OPINION

PROCEDURES FOR CALCULATING THE SHEAR RESPONSE OF REINFORCED CONCRETE ELEMENTS: A DISCUSSION

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ABSTRACT: This paper demonstrates that the two so-called "conceptual errors" which are claimed to exist in the modified compression field theory are actually conceptual errors about the modified compression field theory.

INTRODUCTION

During the last 20 years, a considerable amount of experimental and analytical research has been conducted with the aim of developing analytical procedures capable of predicting the load-deformation response of reinforced concrete elements loaded in shear. At the University of Toronto, a procedure called the compression field theory, CFT (Collins 1978), was first developed. Based on experiments of reinforced concrete panels loaded in pure shear, this procedure was further developed and became called the modified compression field theory, or MCFT (Vecchio and Collins 1981, 1982, 1986).

At the University of Houston, the rotating angle-softened truss model, RA-STM (Belarbi and Hsu 1995), and the fixed angle-softened truss model, FA-STM (Pang and Hsu 1996; Hsu and Zhang 1997), were developed. Though the formulations of these two versions of the softened truss model are considerably more complex than those of the modified compression field theory, it is claimed (Hsu 1996, 1998; Pang and Hsu 1996) that the MCFT contains "two conceptual errors" which cause it to be up to 32% unconservative. Hence, Hsu states that this earlier procedure "can not be considered a rational theory." It is the intention of this short article to clarify the issues that have led to these erroneous conclusions.

CFT, MCFT, RA-STM, FA-STM, AND V_c

The compression field theory neglects the ability of concrete to resist tensile stress. The modified compression field theory accounts for the influence of the tensile stresses in the cracked concrete. This is illustrated in Fig. 1, which shows how the tensile strength of concrete can, in some cases, increase shear strength. The additional shear strength that results from the tensile strength of concrete is called V_c. Note that for panels with 1% longitudinal reinforcement and with from 0.6 to 1.6% transverse reinforcement, this figure, which is from the 1982 Vecchio and Collins report, predicts that V_c will be zero. This is because, for these reinforcement ratios, failure is governed by yielding of both the transverse and the longitudinal reinforcement at the cracks.

Hsu, apparently, is unclear about the distinction between the compression field theory and the modified compression field theory. Referring to the 1981 paper, he states, "Vecchio and Collins called their analysis the compression field theory," and he goes on to state that, in essence, this theory is the same as the rotating angle-softened truss model and that neither of these theories can predict the concrete contribution V_c. To allow for the derivation of V_c, he introduced the fixed angle-softened truss model. It is true that the compression field theory cannot predict V_c, but the modified compression field theory, even as presented in the 1981 paper, can predict V_c. For the MCFT, V_c becomes zero when the reinforcement in both directions has yielded at a crack. For the RA-STM, V_c becomes zero when the reinforcement in either direction yields at a crack.

In many practical situations, it is necessary to investigate the shear capacity of cracked reinforced concrete elements that do not contain reinforcement in both longitudinal and transverse directions (e.g., beams without stirrups). For such cases, it is critical to evaluate V_c accurately. The second major series of experiments conducted on the University of Toronto's membrane element tester involved 30 tests of panels reinforced in only one direction (Bhide and Collins 1987, 1989). In 18 of these tests, the panel was subjected to combined uniaxial tension and shear, simulating the loading experienced by the flexural tension region of the web of a beam. It was found that such elements have considerable postcracking shear capacity and that the 1986 MCFT could predict the V_c values well. The ratio of observed to calculated V_c had an average value of 1.00 with a coefficient of variation of 11%. In some of these tests (e.g., PB21), the principal strain directions were observed to rotate by nearly 50° between first cracking and final failure.

In addition to calculating average stresses and average strains, the MCFT, even as presented in the 1981 paper, checks the stress conditions at a crack. Instead of directly checking stresses at a crack, the RA-STM adjusts the average stress-average strain relationships of the reinforcement to account for the possibility of local yielding at a crack. Unfortunately, the forms of expression chosen involve dividing by the percentage of reinforcement; hence, as the percentage approaches zero, the expressions give unreliable results. For elements with large differences in the amounts of reinforcement in the two directions—that is, elements which will experience a large rotation in principal strain direction—Pang and Hsu (1996) recommend the use of the fixed angle-softened truss model. Unfortunately, the FA-STM equation given for V_c [Eq. (13), Hsu (1998)] again involves dividing by the percentage of transverse reinforcement. Elements with no transverse reinforcement would appear to have a V_c value of infinity.

TRANSMITTING LOADS ACROSS CRACKS

The two so-called "conceptual errors" in the MCFT are claimed to occur in the 1986 procedures for checking that the loads can be transmitted across the cracks. The simpler crack check procedures used in the 1981 and 1982 versions of the MCFT apparently are deemed to be satisfactory. To clarify the specifics of these objections, it is useful to consider a numerical example. The example chosen by Hsu (1998) to illustrate that the 1986 MCFT predicts shear strengths 32% higher than "Hsu and Zhang's accurate set" will be employed. This is an element loaded in pure shear, which contains 1.2% of longitudinal reinforcement (x direction) and 0.6% of transverse reinforcement (y direction). The yield strength of the reinforcement is 400 MPa, and the cylinder strength of the concrete is 40 MPa. The objective is to determine the relationship between the applied shear stress and the resulting shear strain.

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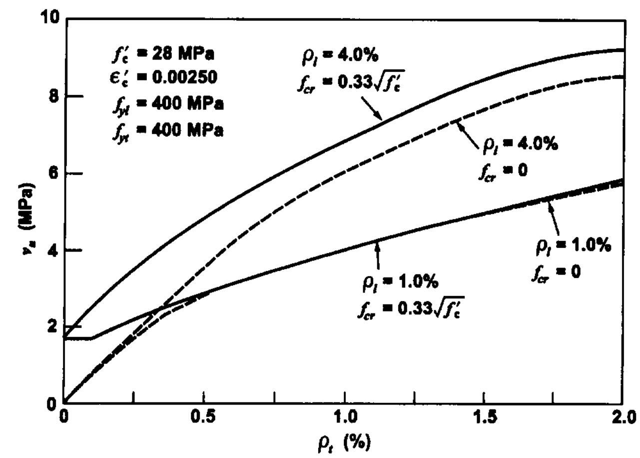

FIG. 1. Predicted Shear Strengths of Four Series of Reinforced Concrete Panels as Given by Vecchio and Collins (1982)

In the MCFT, the shear stress and shear strain associated with a chosen value of principal tensile strain, ε_1, are determined. These calculations are then repeated for a range of values of ε_1 in order to determine the complete load-deformation response of the element. Herein, a case where ε_1 equals 0.0025 is considered. For this strain, the average principal tensile stress in the concrete is given by Vecchio and Collins (1986) as

$$f_1 = \frac{0.33\sqrt{f'_c}}{1 + \sqrt{200\varepsilon_1}} = \frac{0.33\sqrt{40}}{1 + \sqrt{200 \times 0.0025}} = 1.223 \text{ MPa} \quad (1)$$

The basic assumption of both the CFT and the MCFT is "that for the diagonally cracked concrete the direction that is subjected to the largest average compressive stress will coincide with the direction that is subjected to the largest average compressive strain" (Collins 1978). This direction is determined by solving the equilibrium conditions, the compatibility conditions, and the appropriate stress-strain relationships. Using the 120-line program Shear (Collins and Mitchell 1991) and changing the 56th line of this program so that it uses the $\sqrt{200\varepsilon_1}$ recommended by Vecchio and Collins (1986) rather than the $\sqrt{500\varepsilon_1}$ recommended by Collins and Mitchell (1987), one finds that $\theta = 41.4°$ and that the average strains are $\varepsilon_x = 0.00100$; $\varepsilon_y = 0.001333$; and $\gamma = 0.00265$. The applied shear stress is then given by

$$v = f_1 \cot \theta + \rho_y f_{sy} \cot \theta$$

$$= 1.223 \cot 41.4° + 0.006 \times 0.001333 \times 200{,}000 \cot 41.4°$$

$$= 1.387 + 1.814 = 3.20 \text{ MPa} \quad (2)$$

Note that in (2), f_1 represents the average principal tensile stress in the concrete, whereas f_{sy} (=267 MPa) represents the average tensile stress in the y reinforcement. Hsu (1998) is not correct when he states that, in this equation, "the second term represents local steel stress at cracks, while . . . the first term is the average tensile stress of concrete."

Fig. 2 illustrates the equilibrium conditions for the example element when the applied shear stress is 3.20 MPa. To convert stresses to forces, the thickness of the element has been taken as 200 mm. Fig. 2(a) shows the equilibrium in terms of the average stresses. A free-body diagram has been made by cutting a corner off the element at an angle of 41.4°. The average principal tensile stress of 1.223 MPa acting on the 1,333 mm long and 200 mm thick plane will produce a tensile force of 326 kN normal to the plane. The y component (vertical component) of this force will be 245 kN. The total area of vertical reinforcement crossing the diagonal plane will be 0.006 × 1,000 × 200 = 1,200 mm^2. At an average tensile stress of 267 MPa, this reinforcement will produce a tensile force of 320 kN. The 245 kN component from the concrete plus the 320 kN from the vertical reinforcement will balance the 565 kN downwards force produced by the 3.20 MPa shear stress acting on the vertical edge of the element.

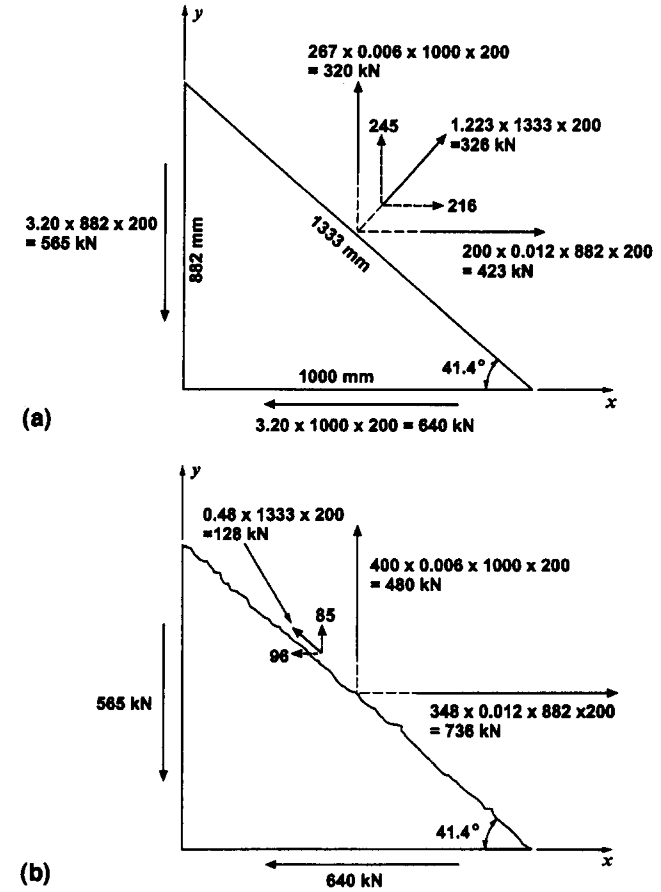

FIG. 2. Equilibrium of 200 mm Thick Example Element when v = 3.20 MPa: (a) Equilibrium in Terms of Average Stress; (b) Equilibrium in Terms of Stresses at Crack

Fig. 2(b) illustrates what happens if a crack forms at 41.4° to the x axis (horizontal axis) of the element. At the crack location, no significant tensile stress normal to the crack can be resisted by the concrete. Hence, all of the normal tensile stress must be resisted by the reinforcement. The maximum tensile force that can be resisted by the vertical reinforcement is 480 kN, when this reinforcement reaches its yield stress at the crack. This 480 kN force is not large enough to resist the 565 kN vertical force produced by the 3.20 MPa shear stress applied to the edge of the element. To hold the corner in equilibrium, a shear stress of 0.48 MPa must act on the crack surface to produce a vertical component of 85 kN. The horizontal reinforcement will need to be stressed to 348 MPa at the crack to resist the 640 kN provided by the applied shear and the 96 kN caused by the shear stress on the crack.

In the modified compression field theory, the "reasonable simplification" is made that the direction of the average principal compression strain, the direction of the average principal compressive concrete stress, and the crack direction all occur at the same angle, θ. For the example case, $\theta = 41.4°$. It can be seen that, in terms of average stresses, Fig. 2(a), there are no concrete shear stresses on this plane. However, in terms of local stresses, Fig. 2(b), equilibrium requires shear stresses on a crack that is inclined at 41.4°. Hsu (1996) is incorrect in his statement that having shear stresses on this crack "violates the basic principle of mechanics."

The requirements of equilibrium illustrated in Fig. 2(b) dictate that, at a crack, the local stress in the x reinforcement, f_{sxcr}, equals 348 MPa. This local stress in the x reinforcement cannot exceed the yield strength of this reinforcement, f_{yx}. Rather than determining the 348 MPa stress by considering the equilibrium of the corner element in the x and y directions, as was done above and in Vecchio and Collins (1986), the stress could have been found by considering the equilibrium of the corner element only in the direction normal to the diagonal crack. This simpler approach, which was used by Vecchio and Collins (1981, 1982), results in the requirement that, to avoid exceeding the yield stress of the reinforcement at a crack

$$f_1 \leq \rho_x(f_{yx} - f_{sx})\sin^2\theta + \rho_y(f_{yy} - f_{sy})\cos^2\theta$$
$$\leq 0.012(400 - 200)\sin^2 41.4° + 0.006(400 - 267)\cos^2 41.4°$$
$$\leq 1.499 \text{ MPa} \quad (3)$$

It can be seen that at the chosen value of ε_1, this limit on f_1 does not control.

Because the 1982 version of the MCFT did not check the ability of the crack surface to resist the shear stresses required by equilibrium, it was cautioned that "the model should only be applied to situations where there is sufficient reinforcement to provide adequate crack control." Otherwise, "a crack may become sufficiently large to destroy aggregate interlock action and lead to a premature shear failure" (Vecchio and Collins 1982). This concern was directly addressed in the 1986 version of the MCFT, which included procedures to calculate both the width of the diagonal cracks and the magnitude of the shear stress that could be resisted by aggregate interlock action.

If the size and spacing of the reinforcement in the example panel is such that if the element was loaded with uniaxial tension in the x direction, the spacing of the cracks, s_x, would be 200 mm; whereas if it was loaded with uniaxial tension in the y direction, the spacing of the cracks, s_y, would be 300 mm; then the spacing of diagonal cracks inclined at 41.4° to the x axis is calculated as

$$s_\theta = \frac{1}{\dfrac{\sin\theta}{s_x} + \dfrac{\cos\theta}{s_y}}$$
$$= \frac{1}{\dfrac{\sin 41.4}{200} + \dfrac{\cos 41.4}{300}} = 172 \text{ mm} \quad (4)$$

For a principal tensile strain of 0.0025, the width of these diagonal cracks is calculated as

$$w = s_\theta \varepsilon_1$$
$$= 172 \times 0.0025 = 0.43 \text{ mm} \quad (5)$$

The interface shear stress capacity, v_{ci}, of a crack 0.43 mm wide in concrete with a maximum aggregate size of 10 mm and with no compressive stress on the crack is calculated as

$$v_{ci} = \frac{0.18\sqrt{f'_c}}{0.31 + \dfrac{24w}{a + 16}}$$
$$= \frac{0.18\sqrt{40}}{0.31 + \dfrac{24 \times 0.43}{10 + 16}} = 1.61 \text{ MPa} \quad (6)$$

As this interface shear stress capacity is considerably greater than the 0.48 MPa shear stress on the crack required for equilibrium, a crack slip failure will not limit the capacity at this strain level.

If the calculations shown above are repeated for different values of ε_1, the predicted shear stress–shear strain response labeled "Vecchio and Collins 1986 according to Collins" in Fig. 3 is obtained. Also shown in Fig. 3 is Hsu's calculated response of this example panel according to Vecchio and Collins (1986). It can be seen that Hsu (1998) calculates a shear capacity for this panel that is about 24% higher than that calculated from the widely available program Shear (Collins and Mitchell 1991). The reason for this discrepancy is that Hsu neglects to apply the limits on f_1 that result from checking the local conditions at a crack. Thus, for an ε_1 value of 0.005, program Shear calculates an f_1 value of 0.26 MPa, rather than the 1.04 MPa given by (1). Angle $\theta = 38.0°$; the average strains are $\varepsilon_x = 0.00171$, $\varepsilon_y = 0.00299$, and $\gamma = 0.00514$; and the shear stress is 3.41 MPa. Eq. (3) then limits f_1 to

$$f_1 < 0.012(400 - 200{,}000 \times 0.00171)\sin^2 38.0$$
$$+ 0.006(400 - 400)\cos^2 38.0 < 0.26 \text{ MPa} \quad (7)$$

It is interesting that the parameters of the example panel chosen by Hsu (1998) are very similar to those of panel B1 tested by Pang and Hsu (1992). This element was loaded in pure shear and contained 1.19% of longitudinal reinforcement and 0.60% of transverse reinforcement. The yield strength of the longitudinal reinforcement was 463 MPa, the yield strength of the transverse reinforcement was 445 MPa, and the cylinder strength of the concrete was 45 MPa. The element was 178

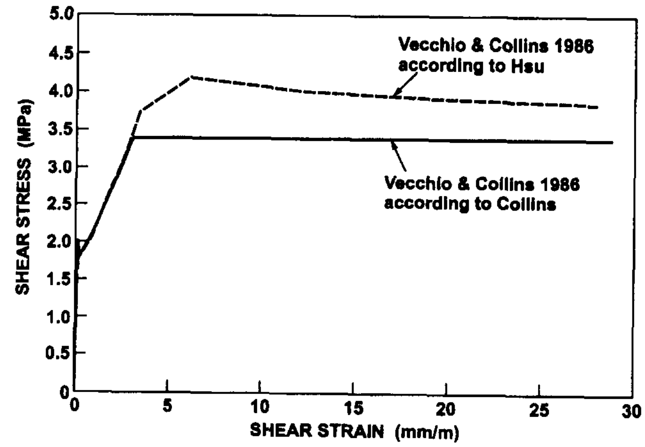

FIG. 3. Predicted Shear Stress–Shear Strain Relationships for Example Element

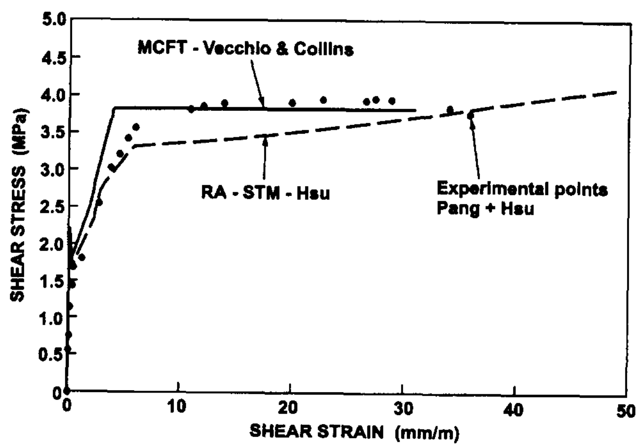

FIG. 4. Shear Stress–Shear Strain Relationships for Panel B1 Tested by Pang and Hsu

mm thick. The observed shear stress–shear strain relationship for the element is compared with the load-deformation relationships calculated from the MCFT and the RA-STM in Fig. 4. From this figure, it would seem that the behavior of this specimen is at least as well calculated by the MCFT as by the RA-STM.

CONCLUDING REMARKS

The two so-called "conceptual errors" that Hsu (1996, 1998; also Pang and Hsu 1996) claims are in the modified compression field theory are actually conceptual errors about the modified compression field theory. As noted by Vecchio and Collins (1986): "In this theoretical model cracked concrete is treated as a new material with its own stress-strain characteristics. Equilibrium, compatibility, and constitutive relationships are formulated in terms of average stresses and average strains. . . . Consideration is also given to local stress conditions at crack locations." The values of the average reinforcement stresses and the average concrete stresses differ from the local values at crack locations. For the average stresses, the angle θ defines the principal stress direction, but, in general, this will not be the principal stress direction for the local stresses. Equilibrium can require that a crack forming at this angle is subjected to shear stress.

The difference than can occur between the average stress–average strain response of a reinforcing bar embedded in concrete and the measured local stress–local strain response of the same bar tested in air are discussed by Collins (1978) and by Vecchio and Collins (1982, 1986). It was concluded that it is reasonable to use the "simplifying approximation" that these two relationships are the same, provided that the ability of the reinforcement to transmit the loads across the cracks is checked. The fact that Hsu neglects this check when using the modified compression field theory has caused him to question the validity of this simplifying assumption.

All theoretical models contain simplifying assumptions, with the test of the theory being that it should be simple enough to be used, but complex enough to capture what happens in reality. As has been said many years ago, "Nothing is more practical than a simple theory." The modified compression field theory makes a number of simplifying assumptions about the complex behavior of cracked reinforced concrete elements subjected to shear. Since being finalized in 1987, this set of assumptions has proved capable of predicting many aspects of observed shear behavior with reasonable accuracy. The results shown in Fig. 4 are typical. Though all theories can and should be improved over time, the cost-benefit ratios of any improvements need to be carefully assessed. In this regard, the softened truss model is more limited in scope than the modified compression field theory, is considerably more complex, and does not seem to offer any improvements in accuracy.

APPENDIX. REFERENCES

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