p	Damping ratio, D_p
24·5	5·15 E − 4	1·96 E4	0·25	0·01

soil medium around it (see Novak and Sheta[15]; Novak and Han[16]; Novak and Aboul-Ella[17]). Development of a boundary zone can be attributed to a number of factors, such as pile installation and large vibration amplitude. Although the boundary zone may be very thin, its influence on the dynamic behaviour of piles is quite pronounced. Inclusion of the boundary zone concept, which provides a more realistic contact between the soil and piles, allows for a closer match to be achieved between theoretical predictions and measured results. With the concept of boundary zone, the applicability of the model of pile vibration was expanded to problems undergoing strong nonlinear effects (see Han and Novak[6]).

With the presence of boundary zone and a layered soil system, the assumption of homogeneity no longer applies. The frequency variation of the interaction factors $F(a_o, s/d)$ shown in eqn (3) can be affected by the nonhomogeneity of the soil (Kaynia[9]). But, the effect occurs at higher dimensionless frequencies, well above those achieved in the experiments, and thus do not have a very significant influence on the interaction factor frequency variations assumed here.

TEST DESCRIPTION AND TEST RESULTS

The site under study is located at the Institute of Engineering Mechanics, Harbin, China. The subsurface investigation indicated that the test site was underlain by a relatively homogeneous layer of silty clay with occasional lenses of sandy clay mixture down to a depth of 30 meters. Both laboratory and *in situ* tests were performed to characterize the dynamic and static properties of the soil. The laboratory experiments included triaxial test to measure the variation of shear modulus and damping ratio with shear strain; specific gravity; bulk density and Atterberg limit tests. The dynamic *in situ* test consisted of seismic crosshole tests for determining the shear wave velocity of the soil. Figure 1 illustrates the arrangement of test piles and the crosshole tests. Some of the measured soil properties used in this study are listed in Table 1.

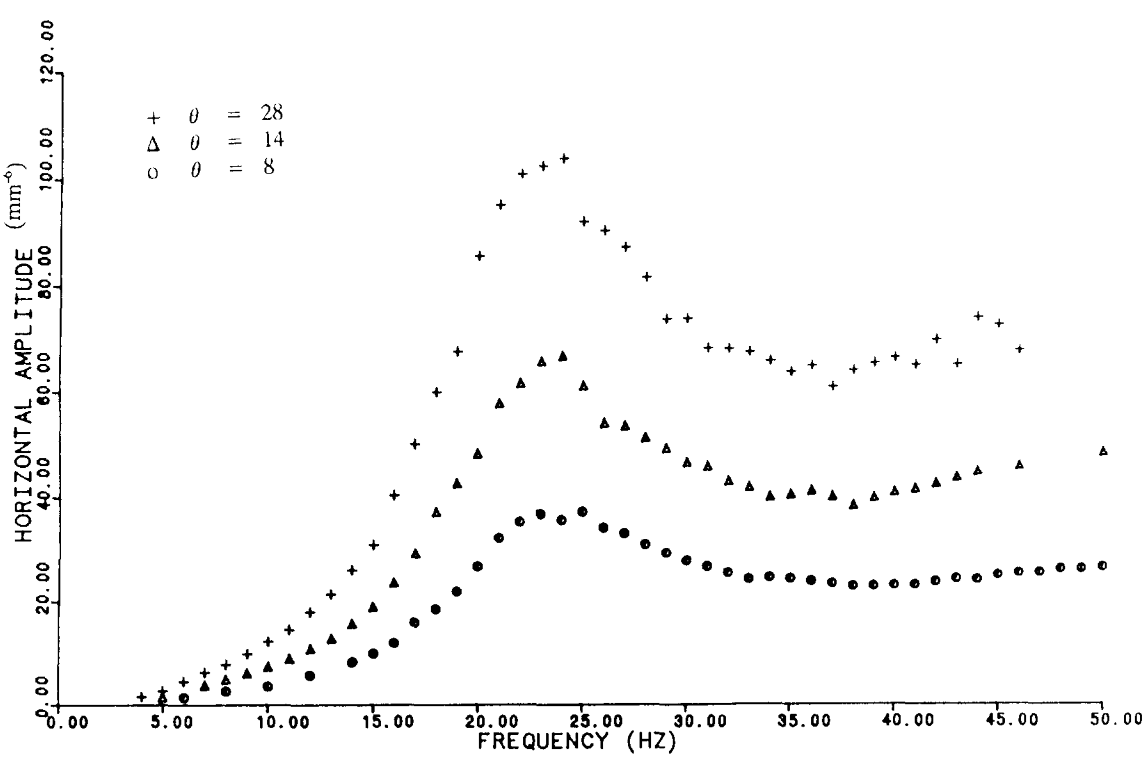

Fig. 3. Measured horizontal response of pile group.

Table 3. Amplitude of exciting force

Angle of eccentric mass, θ	8°	14°	28°
Exciting force, (N)	$3{\cdot}79\,f^2$	$6{\cdot}75\,f^2$	$10{\cdot}23\,f^2$

The pile group under study was comprised of six bored reinforced concrete piles, 7·5 m long and 0·32 m in diameter. The pile slender ratio, L/d = 23·4 and spacing ratio, s/d = 2·81. The concrete cap was 2·5 m long by 1·6 m wide and 0·5 m thick, weighing 49·0 kN, and having a clearance of 0·25 m above the ground surface. Figure 2 shows the scheme of the pile group and the variation of shear wave velocity over the length of the pile. The properties of the pile are shown in Table 2.

An exciter with two counterrotating eccentric masses, weighing 1·18 kN, was fixed on the pile cap by foundation bolt to produce the horizontal harmonic excitation. The active component of the horizontal excitation was situated 0·2 m above the cap surface, and the centre of gravity of the cap-exciter system was 0·24 m below the cap surface. The magnitude of exciting force was changed by adjusting the angle of eccentric mass θ. Several levels of exciting force were used in the experiment, as indicated in Table 3, where f denotes the frequency of the exciting force (Hz).

As shown later, θ is used to represent the excitation intensity.

Two horizontal displacement pick-ups (to measure the horizontal vibration) and two vertical displacement pick-ups (to measure the rocking vibration) were mounted on top of the pile cap. The steady-state dynamic response of the pile group under horizontal excitation was measured under different frequencies and excitation intensities. The horizontal and rocking response curves of the pile group in different excitation intensities are shown in Figs 3 and 4, respectively. The maximum horizontal amplitude of the pile group was measured to be 0·104 mm at top of the pile cap; this is considered to be a small amplitude vibration.

In the winter, the thickness of the frozen soil layer at the time of the test was measured to be 0·35 m. The shear wave velocity of the frozen soil layer was measured by employing the surface wave method, and as it is shown in Fig. 2, this was 540 m/s. The horizontal and rocking response curves of the pile group for different excitation intensities are shown in Figs 5 and 6, respectively.

COMPARISON OF EXPERIMENTAL RESPONSE CURVES WITH THEORETICAL PREDICTIONS

The problem under consideration involves application of a harmonic exciting force $F(t)$ acting in the Y-direction on the pile group, as shown in Fig. 2. The force $F(t)$ is given by

$$F(t) \;=\; (M_c e)\,\omega^2 \cos \omega t \tag{8}$$

where $M_c e$ is the eccentric mass of the exciter multiplied by eccentric distance and t is time.

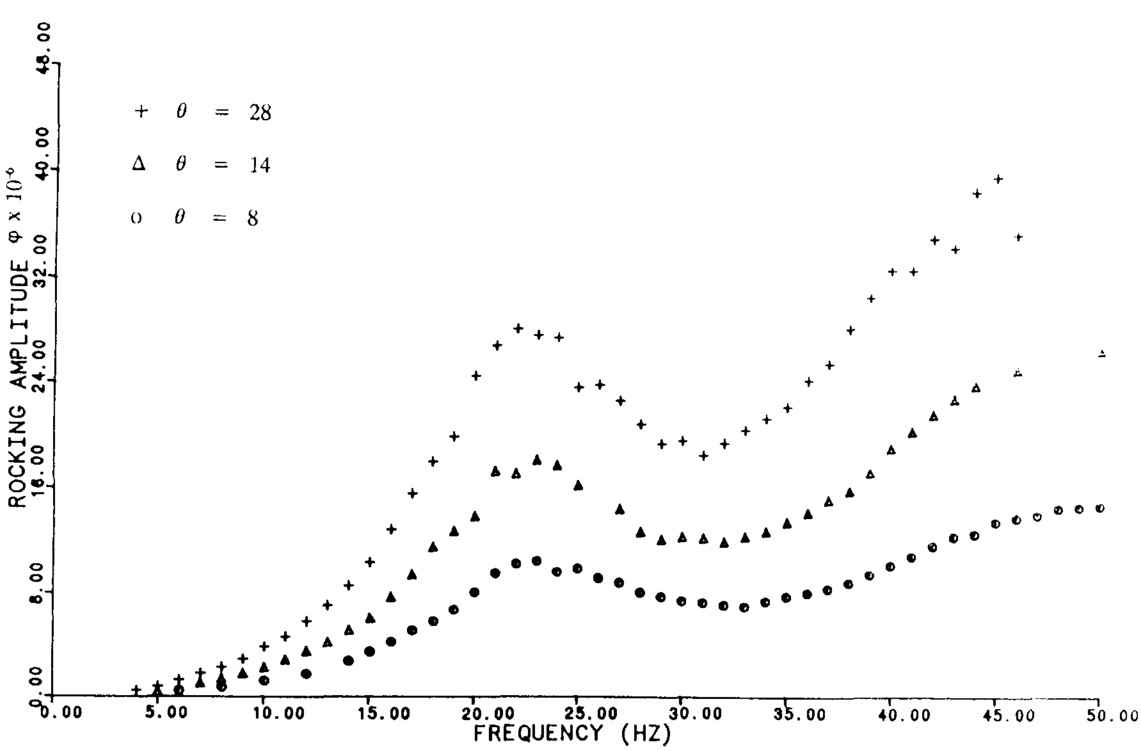

Fig. 4. Measured rocking response of pile group.

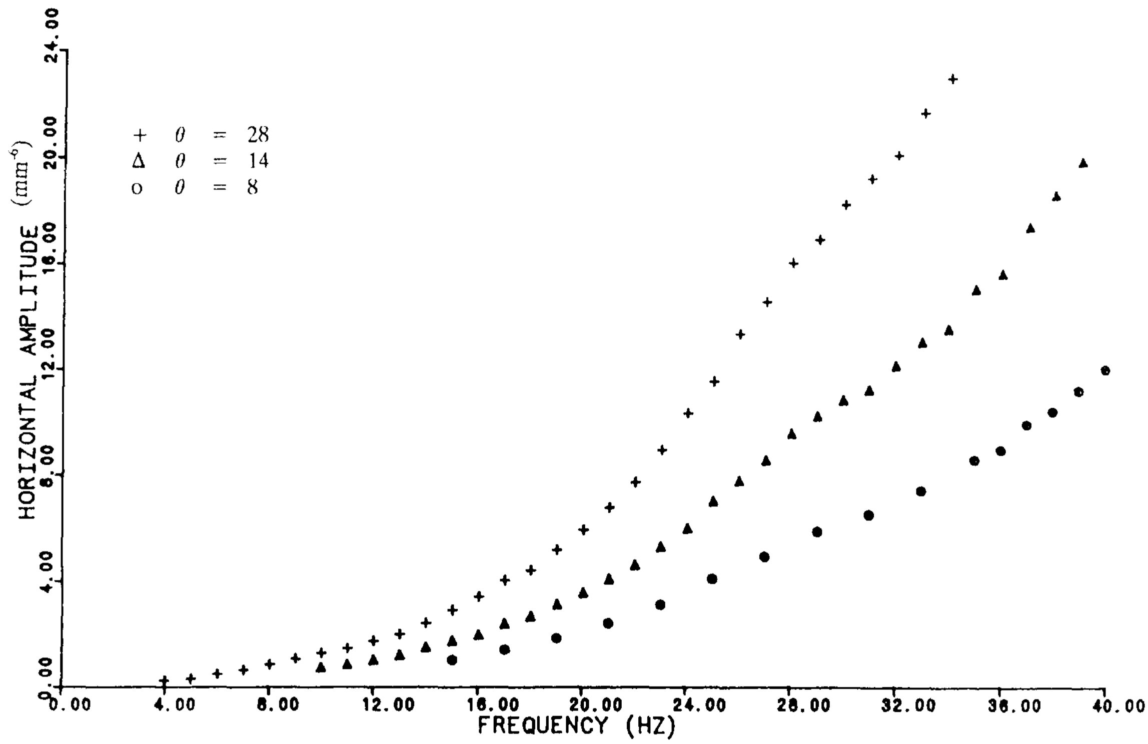

Fig. 5. Measured horizontal response of pile group with frozen layer.

For linear vibration, the response curves of the pile group can be normalized. Normalized response amplitude for translation is defined as:

$$A = (M/M_e e)\ U \tag{9}$$

and for rotation by:

$$\Phi = [I/(M_e e\ Z_e)]\ \Psi \tag{10}$$

where U and Ψ are the real translation and rotation, respectively; M and I are the mass and mass moment of

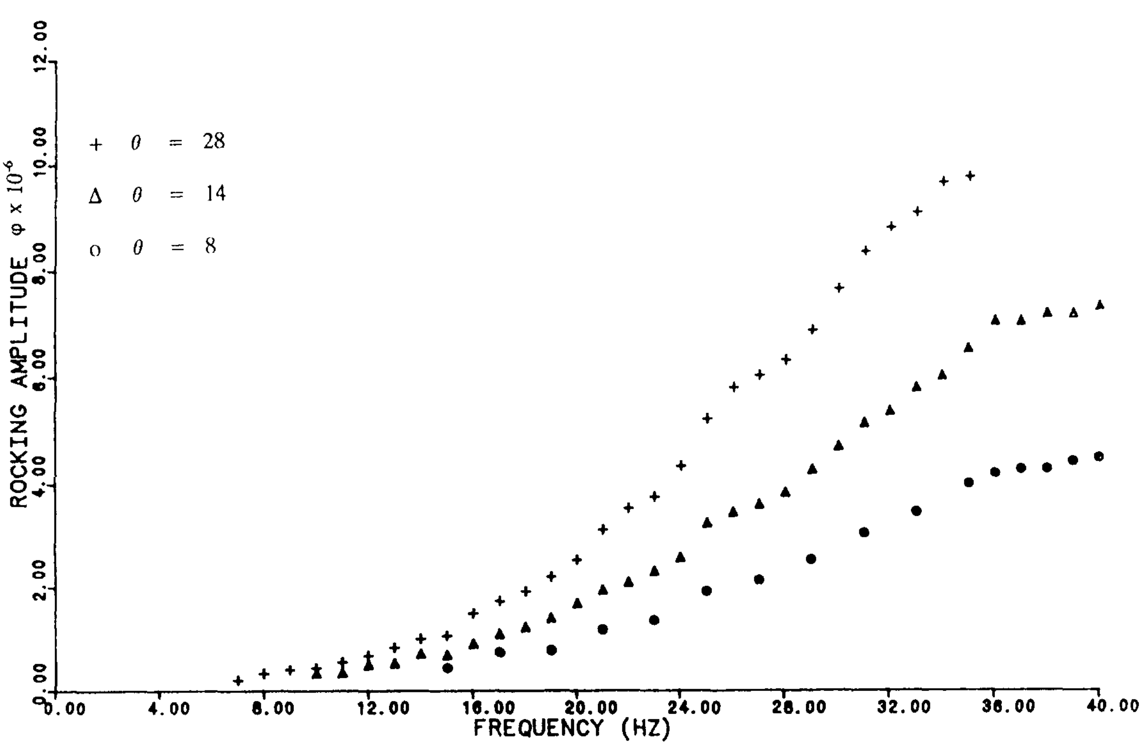

Fig. 6. Measured rocking response of pile group with frozen layer.

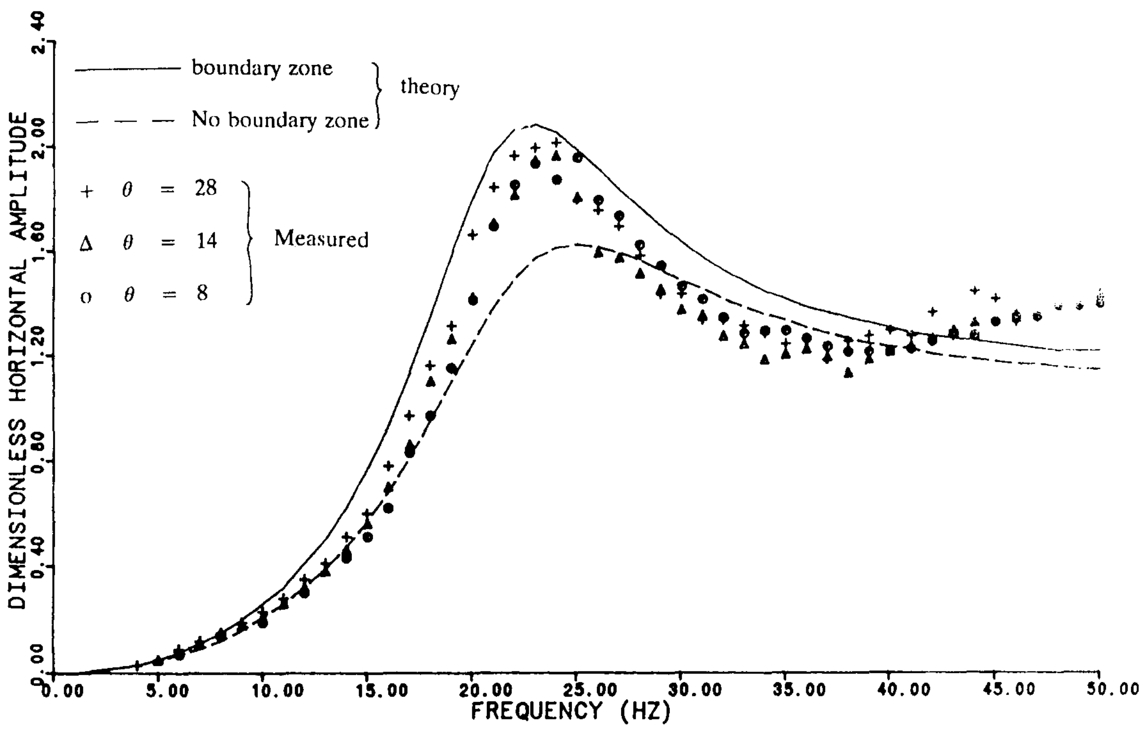

Fig. 7. Horizontal response of pile group.

inertia for the cap; Z_e is the height of the horizontal excitation above the centre of gravity.

For the theoretical analysis, Poisson's ratio of the soil was taken as $v_s = 0{\cdot}3$ and its damping ratio $D_0 = 0{\cdot}035$. The relevant parameters of the boundary zone are as follows: $G_i/G_0 = 0{\cdot}1$, where G_i and G_0 are shear moduli for the inner zone and outer soil, respectively; thickness of the zone, $t_m = 0{\cdot}5r$, where r is the pile radius; damping ratio of inner zone, $D_i = 0{\cdot}07$.

With the properties of soil and pile shown in Tables 1 and 2 and the parameters of boundary zone, the dynamic response of the pile group can be calculated using the dynamic interaction factors approach. Comparison of experimental response curves and the theoretical predic-

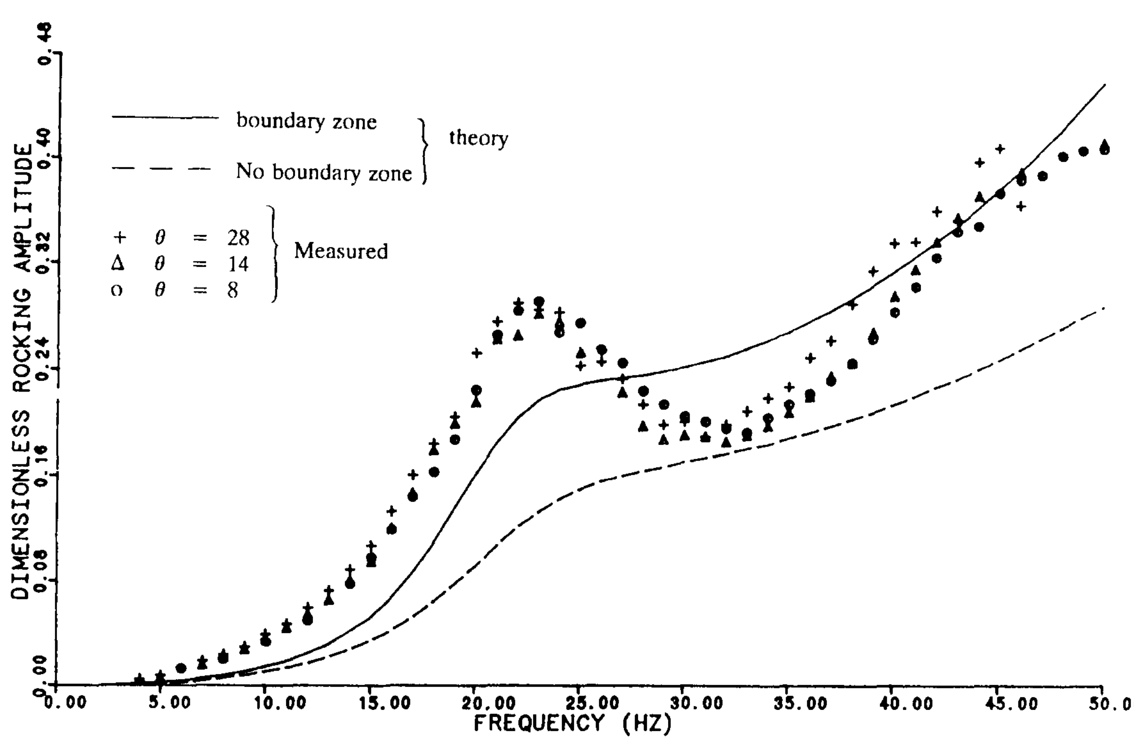

Fig. 8. Rocking response of pile group.

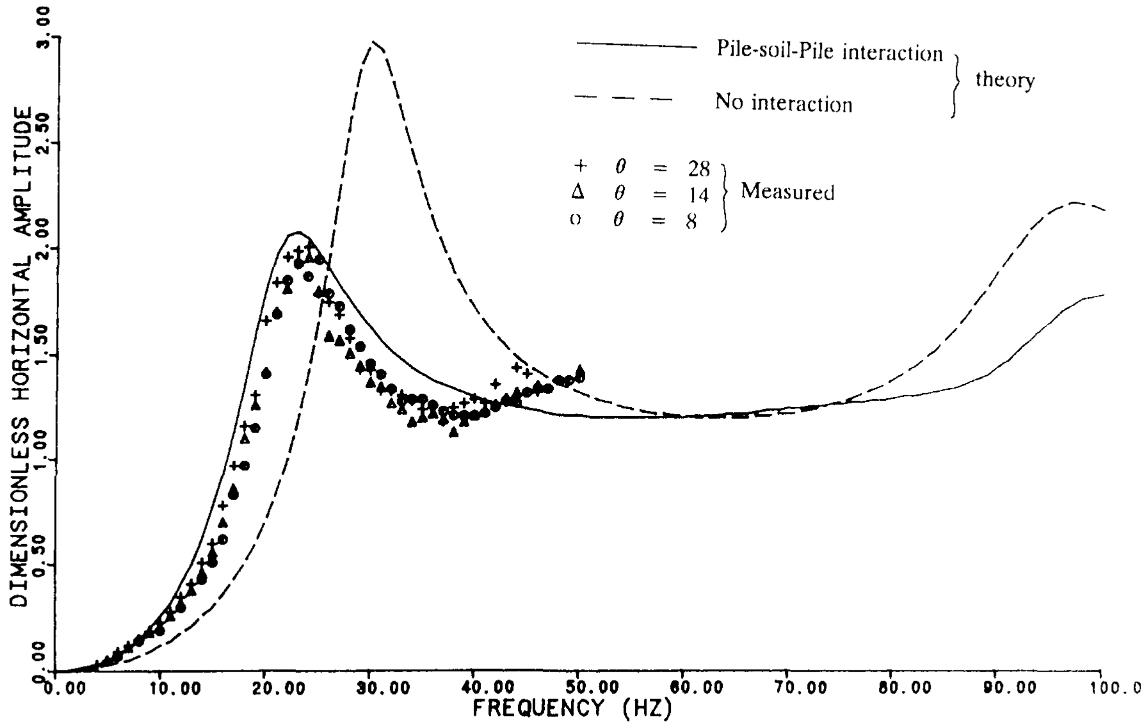

Fig. 9. Comparement of horizontal response for interaction.

tions is shown in Figs 7 and 8 for horizontal and rocking vibration of the pile group, respectively. It can be seen that the theoretical predictions agree with the measured results quite well, particularly for the horizontal vibration.

As another calculation method, the analyses were repeated with no regard to the boundary zone effects (i.e., the boundary zone was omitted). In this case, in order to properly account for the actual soil and pile contact, allowance was made for a free pile length (equivalent separation between pile and soil) at the top of the pile. Separation of the top segment of the pile from the soil can occur because of the very low confining stress that

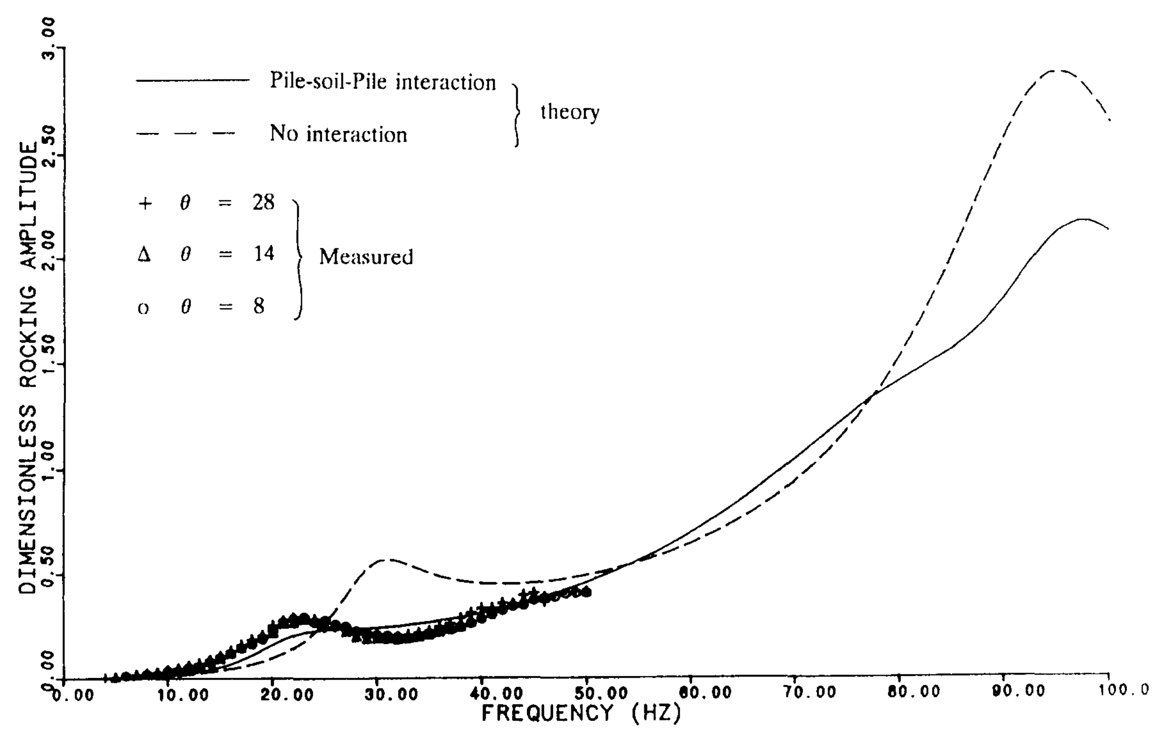

Fig. 10. Comparement of rocking response for interaction.

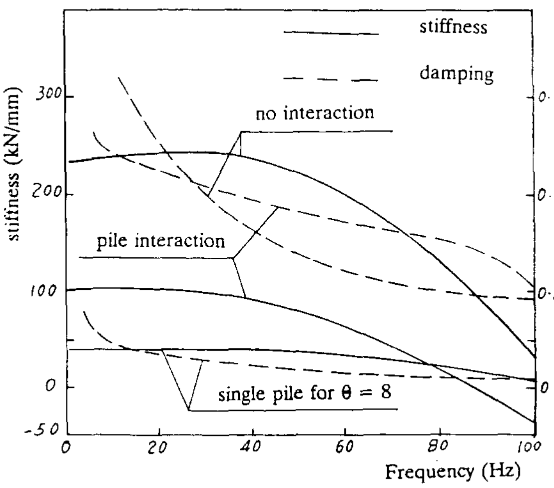

Fig. 11. Horizontal stiffness and damping of pile group.

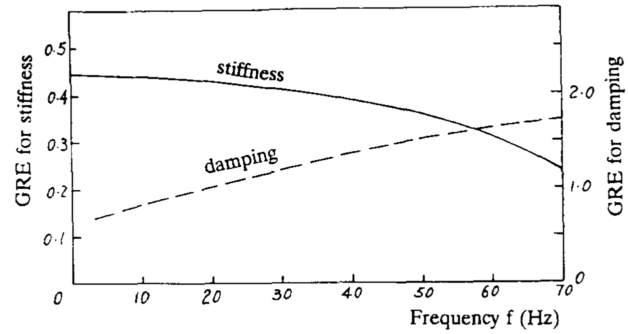

Fig. 13. Group efficiency ratio for horizontal vibration.

exists in the soil layer within the close proximity of the ground surface. Herein, the free pile length is taken as 0·5*d*. The dashed lines in Figs 7 and 8 represent the theoretical calculation without boundary zone. It can be seen that the agreement with this approach, albeit reasonable, is not as good as the case with the boundary zone.

In order to demonstrate the influence of pile-soil-pile interaction on the dynamic response of the pile group, the dynamic response was calculated with interaction effects omitted. The comparison of horizontal and rocking vibration against measured data is shown in Fig 9 and 10, respectively. It can be seen that the theoretical predictions without interaction (dashed lines in the figures) result in a higher resonant frequency and a larger displacement amplitude. It can also be noticed that another resonant peak develops around the frequency of 100 Hz; this is primarily due to the second mode of vibration for the coupled horizontal and rocking vibration of the pile cap.

The stiffness and damping for horizontal and rocking vibration of the pile group are shown in Figs 11 and 12, respectively. From these figures it can be seen that the stiffness of pile group is reduced and the damping is increased by the pile-soil-pile interaction, as was noted previously. More interestingly, however, is that the stiffnesses of the pile group varies with frequency. For comparison, the frequency response curves of stiffness and damping for a single pile are also shown in Fig. 11.

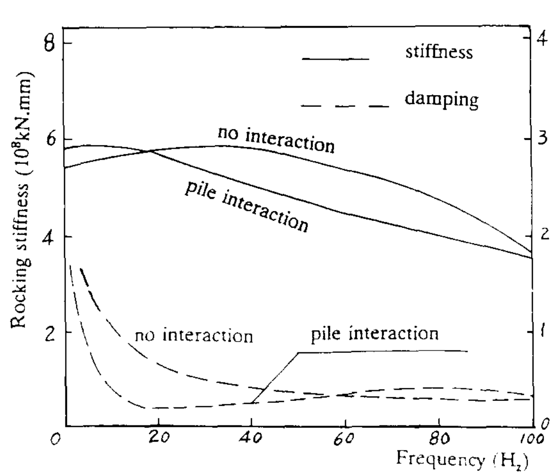

Fig. 12. Rocking stiffness and damping of pile group.

The group efficiency ratio, *GER*, for the horizontal vibration is shown in Fig. 13. It can be noticed that the *GER* for stiffness is less than 1, implying that the stiffness of the pile group is reduced by pile-soil-pile interaction (by more than 50% in this case); and it is larger than 1 for damping, implying that the damping of the pile group is increased by pile-soil-pile interaction. Furthermore, it can be noted that the *GER* for stiffness and damping is not constant, rather it varies with frequency. With the frequency range considered here, *GER* for stiffness is reduced and that for damping is increased with an increase in frequency.

INFLUENCE OF FROZEN SOIL LAYER

To study the influence of a frozen soil layer on the dynamic response of a pile group, a similar set of experiments as described in the preceding sections was performed during the winter months when the ground surface was covered by a layer of frozen soil, 0·35 m thick, for which the measure shear wave velocity was 540 m/s. In this case, due to the increased soil resistance resulting from soil freezing, the maximum horizontal displacement that could be achieved (with the employed exciter system) was only 0·023 mm, as shown in Fig. 5. For such small displacements and hence strains, it was considered inappropriate to allow for either soil separation or yielding in the frozen soil layer for the analysis. The comparison of experimental response curves with the theoretical predictions is shown in Figs 14 and 15 for horizontal and rocking vibration of the pile group, respectively. Because of the much higher resonant frequency and the limitation of the exciter, the resonance peak could not be obtained. However, the results, at least over the portion for which data were available, show a satisfactory agreement with the measured response.

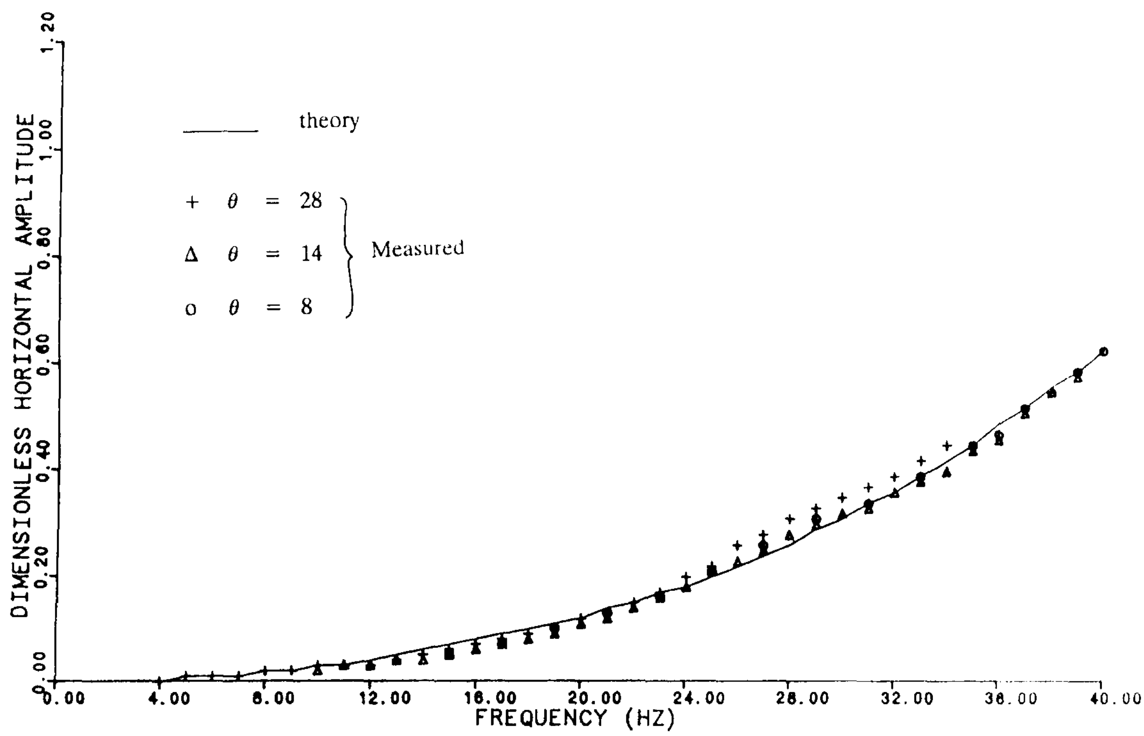

Fig. 14. Horizontal response of pile group with frozen layer.

In order to assess where the resonant frequency for the pile group with a layer of frozen soil would occur, the theoretical calculations were extended to a frequency of 100 Hz. These results are shown in Figs 16 and 17 for the horizontal and rocking vibration, respectively. For comparison, the results for the case with no frozen layer are also plotted along with the measured data. From these figures it can be seen that the resonant frequency for the pile group with frozen soil layer is much higher than that without the frozen soil layer.

The stiffness and damping of the pile group with and without a frozen layer are shown in Figs 18 and 19 for

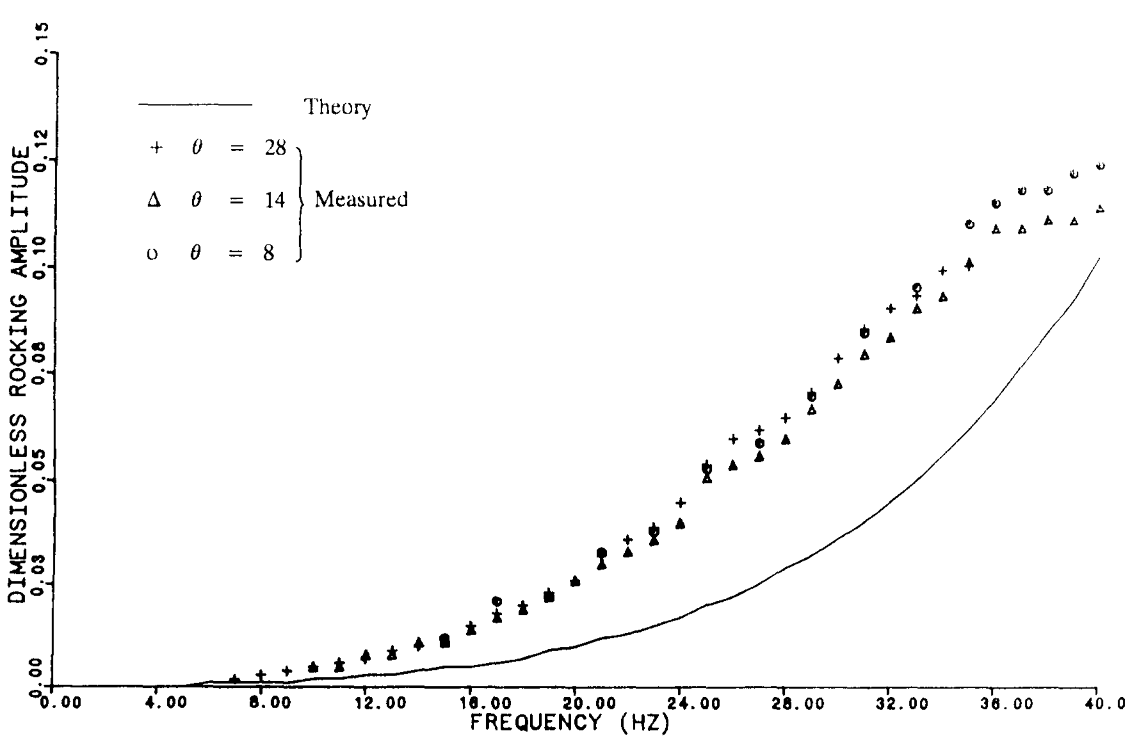

Fig. 15. Rocking response of pile group with frozen layer.

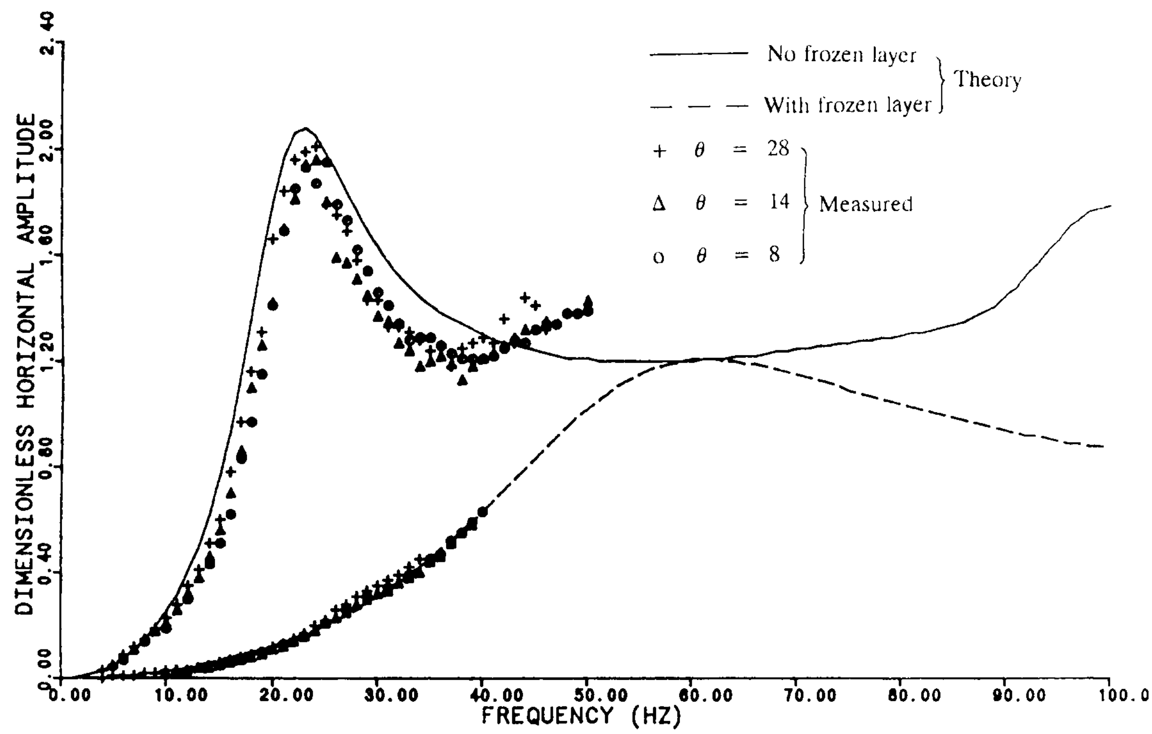

Fig. 16. Frozen soil effect on the horizontal response.

horizontal and rocking vibration, respectively. From these figures it can be seen that the stiffness and damping of the pile group with frozen soil layer are much higher than those without the frozen soil layer. For instance, the horizontal stiffness of the pile group is increased by a factor of eight as compared to the case with no frozen soil. From these observations it can be concluded that the presence of even a thin layer of frozen soil (in the order of 1/20 of the pile length) can have a profound influence on the response of pile group, as was demonstrated by a

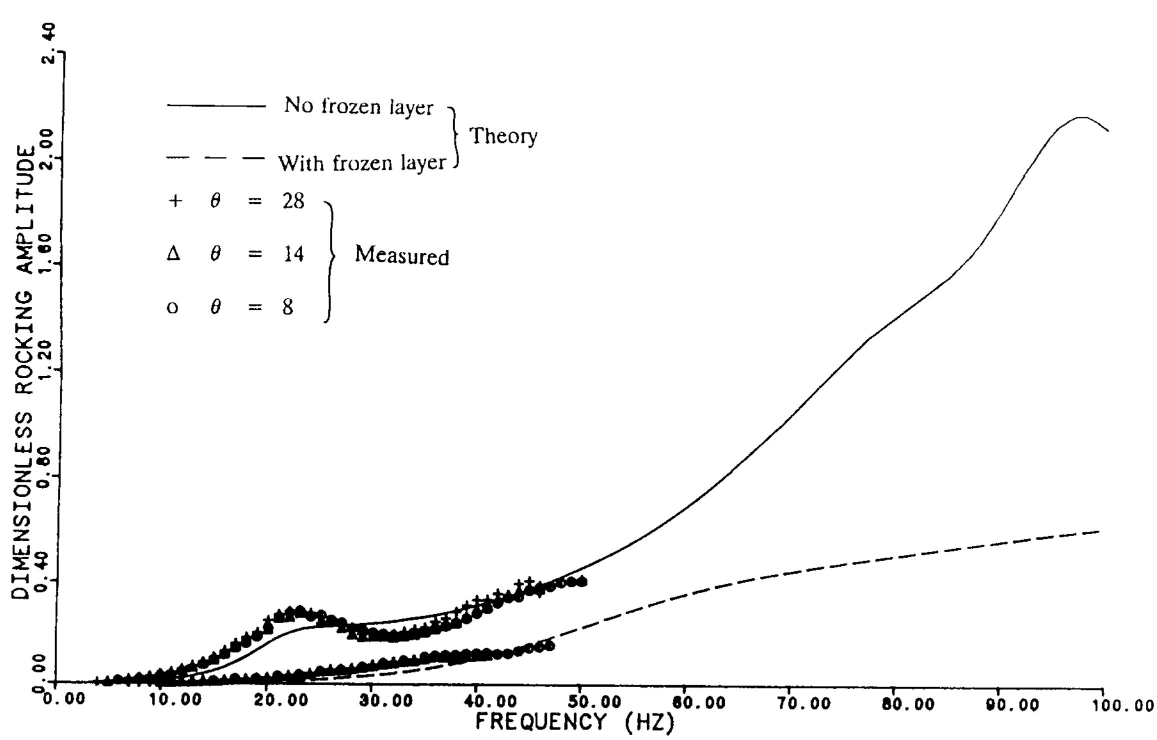

Fig. 17. Frozen soil effect on the rocking response.

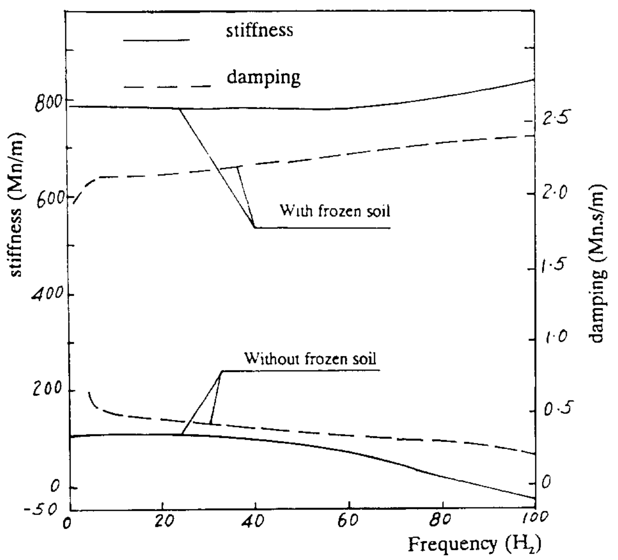

Fig. 18. Frozen soil effect on the horizontal stiffness and damping.

significant reduction in displacements and an increase in the resonant frequency.

SUMMARY AND CONCLUSIONS

The study reported here was concerned with the *in situ* dynamic response of full-scale pile groups and the evaluation of theoretical solutions for matching the observed field data. The influence of pile-soil-pile interaction on the dynamic behaviour of the pile group was investigated using the approach of dynamic interaction factors. Another aspect which was concurrently investigated was the influence of a thin, near-surface frozen soil layer (which is a rather common occurrence in large regions of the world during cold seasons) on the response of the pile group. The field tests were performed on a pile group comprised of six cast-in-place reinforced concrete piles, each 7·5 m long and 0·32 m in diameter, embedded in a relatively homogeneous silty clay formation. Several tests at different excitation intensities were performed in the winter and then in the summer to assess the influence of the frozen soil layer.

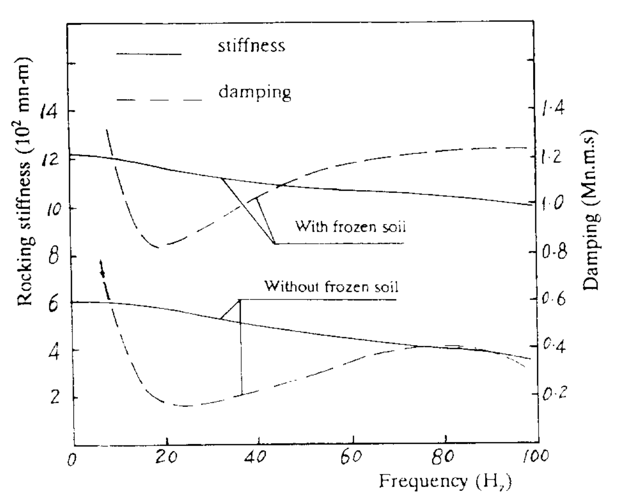

Fig. 19. Frozen soil effect on the rocking stiffness and damping.

Some of the conclusions that can be made from the study performed are as follows:

1. The pile-soil-pile interaction results in a reduction in the stiffness and an increase in the damping of the pile group. The group efficiency ratio is not a constant, showing a reduction for stiffness and an increase for damping as the frequency increases.
2. The presence of a frozen soil layer has a profound influence on the dynamic response of pile groups which is manifested by a significant reduction in the displacement and increase of the resonant frequency; horizontal stiffness of the pile group was increased by a factor of eight in the presence of a 0·35 m thick (less than 1/20 of the pile length) frozen soil layer.
3. The approach based on the dynamic interaction factors predicts the measured response curves satisfactorily. The advantages in using this approach are that the dynamic calculations can be performed on the basis of the single pile theory and the dynamic interaction factors can be derived from the static interaction factors through dimensionless frequency functions. This enables the static interaction factors and the findings from the behaviour of single piles, that have been relatively well studied, to be used in the analysis of pile groups.
4. Incorporation of the boundary zone effects and provisions for the development of a free pile length in the analysis improve the agreement between calculated and measured results.

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REFERENCES

1. Blaney, G.W., Muster, G.L. & O'Nell, M.W. Vertical vibration test of a full-scale pile group, *Geotechnical Special Publication*, ASCE, 1987, **11**, 149–165.
2. Crouse, C.B. & Cheang, L. Dynamic testing and analysis of pile-group foundations, *Geotechnical Special Publication*, ASCE, 1987, **11**, 79–98.
3. Crowther, G.S. Analysis of laterally loaded piles embedded in layered frozen soil, *J. of Geotch. Engrg.*, ASCE, 1990, **116**(7), 1137–1152.
4. El-Sharnouby, B & Novak, M. Flexibility coefficients and

interaction factors for pile group analysis, *Canadian Geotech. J.*, 1986, **23**(4), 441–450.
5. El-Sharnouby, B. & Novak, M. Dynamic experiments with group of piles, *J. of Geotech. Engrg.*, ASCE, 1984, **110**(6) 523–534.
6. Han, Y.C. & Novak, M. Dynamic behaviour of single pile under strong harmonic excitation, *Canadian Geotech. J.*, 1988, **25**(3), 523–534.
7. Hassini, S. & Woods, R.D. Dynamic experiments with model pile foundations, *Proc. 12th ICSMFE*, Rio de Janeiro, 1989, 2, 1135–1138.
8. Kaynia, A.M. & Kausel, E. Dynamic behaviour of pile group, *Conf. on Num. Methods in Offshore Piling.*, Univ. of Texas, Austin, TX. 1982, pp. 509–532.
9. Kaynia, A.M. Characteristic of the dynamic response of pile group in homogeneous and nonhomogeneous media, *Proc. 9th World Conf. on Earthquake Engrg.*, Aug. 2–9, 1988, Tokyo-koyota, Japan, 3, 575–580.
10. Morgenstern N.R., Roggensack, W.D. & Weaver, J.S. The behaviour of friction piles in ice and ice-rich soil, *Canadian Geotech. J.*, 1980, **17**, 405–415.
11. Masuda, K., Saseki, F., Urao, K., Veno K. & Miyamoto, Y. Simulation analysis of forced vibration test of actual pile foundation by thin layer method. *Proc. Annual Meeting of Architectural Institute of Japan*, 1986.
12. Nixon, J.K. Foundation design approaches in permafrost areas, *Canadian Geotech. J.*, 1978, **21**, 431–438.
13. Nixon, J.K. & McRoberts, E.C. A design approach for pile foundations in permafrost, *Canadian Geotech. J.*, 1976, **13**, 40–57.
14. Novak, M. & Mitwally, H. Random response of offshore towers with pile-soil-pile interaction, *J. of Offshore Mechanics and Artic Engrg.*, Feb. 1990, **112**, 35–41.
15. Novak, M. & Sheta, M. Approximate approach to contact effects of piles, *Proc. Dynamic Response of Pile Foundations: Analytical Aspects*, ASCE Florida, 1980, 53–79.
16. Novak, M. & Han, Y.C. Impedances of soil layer with boundary zone *J. of Geotech. Engrg.*, ASCE., 1990, **116**(6), 1008–1014.
17. Novak, M. & Aboul-Ella, F. (1978). Impedance functions of piles in layered media, *J. of Geotech. Engrg.*, ASCE, 1978, **104**(EM3), 643–661.
18. Poulos, H.G. & Davis, E.H. *Pile Foundation Analysis and Design*, John Wiley and Sons, 1980, 397 p.